Four-body calculations of $^4$He tetramer and light hypernuclei using realistic two-body potentials

E. Hiyama (RIKEN)
Outline of my talk

(1) Introduction

In 3-dimension and 2-dimension world

(2)

LM2M2 potential
SAPT potential
TTY potential
HFD-B3-FCI1 potential
CCSAPT07 potential
PCKJS potential
HFD_B potential
Introduction
Many of important subjects in physics come finally to solving few-body (mainly 3- and 4-body) Schrödinger equations accurately.

By solving the equations, we can predict various observables before measurements and can obtain new understandings by analyzing the experimental data.

For this purpose, it is necessary to develop any accurate calculation-method for few-body problems and apply it to many subjects in various fields such as nuclear physics and atomic physics.
Our few-body calculation method

**Gaussian Expansion Method (GEM), since 1987**

- A variational method using Gaussian basis functions
- Take all the sets of Jacobi coordinates

Developed by Kyushu Univ. Group, Kamimura and his collaborators.

Review article:
E. Hiyama, M. Kamimura and Y. Kino,
Prog. Part. Nucl. Phys. 51 (2003), 223.

**High-precision calculations** of various 3- and 4-body systems:

- Exotic atoms / molecules
- 3- and 4-nucleon systems
- Multi-cluster structure of light nuclei
- Light hypernuclei
- 3-quark systems
Advantage of this method: We can treat

1) any kinds of particles (nucleon, electron, quark, .... )

3-body problem 4-body problem Currently, 5-body problem

2) any types of interactions (even if they have strong short-range correlations, spin-dependence or momentum dependence, .......).

Graph: $V(R)$ vs $R$
Few-nucleon systems and hypernuclear physics has been encouraging my method to develop to the above treatments.

Especially, to treat potential to have high repulsive core and long range tail is interesting subject for me.

For this purpose, hypernuclear physics provide us many challenging subjects.

In hypernuclear physics, we have realistic interactions such as Nijmegen model (Nijmegen soft core 97, Extended soft core 08, etc)

- To have high repulsive core

\[ ^3n \]
Another interesting subject is to solve bound states in $^4$He ($^3$He) trimer and tetramer systems. The potential between two $^4$He has high repulsive core and long-ranged tail. To solve these systems encourages us to develop our method, Gaussian Expansion Method.
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Section 2

three-body calculation of $^3\Lambda n$

E. Hiyama, S. Ohnishi, B.F. Gibson, and T. A. Rijken,
The paper will be published in PRC as a Rapid communication soon.
In order to understand the baryon-baryon interaction, two-body scattering experiment is most useful.

YN and YY potential models so far proposed (ex. Nijmegen, Julich, Kyoto-Niigata) have large ambiguity.

1) To understand baryon-baryon interactions

Total number of Nucleon (N) - Nucleon (N) data: 4,000

• Total number of differential cross section Hyperon (Y) - Nucleon (N) data: 40
• NO YY scattering data

Study of NN interaction has been developed.

2) To study the structure of multi-strangeness systems

YN and YY potential models so far proposed (ex. Nijmegen, Julich, Kyoto-Niigata) have large ambiguity.

since it is difficult to perform YN scattering experiment even at J-PARC.
Hypernucleus

No Pauli principle
Between N and \( \Lambda \)

\( \Lambda \) particle can reach deep inside, and attract the surrounding nucleons towards the interior of the nucleus.

Due to the attraction of \( \Lambda N \) interaction, the resultant hypernucleus will become more stable against the neutron decay.
Nuclear chart with strangeness

Multi-strangeness system such as Neutron star

Double-Λ Hypernuclei
Ξ Hypernuclei

Λ, Σ Hypernuclei
Λ ≈ 35
Σ 1

Extending drip-line!

Interesting phenomena concerning the neutron halo have been observed near the neutron drip line of light nuclei.

How is structure change when a particle is injected into neutron-rich nuclei?
Question: How is structure change when a $\Lambda$ particle is injected into neutron-rich nuclei?

$^7$He

$^6_\Lambda$H


What is interesting to study $nn\rangle$ system?

The lightest nucleus to have a bound state is deuteron.

$n+p$ threshold

-2.22 MeV  \( J = 1^+ \)  

Exp.

Lightest hypernucleus to have a bound state  

$^3H$ (hyper-triton)  

$0.13 \text{ MeV } \uparrow_{\text{Exp.}} \downarrow_{\text{Exp.}}  \quad J = 1/2^+$
Observation of nn\rangle system (2013)
Lightest hypernucleus to have a bound state
Any two-body systems are unbound. => nn\rangle system is bound.
Lightest Borromean system.

They did not report the binding energy.

breaking threshold

scattering length: -2.68fm

-23.7fm
Theoretical important issue:
Do we have bound state for $nn\rangle$ system?
If we have a bound state for this system, how much is binding energy?

