Exotic pairing states in 2D Fermi gases with Rashba spin-orbit coupling

An interest in few-body physics from a many-body perspective

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Acknowledgements

- RUC
  - Peng Zhang
  - Ren Zhang

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  - Wei Yi
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Outline

• Background
  • Spin-orbit coupling
  • Realization of spin-orbit coupling (1D, ERD)
  • Single particle property

• 2D Fermi gas with Rashba SOC & B-fields
  • Topological superfluid
  • FFLO pairing within a single branch
  • Topological FFLO state

• Few-body physics from a many-body perspective
  • Quasi-2D versus 2D: When and how the 3rd dimension matters?
  • Validity of a contact interaction
Spin-orbit coupling (SOC)

- SOC is well observed in nature

- SOC in continuous system

\[ H_{SO} = \alpha_x \sigma_x k_x + \alpha_y \sigma_y k_y + \alpha_z \sigma_z k_z \]
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Realization of ERD SOC: NIST-scheme

Lin et. al., Nature 471, 83 (2011)

\[ H = \begin{pmatrix} \frac{k_x^2}{2m} + \frac{\hbar}{2} & \frac{\Omega}{2} e^{i2k_0 x} \\ \frac{\Omega}{2} e^{-i2k_0 x} & \frac{k_x^2}{2m} - \frac{\hbar}{2} \end{pmatrix} \begin{pmatrix} \uparrow \\ \downarrow \end{pmatrix} \]

\[ U = \begin{pmatrix} e^{-ik_0 x} & 0 \\ 0 & e^{ik_0 x} \end{pmatrix} \]
Realization of ERD SOC: NIST-scheme

\[ H_{SO} = U H U^\dagger = \left( \frac{(k_x+k_0)^2}{2m} + \frac{\hbar}{2} \right) \left( \frac{\Omega}{2} \right)^2 - \frac{\hbar}{2} \left( \frac{\Omega}{2} \right)^2 \]

\[ = \frac{1}{2m} (k_x + k_0 \sigma_z)^2 + \frac{\Omega}{2} \sigma_x + \frac{\hbar}{2} \sigma_z. \]

- Spin rotation, x to –z, z to x

\[ H_{SO} = \frac{1}{2m} \left( k_x^2 + k_0^2 + 2\sigma_x k_x \right) - \frac{\Omega}{2} \sigma_z + \frac{\hbar}{2} \sigma_x \]

1D (ERD) SOC

x-field

2-photon detuning

z-field

Rabi Frequency
Single-particle dispersion

- w/o SOC
- w/ SOC
- w/ SOC + z-field
- w/ SOC + z-field + x-field
Implementation in Fermi gases

Shanxi University, Taiyuan, China
Wang et al., PRL 109, 095301 (2012)

MIT, Boston, USA
Cheuk et al., PRL 109, 095302 (2012)
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  • Quasi-2D versus 2D: When and how the 3rd dimension matters?
  • Validity of a contact interaction
Single-particle dispersion

- w/o SOC w/o z-field
- w/ SOC w/o z-field
- w/ SOC w/ z-field

Inversion Sym.
Time Rev. Sym.
Lifshitz transition

- 2D Fermi gas with Rashba SOC + z-field

[Diagrams showing Fermi surfaces and chemical potential]
Now we add in the interaction…

- w/o SOC w/o z-field
- w/o SOC w/ z-field
- BCS pairing
- FFLO pairing
Now we add in the interaction…

w/ SOC + z-field

Single-branch BCS pairing

Single-branch BCS pairing v.s. inter-branch FFLO pairing
Topological Superfluid (TSF)

- Fully gapped in the bulk
- Topologically non-trivial properties
  - Not characterized by local order parameters
  - Topologically protected gapless edge modes
- Majorana fermions at vortex cores
  - Fault-tolerant quantum computing
- 2D Fermi system which breaks inversion and time-reversal symmetries

Sau et. al., PRL 104, 040502 (2010)
Single-particle dispersion

w/o SOC

w/ SOC

w/ z-field

Inversion Sym.

Time Rev. Sym.

Inversion Sym.

Time Rev. Sym.
2D Fermi gas with Rashba SOC & z-field

- Hamiltonian

\[
H - \sum_{\sigma} \mu_\sigma N_\sigma = H_0 + H_{soc} + H_{int} \\
= \sum_{\mathbf{k},\sigma} (\epsilon_{\mathbf{k}} - \mu_\sigma) a_{\mathbf{k},\sigma}^\dagger a_{\mathbf{k},\sigma} + \sum_{\mathbf{k}} \alpha_{\mathbf{k}} \left( e^{-i\varphi_{\mathbf{k}}} a_{\mathbf{k},\uparrow}^\dagger a_{\mathbf{k},\downarrow} + \text{H.C.} \right) \\
+ \frac{U}{V} \sum_{\mathbf{k},\mathbf{k}'\uparrow,\downarrow} a_{\mathbf{k},\uparrow}^\dagger a_{-\mathbf{k}',\downarrow}^\dagger a_{-\mathbf{k}',\downarrow} a_{\mathbf{k},\uparrow} \tag{1}
\]

