INT Workshop 2014

May 13, 2014

Geometric scaling of three-body collision resonances for a $^6\text{Li} - ^{133}\text{Cs}$ mixture in the Efimov scenario

Funding:

Colin V. Parker
University of Chicago
WEAKLY-BOUND STATES OF THREE RESONANTLY-INTERACTING PARTICLES

V. N. EFIMOV

A. F. Ioffe Physico-technical Institute, USSR Academy of Sciences

Submitted February 16, 1970
Yad. Fiz. 12, 1080–1091 (November, 1970)

It is shown that if the pair forces of three identical particles are sufficiently resonant, a family of bound states of low energy is produced. The quantum numbers of all the states are the same: for spinless bosons $0^+$ and for nucleons $1/2^+$, $T = 1/2$. The dimension of the states is larger than the radius of the pair forces. The most favorable conditions for the appearance of a family of levels occur for three spinless neutral bosons: the conditions are less favorable for charged particles and particles with spin and isospin. The possibility of existence of such levels in a system of three particles (in the $^{12}$C nucleus) and of three nucleons ($^3$H) is considered.

Hyperspherical equation:

$$i\hbar \frac{\partial \phi}{\partial t} = -\frac{\hbar^2}{m} \frac{\partial^2 \phi}{\partial R^2} - \frac{\hbar^2}{m} \frac{s_0^2 + 1/4}{R^2} \phi$$

⇒ Scale invariant under dilation $R \rightarrow \lambda R$ and $t \rightarrow \lambda^2 t$

$\lambda = \exp(\pi/s_0) = 22.7$ for 3 identical bosons

Vitaly Efimov
Recombination of Three Atoms in the Ultracold Limit

B. D. Esry

Institute for Theoretical Atomic and Molecular Physics, Harvard-Smithsonian Center for Astrophysics, Cambridge, Massachusetts 02138

Chris H. Greene and James P. Burke, Jr.
Department of Physics and JILA, University of Colorado, Boulder, Colorado 80309-0440
(Received 19 May 1999)
Observation of Efimov Resonance in Cesium

Also found in Li6 (Penn State, Heidelberg), Li7 (Rice, Bar-Ilan), K39 (LENS), Rb85 (JILA)
Efimov state structure near a Feshbach resonance
Excited Efimov state in 3-component Fermi gas

O’Hara Group (Penn State): Williams et al., PRL 103, 130404 (2009)
Second Efimov resonance in Cs!

Length scale ratio = 21.1(1.4)
Theory = 22.7

Grimm group (Innsbruck Univ.) B. Huang et al., arXive: 1402.6161
Second Efimov resonance in $^7\text{Li}$?

Hulet group (Rice University): Science 326, 1683 (2009)
Second Efimov resonance in $^7$Li?

Hulet group (Rice University): Science 326, 1683 (2009)
Homonuclear vs. Heteronuclear Trimers

Dynamic range of measurement

$\lambda$: Determined by
(1) Mass ratio of constituent atoms
(2) Number of resonant pair interactions
(3) Symmetry of particle statistics
Mass Dependence

(MIT)  
$^{23}\text{Na}^{23}\text{Na}^{40}\text{K}$

(JILA)  
$^{87}\text{Rb}^{87}\text{Rb}^{40}\text{K}$

(U. Chicago)  
$^{133}\text{Cs}^{133}\text{Cs}^{6}\text{Li}$

Geometric scaling constant

Homonuclear case

Mass ratio ($m_2/m_1$)
Efimov states in atomic mixtures

2 heavy Bosons + 1 light atom

| B-F         | \(e^{\pi l_{s0}}\) | \(|a_{min}|\) | \(E_{max}(nK)\) | \(|a_{min}|\) | \(E_{max}(nK)\) |
|-------------|---------------------|----------------|------------------|----------------|------------------|
| \(^{133}\text{Cs}-^6\text{Li}\) | 4.877               | \(3 \times 10^3\) | 1500             | \(2 \times 10^4\) | 60.0             |
| \(^{87}\text{Rb}-^6\text{Li}\) | 6.856               | \(8 \times 10^3\) | 230              | \(6 \times 10^4\) | 5.00             |
| \(^{23}\text{Na}-^6\text{Li}\) | 36.28               | \(9 \times 10^5\) | \(\leqslant 0.1\) | \(3 \times 10^7\) | \(\leqslant 0.1\) |
| \(^7\text{Li}-^6\text{Li}\)     | \(> 10^2\)          | \(\geqslant 10^8\) | \(\leqslant 0.1\) | \(\geqslant 10^8\) | \(\leqslant 0.1\) |
| \(^{133}\text{Cs}-^{40}\text{K}\) | 47.02               | \(2 \times 10^6\) | \(\leqslant 0.1\) | \(9 \times 10^7\) | \(\leqslant 0.1\) |
| \(^{87}\text{Rb}-^{40}\text{K}\) | \(> 10^2\)          | \(\geqslant 10^8\) | \(\leqslant 0.1\) | \(\geqslant 10^8\) | \(\leqslant 0.1\) |
| \(^{23}\text{Na}-^{40}\text{K}\) | \(> 10^2\)          | \(\geqslant 10^8\) | \(\leqslant 0.1\) | \(\geqslant 10^8\) | \(\leqslant 0.1\) |
| \(^7\text{Li}-^{40}\text{K}\)   | \(> 10^2\)          | \(\geqslant 10^8\) | \(\leqslant 0.1\) | \(\geqslant 10^8\) | \(\leqslant 0.1\) |

