Dressed impurities in an ideal Fermi gas: an \((N+1)\)-body problem, with \(N \gg 1\)

Pietro Massignan
Introduction

Quantum Mixtures in CondMat

- Mixtures of fermionic/bosonic atoms
  
  \(^{3}\text{He}-^{4}\text{He}, \text{ultracold gases, neutron stars, Quark-Gluon Plasma, ...}\)

- Spinor gases, SU(N) invariant systems, ...

- Quantum magnets, quantum Hall systems, and spin-liquids

- Unconventional and multi-band superconductors

Despite different microscopic origins, at low energies these systems can be described by emergent many-body theories exhibiting a significant degree of universality.
Universality in Quantum Mixtures

but similar transport properties!
example: 
(shear viscosity/entropy density) close to Tc:
Quantum simulation with ultracold atoms

- chemical composition
- temperature
- interaction strength
- periodic potentials
- physical dimension
- atom-light coupling
- exotic couplings (x-wave, spin-orbit, ...)
- dynamics
- disorder
- periodic driving (shaken optical lattices, ...)
Attractive Fermi Mixtures

\[ N = N : \text{BCS-BEC crossover} \]
Population-imbalanced attractive Fermi Mixtures

SF-normal transition

Zwierlein et al., Nature 2005
Very imbalanced attractive Fermi mixtures

$N \gg N$

polarons

Schirotzek et al., PRL 2009
Repulsive Fermi Mixtures

Repulsion

repulsion vs. Fermi pressure

Stoner’s Itinerant Ferromagnetism

predicted in 1933, not yet realized...
Outline of this talk

one impurity (↓)

ideal Fermi sea at $T \ll T_F$ (↑)

s-wave

p-wave

IFM
Motivation

Understanding the properties of a single impurity in a Fermi gas provides insight on:

- phase diagram of imbalanced Fermi gases
- coherence properties of fundamental quasiparticles
- their decay mechanisms

With p-wave interactions, superfluids may be polar, chiral, topological, ...

Routes towards Itinerant Ferromagnetism?
Report on Progress

Polarons, dressed molecules and itinerant ferromagnetism in ultracold Fermi gases

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a detailed review on:
  • theoretical methods
  • experimental probes and results
  • mass imbalance
  • reduced dimensionality
  • decay processes
one impurity (↓)

ideal Fermi sea at $T \ll T_F$ (↑)

$s$-wave

$p$-wave

IFM
Many-body systems
(from Richard Mattuck's book)

Fig. 0.1 Some Many-body Systems
Quasi-Particles

Landau’s idea:  
why care about real particles?

Of importance are the excitations, which behave as quasi-particles!

A QP is a “free particle” with:
@ q. numbers (charge, spin, ...)
@ renormalized mass
@ chemical potential
@ shielded interactions
@ lifetime
The impurity problem

Switch on interactions

Repulsive polaron

Molecule-Hole

Attractive polaron

Real particle

Real horse

Quasi particle

Quasi horse

RDM
The impurity problem

At zero momentum of the impurity:

- Repulsive polaron
- Attractive polaron

K impurity in a Li Fermi gas

\[ \frac{m_\downarrow}{m_\uparrow} = \frac{40}{6} \]

\[ k_F R^* \sim 1 \]
The polaron: a dressed impurity

\[ |\psi_p\rangle = \phi_p c^\dagger_p |FS_N\rangle + \sum_{q < k_F}^{k > k_F} \phi_{pqk} c^\dagger_{p+q-k} c^\dagger_{k\uparrow} c_{q\uparrow} |FS_N\rangle \]

(and a similar variational w.f. may be written for the molecule)
Quasiparticle properties

self-energy of the impurity: $\Sigma_P(p, E) = \sum_{q < k_F} T(p + q, E + \xi_{q\uparrow})$

energies of the two polarons: $E_\pm = \Re[\Sigma_P(p, E_\pm + i0^+)]$

residues: $Z_\pm = \frac{1}{1 - \partial_\omega \Re(\Sigma_P)}$

effective masses: $\frac{m^*}{m_\downarrow} = \frac{1}{Z_\pm} \left[ 1 + \frac{\partial \Re(\Sigma_P)}{\partial(p^2/2m_\downarrow)} \right]^{-1}$

self-consistent equation for the molecule energy:

$$\sum_{k'} \frac{K_{k'pq}}{E_{kk'pq}^{(2)}} - \sum_{q'} \frac{K_{k'qp}}{E_{kp}^{(1)}} - \frac{T(p, 0)}{E_{kp}^{(1)}} \sum_{k'q'} \frac{K_{k'qp}}{E_{k'p}^{(1)}} + \frac{K_{kqp}}{T(q + p - k, \xi_{q\uparrow} - \xi_{k\uparrow})} = -\frac{T(p, 0)}{E_{kp}^{(1)}}$$
Comparison with Diagrammatic QMC

\[ \frac{E - E_b}{\varepsilon_F} \]

Polaron DiagramMC-this work
Molecule DiagramMC-this work
Chevy's ansatz
Molecular ansatz
FN-DMC
Prokof'ev-Svistunov
Prokof'ev-Svistunov