NN interaction: to reproduce the observed binding energies of $^3H$ and $^3He$

NN: AV8 potential
We do not include 3-body force for nuclear sector.

How about YN interaction?
Non-strangeness nuclei

On the other hand, the mass difference between $\rightarrow$ and £ is much smaller, then $\rightarrow$ can be converted into £ particle easily.

Nucleon can be converted into $\sim$. However, since mass difference between nucleon and $\sim$ is large, then probability of $\sim$ in nucleus is not large.
To take into account of a particle to be converted into £ particle, we should perform below calculation using realistic hyperon(Y)-nucleon(N) interaction.

YN interaction: Nijmegen soft core ‘97f potential (NSC97f) proposed by Nijmegen group reproduce the observed binding energies of \(^3\text{H}\), \(^4\text{H}\) and \(^4\text{He}\).
\[ ^3\text{H} \] 

\[ {\text{-B}}_\text{>} \]

0 MeV

\[ 1/2^+ \]

\[-0.13 \pm 0.05 \text{ MeV} \]

Exp.

-0.19 MeV

Cal.

\[ 1/2^+ \]

d+< >
What is binding energy of \( nn \)?
We have no bound state in nn\(^{>}\) system. This is inconsistent with the data.

Now, we have a question.

Do we have a possibility to have a bound state in nn\(^{>}\) system tuning strength of YN potential?

It should be noted to maintain consistency with the binding energies of \(^3\)H and \(^4\)H and \(^4\)He.

\[ V_T^{\gamma N} \times 1.1, 1.2 \]
When we have a bound state in $nn\Lambda$ system, what are binding energies of $^3\text{H}$ and $A=4$ hypernuclei?
We have no possibility to have a bound state in $nn^\Lambda$ system.
Question: If we tune $^1S_0$ state of nn interaction, Do we have a possibility to have a bound state in nn? In this case, the binding energies of $^3$H and $^3$He reproduce the observed data?

Some authors pointed out to have dineutron bound state in nn system. Ex. H. Witala and W. Gloeckle, Phys. Rev. C85, 064003 (2012).

T=1, $^1S_0$ state
I multiply component of $^1S_0$ state by 1.13 and 1.35. What is the binding energies of nn? 

PHYSICAL REVIEW C 85, 064003 (2012)

Di-neutron and the three-nucleon continuum observables

H. Witala
M. Smoluchowski Institute of Physics, Jagiellonian University, PL-30059 Kraków, Poland

W. Glöckle
Institut für Theoretische Physik II, Ruhr-Universität Bochum, D-44780 Bochum, Germany
(Received 24 April 2012; published 25 June 2012)

We investigate how strongly a hypothetical $^1S_0$ bound state of two neutrons would affect observables in neutron-deuteron reactions. To that aim we extend our momentum-space scheme of solving the three-nucleon Faddeev equations and incorporate in addition to the deuteron also a $^1S_0$ di-neutron bound state. We discuss effects induced by a di-neutron on the angular distributions of the neutron-deuteron elastic scattering and deuteron breakup cross sections. A comparison to the available data for the neutron-deuteron total cross section and elastic scattering angular distributions cannot decisively exclude the possibility that two neutrons can form a $^1S_0$ bound state. However, strong modifications of the final-state-interaction peaks in the neutron-deuteron breakup reaction seem to disallow the existence of a di-neutron.
We do not find any possibility to have a bound state in nn\>

\[\begin{array}{ccc}
\text{nn} & \text{unbound} & 0 \text{ MeV} \\
-0.066 \text{MeV} & 1S_0 \times 1.13 & -1.269 \text{ MeV} \\
1S_0 \times 1.35 & \\
\text{nn\>} & \text{unbound} & \text{unbound} & 0 \text{ MeV} \\
1/2^+ & -1.272 \text{ MeV} \\
\end{array}\]
Summary of hypernuclear part:

Motivated by the reported observation of data suggesting a bound state \( nn' \), we have calculated the binding energy of this hypernucleus taking into account \( N-\£N \) explicitly. We did not find any possibility to have a bound state in this system. However, the experimentally they reported evidence for a bound state. As long as we believe the data, we should consider additional missing elements in the present calculation. But, I have no idea. Unfortunately, they did not report binding energy. I hope that further experimental data is needed.
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HFD_B potential
Efimov effects in 4He atoms

One of the most fundamental theoretical issues from the view point of few-body problems:

to perform accurate calculations of the 3- and 4-body 4He-atom systems using realistic 4He-4He potentials.

This subject has been intensively studied by nuclear physicists, atomic physicists and quantum chemists.
There were a few calculation or estimation for very weakly bound state near the trimer +4He threshold.

