Universal Aspects of Dipolar Scattering
Christopher Ticknor

- May 15 at INT
Outline of Talk

- Universal Dipolar Scattering
  - Theory of a long range scattering potential
- 2D Universal and tilted dipolar scattering
  - Interplay of interaction and geometry
- Universal 3-body dipolar recombination (not included)
  \[ L_3 \propto D^4, a^2 D^2 \]
Ultracold Scattering

- Feshbach resonances
- s-wave
- p-wave \[ \sigma = \frac{4\pi}{k^2} \sin^2(\delta) \]

Regal et al. PRL 90 053201 (2003)
Ultracold Scattering

- Threshold laws for partial waves
- $s$-wave, energy independent
- $p$-wave, suppressed

\[ \delta \propto \langle \psi | \frac{1}{r^s} | \psi \rangle \]
\[ \delta \sim \alpha k^{2l+1} + \beta k^{s-2} \]
\[ \sigma_l \propto E^p \]
\[ p = \min(2l, s-3) \]
Ultracold Scattering

- But dipoles are different
- Long range
- Anisotropic
- Strong

\[ V_{dd} = \frac{\vec{d} \cdot \vec{d} - 3 (\hat{R} \cdot \vec{d})^2}{R^3} \]

\[ V_{dd} \sim \langle d \rangle^2 \frac{1 - 3 \cos^2(\theta)}{R^3} \]

\[ s = 3 \]

\[ \sigma_l \propto E^0 \]
Dipolar Scattering

- The system can be magnetic or electric.
- Dipolar length scale
  \[ D = \langle d \rangle^2 \frac{m_r}{\hbar^2} \]
- Energy scale
  \[ E_D = \frac{\langle d \rangle^2}{D^3} = \frac{\hbar^6}{\langle d \rangle^4 m_r^3} \]
- Strong interactions, many exotic theories.
Ultracold Scattering

- Long Range

Graph showing the relationship between energy and $R/D$ with a curve for $1/R^3$ and another for a full range.
Dipolar Scattering

- Equation of Motion

\[
\left(-\frac{\nabla^2}{2m_r} + \langle \mu \rangle^2 \frac{1 - 3\cos^2(\theta)}{R^3}\right)\psi = E \psi
\]

\[
y = \frac{R}{D}
\]

\[
R \psi = \sum_l Y_{lm} F_l(R)
\]
Dipolar Scattering

- Equation of Motion → Universal!

- Except for short range \( y_0 = R_0 / D \ll 1 \)

\[
\left(-\frac{d^2}{2dy^2} + \frac{l(l+1)}{2 y^2}\right)F_l + \sum_{l'} \frac{C_{ll'}}{y^3} F_{l'} = \frac{E}{E_D} F_l
\]

\[
C_{ll'} = \langle lm | 1 - 3\cos^2(\theta) | l' m \rangle
\]
Dipolar Scattering

- Semi-classical solution from Eikonal approximation

\[ \sigma \leftarrow \int_{\text{line}} V_{dd} (z, \vec{b}, \vec{\mu}) \, dz \]

\[ \frac{E}{E_D} \gg 1 \]

\[ \sigma = \frac{8}{\pi} \frac{D}{k} \]

Bohn, Cavagnero, and Ticknor NJP 11 055039 (2009)

Ticknor PRL 100, 133202 (2008)
Dipolar Scattering

- **Threshold behavior**
  \[
  \frac{E}{E_D} \ll 1
  \]

- **Born Approximation**: all dipolar coupled partial waves have an energy independent cross section!

  \[
  \sigma_{ll'} \propto |\langle F_{l} | 1/R^3 | F_{l'} \rangle \langle lm | 1 - 3\cos^2(\theta) | l' m \rangle|^2
  \]

  \[
  \sigma_e = 1.117 \, D^2
  \]

  \[
  \sigma_o = 3.351 \, D^2
  \]

  Bohn, Cavagnero, and Ticknor NJP 11 055039 (2009)
Dipolar Scattering

- Experimental observation from Innsbruck.
  - Threshold universality

\[ D = 100 \, a_0 \]
\[ E_D = 200 \, \mu K \]