A Translatable Crossed Dipole Trap

Li: $T = 3 \mu K; N_{Li} = \sim 10^5$

Cs: $T = 16 \mu K; N_{Cs} = \sim 10^5$

S. Tung et. al., PRA 87, 010702(R) (2013)
### Trap Loss of Li-a State + Cs-a State Mixture

<table>
<thead>
<tr>
<th>Spin state</th>
<th>Experiment</th>
<th>Theory</th>
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<tbody>
<tr>
<td></td>
<td>$B_0$ (G)</td>
<td>$\delta$ (G)</td>
</tr>
<tr>
<td>$</td>
<td>\text{Li}:a\rangle +$</td>
<td>843.4(2)</td>
</tr>
<tr>
<td>$</td>
<td>\text{Cs}:a\rangle$</td>
<td>892.9(2)</td>
</tr>
<tr>
<td>$</td>
<td>\text{Li}:b\rangle +$</td>
<td>816.1(2)</td>
</tr>
<tr>
<td>$</td>
<td>\text{Cs}:a\rangle$</td>
<td>889.0(2)</td>
</tr>
<tr>
<td>$</td>
<td>\text{Cs}:a\rangle$</td>
<td>943.4(2)</td>
</tr>
</tbody>
</table>

Tung et al., PRA 87 010702 (2013)
Repp et al., PRA 87 010701 (2013)
Experiment Procedures

1. Loading Li MOT
2. Translate the Li cloud to the hiding position
3. Loading Cs MOT
4. Loading Cs atoms into dipole trap 2
5. Merge two traps

Translating Cs atoms to the final position 2
Dual Resonant Absorption Imaging

The time of flight method

Resonant absorption imaging

y position

\[
t = t_0
\]

Probe atoms after expansion

Switch-off trapping potential

Li imaging

Cs imaging

\[
t = 0
\]

Atoms in trap

Li atoms (in-situ)

30 x 10^3

Cs atoms (TOF)

15 ms

30 x 10^3
Trap loss measurement: $Cs+Cs+Cs$ vs. $Li+Cs+Cs$

5G away (-300 $a_0$)

Feshbach resonance
Trap loss measurement: Field scans
Temperature dependence?

\[ <T> = 405 \text{ nK} \]

209 averages

\[ <T> = 250 \text{ nK} \]

328 averages
Result

- Three Efimov resonances:
  - First resonance: $+5.6(2)\,\text{G}$, $a_1 = -337(9)\,\text{Bohr}$
  - Second resonance: $+1.07(2)\,\text{G}$, $a_2 = -1650(30)\,\text{Bohr}$
  - Third resonance: $+0.22(4)\,\text{G}$, $a_3 = -7900(1400)\,\text{Bohr}$
  - Feshbach: $842.75(1)\,\text{G}$

- Scaling ratio: $a_1 : a_2 = 1 : 4.90(16)$
  $a_2 : a_3 = 1 : 4.79(87)$
  Weighted ratio: $4.85(44)$

Systematics (Thanks to R. Grimm and C. Salomon)

- Finite temperature shift: $<8\%$ in $0 \sim 1\mu\text{K}$  (Y. Wang)
- Feshbach resonance position: $<10\,\text{mG}$  (Y. Wang)
- Finite size: $E_3 = 500\text{nK}$, Cs trap freq. = 4 nK, LiCs freq. = 4.5 nK
Trap independent coefficients (a best guess)

\[
\frac{dN_{Li}}{dt} = -K_3 X(T)N_{Cs}^2 N_{Li} - AN_{Li} - BN_{Li}e^{-t/C}
\]

\[
\frac{dN_{Cs}}{dt} = -2K_3' X(T)N_{Cs}^2 N_{Li} - DN_{Cs}^3 T^{-3}
\]

\[
\frac{dT_{Cs}}{dt} = -FN_{Cs}^2 T^{-3/2}
\]

\[
X(T)N_{Cs}^2 N_{Li} = \int [n_{Cs}(x, T)]^2 [n_{Li}(x, T)] \, d^3x,
\]