Vlietinck et al., PRB (2013)
Narrow Feshbach Resonances

Scattering amplitude: \( f = - \left[ a^{-1} + ik + R^* k^2 + \ldots \right]^{-1} \)

close to resonance: \( R^* = -\frac{r_s}{2} = \frac{1}{2m_r a_{bg} \Delta B \delta \mu} > 0 \)

(Petrov, PRL 2004)

a FR is broad if \( R^* \ll R_{VdW} \) or \( k_F R^* \ll 1 \)

most heteronuclear FR are narrow

Molecule energy: \[
E_M = -\frac{\hbar^2}{2m_r (a^*_*)^2} \quad \text{with} \quad a^* = \frac{2R^*}{\sqrt{1 + 4R^*/a} - 1}
\]

\[
a \gg R^* : \quad a^* \sim a
\]

\[
a \ll R^* : \quad a^* \sim \sqrt{a R^*}
\]
many-body + narrow FR + bg scatt.

\[ T(\mathbf{P}, \omega) = T_{\text{open}}(\mathbf{P}, \omega) + T_{\text{closed}}(\mathbf{P}, \omega) \]

\[
T_{\text{open}}(\mathbf{P}, \omega) = \frac{1}{T_{bg}^{-1} - \Pi(\mathbf{P}, \omega)}
\]

\[ T_{bg} = \frac{2\pi a_{bg}}{m_r} \]

\[ \Pi(\mathbf{P}, \omega) = \int \frac{dk}{(2\pi)^3} \left[ \frac{1 - f_\uparrow(k) - f_\downarrow(\mathbf{P} + k)}{\omega + i0^+ - \xi_\uparrow k - \xi_\downarrow k + \frac{2m_r}{k^2}} \right] \]

\[ T_{\text{closed}}(\mathbf{P}, \omega) = V(\mathbf{P}, \omega)D(\mathbf{P}, \omega)V(\mathbf{P}, \omega) \]

\[ V(\mathbf{P}, \omega) = g[1 - T_{bg}\Pi(\mathbf{P}, \omega)]^{-1} \]

\[ D(\mathbf{P}, \omega) = [E_{CM} - \delta\mu(B - B_0) - \Sigma_{\text{mol}}(\mathbf{P}, \omega)]^{-1} \]

\[ \Sigma_{\text{mol}}(\mathbf{P}, \omega) = g \Pi(\mathbf{P}, \omega) V(\mathbf{P}, \omega) \]

Bruun, Jackson & Kolomeitsev, PRA 2005
PM & Stoof, PRA 2008
many-body + narrow FR + bg scatt.

\[ T(P, \omega) = T_{\text{open}}(P, \omega) + T_{\text{closed}}(P, \omega) \]

\[ T(P, \omega) = \frac{1}{2\pi \hat{a}(E_{CM})} - \Pi(P, \omega) \]

energy-dependent “scattering length”: \( \hat{a}(E_{CM}) \equiv a_{bg} \left( 1 - \frac{\Delta B}{B - B_0 - E_{CM}/\delta \mu} \right) \)

low energy expansion in vacuum: \( - \frac{1}{f_{\text{vac}}} = a^{-1} + ik + \tilde{R}^* + \ldots \)

\[ a = a_{bg} \left( 1 - \frac{\Delta B}{B - B_0} \right) \]

\[ \tilde{R}^* = R^* \left( 1 - \frac{a_{bg}}{a} \right)^2 \]
RF spectroscopy
RF spectroscopy

low power RF:

- High power RF is needed to couple to the MH continuum, due to a small FC overlap.

high power RF:

- Repulsive polarons exist as well-defined quasiparticles even in the strongly-interacting regime.

- P. Massignan, EPL (2012)
Decay of repulsive polarons

exp. data vs. theory for \( \text{Pol}^+ \rightarrow \text{Pol}^- \) and \( \text{Pol}^+ \rightarrow \text{Mol} \)

long lifetimes!
10 times more than in the MIT expmt. (Science 2009)
Rabi oscillations

\[ \hat{R} \propto \Omega_0 \sum_q (\hat{a}_{1q}^\dagger \hat{a}_{0q} + h.c.) \]

\[ |I\rangle = \hat{a}_{0q=0}^\dagger |\text{FS}\rangle \]

\[ |F\rangle = \sqrt{Z} \hat{a}_{1q=0}^\dagger |\text{FS}\rangle + \sum_{p < \hbar \kappa_F} \phi_{q,p} \hat{a}_{1p-q}^\dagger \hat{b}_q^\dagger \hat{b}_p |\text{FS}\rangle + \ldots \]

\[ \langle F | \hat{R} | I \rangle = \sqrt{Z} \Omega_0 \]

Rabi frequency as a measure of polaron residues

regime of very high RF power, well beyond linear response regime:
fast oscillations, and quasiparticle decay may be ignored

collision-induced decoherence is the main damping mechanism
Equation of state

A strongly-interacting system, described as an ensemble of weakly-interacting quasi-particles (a Fermi liquid)

\[ E = \frac{3}{5} \epsilon_F N_\uparrow \left[ 1 + \frac{m}{m^*} \left( \frac{N_\downarrow}{N_\uparrow} \right)^{5/3} \right] + N_\downarrow E_p + \ldots, \]

- kinetic energy of the Fermi sea
- kinetic energy of the polarons
- chemical potential of one polaron

(m* is their effective mass)
How many ↑ in the dressing cloud?