- Thermodynamic potential

\[
\Omega = -\frac{1}{\beta} \ln \text{tr} \left[ e^{-\beta (H_m - \sum_\sigma \mu_\sigma N_\sigma)} \right] \bigg|_{T \to 0} \\
= \frac{1}{2} \sum_{\mathbf{k},\lambda = \pm} (\xi_\lambda - E_{\mathbf{k},\lambda}) - V \frac{|\Delta|^2}{U}.
\]
Zero-T phase diagram

Zhou, WZ, and Yi, PRA 84 063603 (2011)
BCS-BEC crossover

Zhou, WZ, and Yi, PRA 84 063603 (2011)
Zero-T phase diagram

Zhou, WZ, and Yi, PRA 84 063603 (2011)
Zero-T phase diagram

- Pairing physics at large magnetic field limit
- Polaron-molecule transition
- Normal gas in the large $h$ limit

Yi and WZ, PRL 109, 140402 (2012)
FFLO pairing

- BCS pairing becomes unstable
- SOC-induced spin mixing
- In-plane field induced Fermi surface asymmetry
Competition of FFLO states

FFLOx

FFLOy
Topological FF state

w/ SOC + z-field + x-field

w/ SOC + z-field

WZ and Yi, Nat. Comm. 4, 2711 (2013)
Detection of tFF

- Momentum distribution of minority fermions

WZ and Yi, Nat. Comm. 4, 2711, (2013)
Summary

- 2D Fermi gas with Rashba SOC
  + z-field
  + z-field + x-field

- Topological superfluid
- Pairing instability at large z-field limit
- Breakdown of BCS pairing
- Topological FFLO state

Zhou, WZ, Yi, PRA 84 063603 (2011)
Yi and WZ, PRL 109, 140402 (2012)
WZ and Yi, Nat. Comm. 4, 2711, (2013)
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A typical 2D system in cold atoms

- Trapping potential along $z \sim 10^4 - 10^5$ Hz
- Trap size $\sim 10^{-7}$ m
- Particle separation in the radial plane $\sim 10^{-6}$ m
- Interatomic potential range $\sim 10^{-9}$ m
Two-body bound state (Q2D)

- Two-channel model

\[ H = H_0 + H_{\text{soc}} + H_{\text{bf}} + H_{\text{int}}. \]

\[ |\Psi\rangle_{\ell,q} = \left( \beta_{\ell,q} b_{\ell,q}^\dagger \right. \]

\[ + \sum_{m,n,k} \sum_{\sigma,\sigma'} \eta_{\sigma\sigma'}^{\sigma\sigma'} c_{m,k,q}^{\dagger} c_{n,-k+q/2,\sigma'}^{\dagger} \left|0\rightrangle \]
Two-body bound state (Q2D)

- $q=0$

Zhang, Wu, Tang, Guo, Yi, WZ, PRA 87, 033629 (2013)
Excitations along the z-direction

\[ |\Psi\rangle_{\ell,q} = \left( \beta_{\ell,q} b^\dagger_{\ell,q} + \sum_{m,n,k} \eta_{m,n,k,q} c^\dagger_{m,k+q/2,\sigma} c^\dagger_{n,-k+q/2,\sigma} \right) |0\rangle \]
• When a Q2D Fermi gas can be looked as 2D?
  • Only on the BCS side.

• What if we introduce Rashba SOC?
  • It makes things worse.
  • SOC tends to increase the two-body binding energy.

• Can we still simulate 2D physics with Q2D system?
  • Yes, but need another way.
Separation of energy scales

• DOF in Q2D model:
  - fermions in ground state $n=0$
  - fermions in excited states $n=1,2,3…$
  - Feshbach molecules

• Effective 2D Hamiltonian (2-channel model)
  - 2D Fermions
  - dressed molecules (structureless bosons)
Effective 2D Hamiltonian

\[ H_{\text{eff}} = \sum_{k,\sigma} \varepsilon_k a_{k,\sigma}^\dagger a_{k,\sigma} + \delta_B d_0^\dagger d_0 + \frac{a_b}{L} \sum_k \left( d_0^\dagger a_{k,\uparrow} a_{-k,\downarrow} + \text{H.C.} \right) \]
\[ + \frac{V_b}{L^2} \sum_{k,k'} a_{k,\uparrow}^\dagger a_{-k,\downarrow}^\dagger a_{-k',\downarrow} a_{k',\uparrow} + \gamma' \sum_k \left[ (k_x - ik_y) a_{k,\uparrow}^\dagger a_{k,\downarrow} + (k_x + ik_y) a_{k,\downarrow}^\dagger a_{k,\uparrow} \right] \]