From K. Aikawa et al. PRL 112 010404 (2013)
2D dipolar scattering with a tilt
Christopher Ticknor

- May 15 at INT
2D dipolar scattering

- Long range
- Anisotropic
- Strong

\[ V_{dd} = \frac{\hat{d} \cdot \hat{d} - 3 (\hat{R} \cdot \hat{d})^2}{R^3} \]

\[ R \rightarrow \rho \]
\[ \hat{d} \rightarrow \cos(\alpha) \hat{z} + \sin(\alpha) \hat{x} \]

\[ V_{dd} = \langle d \rangle^2 \frac{1 - 3 \cos(\theta)^2 \sin^2(\alpha)}{\rho^3} \]
2D Universal Dipolar Scattering

\[ \hat{d} \cdot \hat{\rho} = 0 \quad \tilde{\rho} = \rho / D \]

\[
\left( -\frac{d^2}{2d \tilde{\rho}^2} + \frac{m^2 - 1/4}{2 \tilde{\rho}^2} + \frac{1}{\tilde{\rho}^3} \right) \Phi_m = \frac{E}{E_D} \Phi_m
\]

- Universal
  - Repulsive diagonal potential for \( m=0 \)
  - Diagonal in partial wave
  - Short range not really important
2D Universal Dipolar Scattering

Semi-classical regime

\[ \frac{E}{E_D} \gg 1 \]

\[ \sigma_{sc} = \frac{4}{k} \sqrt{\pi Dk} \]

Ticknor, PRA 80 052702 (2009)
2D Universal Dipolar Scattering

Threshold regime

\[
\frac{E}{E_D} \ll 1
\]

\[
m = 0 \quad a/D = e^{2\gamma + \ln 2} \approx 6.344
\]

\[
m^2 > 0 \quad \sigma_m = \frac{4}{k} \frac{(Dk)^2}{(m^2 - 1/4)^2}
\]

Ticknor, PRA 80 052702 (2009)

\[
\sigma_m \propto k^{4m-1}
\]
Interplay of geometry and interaction

\[ \hat{d} \cdot \hat{\rho} \neq 0 \]

\[ V_{dd} = \langle d \rangle^2 \frac{1 - 3\cos^2(\theta) \sin^2(\alpha)}{\rho^3} \]
Breakdown of superfluidity in a BEC

An amazing BEC experiment.

Observation of Vortex Dipoles in an Oblate Bose-Einstein Condensate


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Interplay of geometry and interaction

- 2D dipolar gas with tilted polarization
- Anisotropic superfluid character!

Ticknor et al. PRL 106 065301
Interplay of geometry and interaction

\[ V_{dd} = \frac{1 - 3\cos^2(\theta)\sin^2(\alpha)}{\rho^3} \rightarrow \frac{1 - 3\cos^2(\theta)}{\rho^3} \]

\[ \alpha = \pi / 2 \]
Interplay of geometry and interaction

\[ V_{dd} \rightarrow \frac{1 - 1.5 \cos^2(\theta)}{\rho^3} \]

\[ \alpha = \frac{\pi}{4} \]
Interplay of geometry and interaction

Binding Energies as function of $a/D$ and $\alpha$

\[ \alpha = \frac{\pi}{4} \rightarrow \]

\[ \alpha = \frac{\pi}{2} \rightarrow \]

\[ \frac{1}{a^2} \]
Interplay of geometry and interaction

\[ f (\theta_i, \theta_f) = \frac{e^{i \pi/4}}{\sqrt{2\pi k}} \sum_{mm'} e^{-im\theta_i} T_{mm'} e^{im'\theta_f} \]

\[ \sigma (\theta_i) = \int d\theta_f |f (\theta_i, \theta_f)|^2 \]

\[ \sigma = \frac{1}{k} \sum_{mm'} |T_{mm'}|^2 \]
Interplay of geometry and interaction

Fermions with tilted polarization

\[ \alpha/\pi = 0.315 \]

\[ \cos(m \theta) \]

\[ \sin(m \theta) \]
Interplay of geometry and interaction

\[ \sigma(\theta_i) \]

\[ \alpha/\pi = 0.315 \]
Conclusions

- Universal dipolar scattering in 3D.
  - Many partial waves always contribute.
- Dipolar scattering in 2D is universal.
- If polarization is tilted then:
  - Tune anisotropy of system.