(a) \( B = 844.5 \text{ G} \)

(b) \( B = 844.5 \text{ G} \)

(c) \( B = 844.5 \text{ G} \)

(d) \( B = 848.7 \text{ G} \)

(e) \( B = 848.7 \text{ G} \)

(f) \( B = 848.7 \text{ G} \)
Trap independent coefficients (a best guess)
Theory:

(Yujun Wang, NIST and Kansas State)
Scattering length determination

\[
\begin{align*}
\text{FB: } & \quad 842.75(1) \text{ G} \\
\text{E3: } & \quad +0.22(4) \text{ G} \\
\text{E2: } & \quad +1.07(2) \text{ G} \\
\text{E1: } & \quad +5.60(20) \text{ G}
\end{align*}
\]

Within the resonance width, scattering length \( \sim \frac{1}{(B-B_0)} \)

\[
\begin{align*}
\frac{(E1-FB)}{(E2-FB)}: & \quad 5.2(2) \\
\frac{(E2-FB)}{(E3-FB)}: & \quad 4.9(9)
\end{align*}
\]
Scattering length determination

\( E_4: 842.75(1) \) G
\( E_3: +0.22(4) \) G
\( E_2: +1.07(2) \) G
\( E_1: +5.60(20) \) G

Within the resonance width, scattering length \( \sim 1/(B-B_0) \)

\[
\frac{(E_1-E_2)}{(E_2-E_3)}: 5.3(4) \\
\frac{(E_2-E_3)}{(E_3-E_4)}: 3.9(7)
\]
Back to Theory: (Yujun Wang, NIST and Kansas State)

\[ K_3 \propto \text{const} + \frac{1}{(B - B_0)^2 + \Delta B^2} \]
# The Crew

<table>
<thead>
<tr>
<th><strong>PI</strong></th>
<th><strong>Postdocs</strong></th>
<th><strong>Undergrads</strong></th>
</tr>
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<tbody>
<tr>
<td>Cheng Chin</td>
<td>Shih-Kuang Tung</td>
<td>Nicholas Kowalski</td>
</tr>
<tr>
<td>Eric Hazlett</td>
<td>Karina Jiménez-García</td>
<td>Dylan O.A.B. Sabulsky</td>
</tr>
</tbody>
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<table>
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<th><strong>Grads</strong></th>
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<tbody>
<tr>
<td>Logan Clark</td>
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<tr>
<td>Gustaf Downs</td>
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<tr>
<td>Lei Fang</td>
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<tr>
<td>Jacob Johansen</td>
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<tr>
<td>Li-Chung Ha</td>
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</tbody>
</table>
Quantum criticality

Science 335, 1070 (2012)

Transport

Postdoc positions available

Sakharov oscillations

Science 341, 1213 (2013)

Arbitrary potential

Nature Physics 9, 769 (2013)

Postdoc positions available
Which resonance should we choose?

Choose the one with \( a_{BB} < 0 \). J. P. D’Incao and B. D. Esry, PRL 103 083202 (2009)
Broad (62G) $^6\text{Li}$-$^{133}\text{Cs}$ Feshbach resonance

Theory: Y. Wang and P. Julienne (NIST)
S.K. Tung et al., PRA 87, 010702 (2013)
Picture of Efimov potential

\[ \psi_\pm (r) = \psi_L (r) \pm \psi_R (r) \]

\[ - \frac{\hbar^2}{2m} \dddot{E}(r) + V \dot{E}(r) = E \ddot{E}(r) \Rightarrow \frac{\hbar^2}{2m} \frac{\dddot{E}(r)}{R^2} + V_{\text{eff}} \ddot{E}(r) = 0 \]

\[ \Rightarrow V_{\text{eff}} \propto - \frac{\hbar^2}{2mR^2} \quad \text{when } R < |a| \]

\[ \approx 0 \quad \text{when } R > |a| \]
Geometric scaling of Efimov states

\[ e^{\pi/s_0} = 22.7 \]

\[ e^{2\pi/s_0} = 515 \]

\[ \psi(R) \]

\[ V_{\text{Efimov}}(R) = -\frac{(s_0^2 + \frac{1}{4})\hbar^2}{2mR^2} \quad \text{for } a = \infty \]

r : Molecular potential range