The density of the majority atoms far away from the impurity should remain unchanged when adding one impurity:

$$\Delta N = \left( \frac{\partial \mu \downarrow}{\partial n \uparrow} \right)_{n \downarrow} / \left( \frac{\partial \mu \uparrow}{\partial n \uparrow} \right)_{n \downarrow} \approx -\left( \frac{\partial \mu \downarrow}{\partial \epsilon_F} \right)_{n \downarrow}$$

$$\delta \mu \uparrow = \frac{\partial^2 \epsilon}{\partial n \uparrow \partial n \downarrow} + \frac{\partial^2 \epsilon}{(\partial n \uparrow)^2} \Delta N = 0$$
Contact density

\[
\frac{C}{8\pi m_r} = -\frac{d\epsilon}{d(1/a)} = -\frac{dE_\downarrow}{d(1/a)}
\]

link with the cloud “weight”: \[
\frac{C}{16\pi m_r a} = -n_\downarrow E_\downarrow - n_\downarrow \Delta N\epsilon_F
\]

link with Tan’s pressure relation: \[
\mathcal{P}_0 = 2\epsilon_F n_\uparrow / 5
\]

\[
\Delta \mathcal{P} = \mathcal{P} - \mathcal{P}_0 = 2\epsilon_F (\Delta n_\uparrow) / 3
\]

\[
\Delta n_\uparrow = -n_\downarrow \Delta N
\]

\[
\frac{C}{16\pi m_r a} = -\Delta \epsilon + \epsilon_F (\Delta n_\uparrow) = -\Delta \epsilon + \frac{3}{2} (\mathcal{P} - \mathcal{P}_0)
\]

adding the contribution of the non-int. Fermi sea: \[
\frac{C}{16\pi m_r a} = \frac{3}{2} \mathcal{P} - \epsilon
\]

Punk et al., PRA 2009
PM and G. Bruun, EPJD 2011
one impurity (↓)

ideal Fermi sea at $T \ll T_F$ (↑)
p-wave scattering

p-wave molecules with $m=\pm 1, 0, -1$ in an external magnetic field have different energies due to dipole-dipole interactions:

$$E_{\pm 1} > E_0$$

(Ticknor, Regal, Jin, and Bohn, PRA 2004)
p-wave polarons

two-channel Hamiltonian

\[ H = \sum_{p, \sigma} \frac{p^2}{2m} a_\sigma^p a_\sigma - \sum_{q, \mu} \left( \epsilon_\mu + \frac{q^2}{4m} \right) b_\mu q b_\mu q + \sum_{p, q, \mu} \frac{g(p)}{\sqrt{V}} p_\mu \left( b_\mu q a_{\frac{q}{2}+p} a_{\frac{q}{2}-p} + a_{\frac{q}{2}-p} a_{\frac{q}{2}+p} b_\mu q \right) \]

cut-off: \( g(p) = g\Theta(\Lambda - p) \)

(a) dressed molecule

(b) \( T \)-matrix

self-energy:

upon proper renormalization, the theory depends on: scattering volumes \( v_\mu \), and effective range \( k_0 \sim -1/R_{vdW} \)
p-wave polarons

p=0 polaron spectra for various resonance splittings $\delta$:

$\delta = 0$

$Z_{\pm 1} = 2Z_0$

$k_0 = -10k_F$

J. Levinsen, PM, F. Chevy, and C. Lobo, PRL 2012
few dilute impurities (↓)

ideal Fermi sea at $T \ll T_F$ (↑)
Itinerant FerroMagnetism

Thermodynamic analysis at $T \geq 0$:
Maxwell construction on the free energy of the mixed phase
The gas is mixed above the lines, and phase separated below.

Polarization: $P = \frac{N_1 - N_2}{N_1 + N_2}$

earlier predictions at $T=0$

Itinerant Ferromagnetism

PM, Z. Yu, and G. Bruun, PRL 2013
Critical interaction for IFM and decay rates

long lifetimes for K impurities in a bath of Li atoms at a narrow Feshbach resonance!
(as in the Innsbruck FeLiKx expmt.)
in collaboration with:

Theory:
Georg Bruun
Frederic Chevy
Carlos Lobo
Jesper Levinsen
Zhenhua Yu

Experiment:
Christoph Kohstall
Matteo Zaccanti
Michael Jag
Andreas Trenkwalder
Florian Schreck
Rudi Grimm
Conclusions

- The properties of one dressed impurity give important insights in the many-body behavior of a complex system.

- A new strongly interacting quantum state: the repulsive polaron (meta-stable, but very long-lived).

- Rabi oscillations confirm the coherence of the quasi-particles.

- New polaron/molecule branches appear in the p-wave case.

- Smaller losses at narrow resonances may open the way to IFM.