- Money from LANL and LDRD.
Interplay of geometry and interaction

\[ |f(\theta_i, \theta_f)|^2 \]
Interplay of geometry and interaction

\[ |f(\theta_i, \theta_f)|^2 \]

\[ \alpha / \pi = 0.315 \]
3-body dipolar recombination

- Studied formation of 2 body molecule from 3 body dipolar system.

Ticknor and Rittenhouse, PRL 105 013201 (2010)
3-body dipolar recombination

- a/D
- bound states
- Shape

Also see: Wang and Greene PRA 85 022704 (2012)
3-body dipolar recombination

- Example wave functions.
3-body dipolar recombination

- Now use Fermi Golden Rule (FGR)

\[
\langle 3\text{-body plane wave}|V_{dd}|\text{molecule + third} \rangle
\]

\[
V_{dd} \propto \sum_{k \ m_1 m_2} \frac{R_k^k}{R^{3+k}} \left[ Y_{k,m_1} \otimes Y_{k+2,m_2} \right]_{20}
\]
3-body dipolar recombination

\[ a^4 \]

\[ a^2 D^2 \]

\[ D^4 \]
Interplay of geometry and interaction

$\alpha/\pi = 0.25$

$\alpha/\pi = 0.35$

$\alpha/\pi = 0.5$

$\alpha/\pi = 0.5$

$\alpha/\pi = 0.5$

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$\alpha/\pi = 0.5$

$\alpha/\pi = 0.5$
Interplay of geometry and interaction

Fermions in 2D with tilted polarization

\[ e^{\pm i m\theta} \]

\[ \alpha/\pi = 0.315 \]

\[ \rho_0/D \]

\[ k\sigma \]
Interplay of geometry and interaction

\[ \hat{d} \cdot \hat{\rho} \neq 0 \]

\[ \langle m | 1 - 3 (\hat{d} \cdot \hat{\rho})^2 | m' \rangle \]

\[ = \left( 1 - \frac{3}{2} \sin^2(\alpha) \right) \delta_{mm'} - \frac{3}{4} \sin^2(\alpha) \delta_{mm' \pm 2} \]
Interplay of geometry and interaction

\[ \langle m | 1 - 3 ( \hat{d} \cdot \hat{r} )^2 | m' \rangle \]

\[
\begin{align*}
\cos(m \phi) &= \left( 1 - \frac{9}{4} \sin^2(\alpha) \right) \delta_{mm'}, \\
\sin(m \phi) &= \left( 1 - \frac{3}{4} \sin^2(\alpha) \right) \delta_{mm'}, \\
- \frac{3}{4} \sin^2(\alpha) \delta_{mm' \pm 2} &= - \frac{3}{4} \sin^2(\alpha) \delta_{mm' \pm 2}
\end{align*}
\]
Dipolar Scattering

- Semiclassical Universality first predicted numerically
- True of Fermions and Bosons and distinguishable particles.

Ticknor PRL 100, 133202 (2008)
Interplay of geometry and interaction

\[ k \sigma_{sc} = 4 \sqrt{\pi Dk} \]
Interplay of geometry and interaction

\[ \hat{d} \cdot \hat{\rho} \neq 0 \]

\[
k \sigma_{m \rightarrow m'} = \frac{4(Dk)^2}{(m^2 - 1/4)^2} \left( 1 - \frac{3}{4} \sin(\alpha)^2 \right)^2
\]

\[
k \sigma_{m' \rightarrow m+2} = \frac{4(Dk)^2}{(m - 1/2)(m+3/2)} \left( \frac{3}{4} \sin(\alpha)^2 \right)^2
\]

\[
k \sigma_{\pm 1 \rightarrow \pm 1} = \frac{4(Dk)^2}{(m^2 - 1/4)^2} \left( \frac{3}{4} \sin(\alpha)^2 \right)^2
\]