- Single-particle physics
  - Single particle dispersion
- Two-particle physics
  - background scattering
  - two-body binding energy
  - # of fermions in ground state

\[ \Delta \varepsilon = \mathcal{O} \left( \frac{\mu - E_b / 2}{\hbar \omega_z} \right)^2 \]
Q2D Fermi gas with Rashba SOC

![Graph showing Fermi gas with Rashba SOC](image)
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Two-body scattering state (2D)

\[ H^{(2D)} = H_0^{(2D)} + V_{2D}(\rho), \]

\[ \perp\langle \rho | \psi_0^{(0)} \rangle = \frac{e^{i\mathbf{k} \cdot \rho}}{2^{3/2}/2\pi} |\alpha(q,k)\rangle_S - \frac{e^{-i\mathbf{k} \cdot \rho}}{2^{3/2}/2\pi} |\bar{\alpha}(q,-k)\rangle_S. \]

\[ \perp\langle \rho | \psi_0^{(+)} \rangle \approx \perp\langle \rho | \psi_0^{(0)} \rangle + A(c) \perp\langle \rho | g(\varepsilon_c) |0\rangle |0,0\rangle_S, \]

\[ f^{(2D)}(c' \leftarrow c) = -2\pi^2 \langle \psi_0^{(0)} | 0 \rangle \perp |0,0\rangle_S A(c). \]
Two-body scattering state (2D)

\[ A(c) = \frac{(2\pi)S\langle 00|_\perp\langle 0|\psi_c^{(+)}\rangle}{i\pi/2 - C - \ln \left( d\sqrt{\varepsilon_c/2} \right) - (2\pi)\lambda(\varepsilon_c, q)}. \]

- Scattering amplitude is \( q \)-dependent
- Qualitative change of behavior at low-energy limit

**w/o SOC**

\[ \lim_{\varepsilon \to 0} f_0^{(2D)} \propto \frac{1}{\ln \varepsilon_c}. \]

**w/ SOC**

\[ \lim_{\varepsilon_c \to \varepsilon_{\text{thre}}(q)} f^{(2D)} \propto \sqrt{\varepsilon_c - \varepsilon_{\text{thre}}(q)}. \]
Two-body scattering state (2D)

\[ F \equiv \frac{f^{(2D)}(c' \leftarrow c)}{\langle \psi^{(0)}_{c'} | 0 \rangle \perp \langle 0 | 0 \rangle \ S \langle 0 | \perp \langle 0 | \psi^{(+)}_{c} \rangle} \]

Zhang, Zhang, WZ, PRA 86 042707 (2012)
Two-body scattering state (Q2D)

\[ H = H_0^{(2D)} + H_z + V_{3D}(r). \]

\[ H_z = -\frac{\partial^2}{\partial z^2} + \frac{\omega^2 z^2}{4} - \frac{\omega}{2} \]

\[ A_{\text{eff}}(c) = \frac{(2\pi)_S \langle 00|_\perp \langle 0|\psi_c^{(+)})}{i\pi/2 - C - \ln \{d_{\text{eff}}(\varepsilon_c, q) \sqrt{\varepsilon_c/2}\} - (2\pi)\lambda(\varepsilon_c, q)}. \]

\[ A(c) = \frac{(2\pi)_S \langle 00|_\perp \langle 0|\psi_c^{(+)})}{i\pi/2 - C - \ln (d\sqrt{\varepsilon_c/2}) - (2\pi)\lambda(\varepsilon_c, q)}. \]
Effective Hamiltonian

- Around threshold

\[
\hat{V}_o = \frac{1}{S} \sum_{k,k',k''} \Gamma g(k+k') a_{k,\uparrow} a_{k',\uparrow} a_{k''+k',\downarrow} a_{k'-k'',\downarrow}, \uparrow.
\]

\[
\ln \kappa_c - \frac{1}{2\pi g(q)} = -C - \ln \left[ \frac{d_{\text{eff}} \left( -\frac{\xi^2}{4}, q \right)}{2} \right]
\]

- \(q\)-dependent
Effective Hamiltonian

- Around 2-body binding energy

\[
\hat{V}_b = \hat{V}_o + \sum_q \left( \frac{q^2}{4} + v(q) \right) b_q^\dagger b_q \\
+ \frac{1}{\sqrt{S}} \sum_{k,k'} \left[ \begin{array}{c} \nu(k+k') a_{k,\uparrow}^\dagger a_{k',\downarrow}^\dagger b_{k+k'} + h.c. \end{array} \right]
\]
A contact interaction is valid provided that...

- The interaction is short-range
- The SOC intensity is weak
- The effective contact interaction strength is $q$-dependent