Numerical difficulty:

There are many 3-body calculations for 4He trimer.
The difficulty for the 4-body calculations of the $^4$He tetramer is

1) the realistic $^4$He–$^4$He potential has an extremely strong repulsive core.
   The core part of the atomic potential is $\sim 1000$ times higher than that of the nucleon-nucleon potential.

2) the very weakly-bound excited state should have a very long-range tail, which is also difficult to treat.
There are 5 calculations of tetramer using realistic pair potentials (LM2M2, TTY).

<table>
<thead>
<tr>
<th>Method</th>
<th>Reference</th>
<th>Ground state</th>
<th>Excited state</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monte Carlo</td>
<td>Lewerenz (1977)</td>
<td>TTY</td>
<td>558</td>
</tr>
<tr>
<td>Monte Carlo</td>
<td>Bressanini et al. (2000)</td>
<td>TTY</td>
<td>559.1</td>
</tr>
<tr>
<td>Monte Carlo</td>
<td>Blume and Greene (2000)</td>
<td>LM2M2</td>
<td>557</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>133</td>
</tr>
<tr>
<td></td>
<td>Das et al. (2011)</td>
<td>TTY</td>
<td>558</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>178</td>
</tr>
</tbody>
</table>

This value was not obtained by bound-state calculation, but was extrapolated from atom-trimer scattering calculation.

So, we intended to confirm this value by our bound-state calculation.
We confirmed this level structure of $^4$He-atom clusters (2012).

$^4$He + $^4$He + $^4$He + $^4$He

$^4$He + $^4$He + $^4$He + $^4$He

$^4$He + $^4$He

0.0 mK

-1.30348 mK

-2.2706 mK

-126.40 mK

-127.33 mK

-558.98 mK

We found that this state is a typical halo state having this configuration.

E. Hiyama et al., PRC 70 (2004)

We found that this state is a typical halo state having this configuration.
How about 2-dimensions?

In the terms of using realistic potential,


Variation and diffusion Monte Carlo method => \( N=2 \) to \( 6 \)-body problems
Potential: SAPT96 potential
I am interested in calculating trimers and tetramer systems using various types of realistic potentials. I will show you our preliminary results here.

LM2M2 potential
SAPT potential
TTY potential
HFD-B3-FCI1 potential
CCSAPT07 potential
PCKJS potential
HFD_B potential
### Dimer (2-body problem)

<table>
<thead>
<tr>
<th>Potential</th>
<th>2 dimensions</th>
<th>3D</th>
</tr>
</thead>
<tbody>
<tr>
<td>SAPT96</td>
<td>-40.84 mK</td>
<td>-41 mK</td>
</tr>
<tr>
<td>LM2M2</td>
<td>-38.39 mK</td>
<td>-1.744 mK</td>
</tr>
<tr>
<td>TTY</td>
<td>-38.35 mK</td>
<td>-1.309 mK</td>
</tr>
<tr>
<td>HFD-B3-FCI1</td>
<td>-38.99 mK</td>
<td>-1.316 mK</td>
</tr>
<tr>
<td>CCSAPT07</td>
<td>-39.49 mK</td>
<td>-1.448 mK</td>
</tr>
<tr>
<td>PCKJS</td>
<td>-39.69 mK</td>
<td>-1.564 mK</td>
</tr>
<tr>
<td>HFD_B</td>
<td>-40.00 mK</td>
<td>-1.615 mK</td>
</tr>
</tbody>
</table>

You see that the dimer binding energy in 2D is less bound than that in 3D. Because Kinetic energy in 2D is smaller than that in 3D. Also, scattering length in 2D is much smaller than that in 3D.

Scattering length in 2D: 41.90 ú effective range: 21.96 ú
Scattering length in 3D: ~190 ú
<table>
<thead>
<tr>
<th>Potential</th>
<th>Our result</th>
<th>Kilic et al. (2004)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SAPT96</td>
<td>-0.02 mK</td>
<td>-0.02 mK</td>
</tr>
<tr>
<td>LM2M2</td>
<td>-0.013 mK</td>
<td></td>
</tr>
<tr>
<td>TTY</td>
<td>-0.013 mK</td>
<td></td>
</tr>
<tr>
<td>HFD-B3-FCI1</td>
<td>-0.014 mK</td>
<td></td>
</tr>
<tr>
<td>CCSAPT07</td>
<td>-0.016 mK</td>
<td></td>
</tr>
<tr>
<td>PCKJS</td>
<td>-0.016 mK</td>
<td></td>
</tr>
<tr>
<td>HFD_B</td>
<td>-0.018 mK</td>
<td></td>
</tr>
</tbody>
</table>

2 dimensions

3D

No bound

No bound
We calculated trimer states of $^4\text{He}$ using LM2M2 potential et al.

We have two bound states.

L.W. Brush and L.A. Tjon, PRA19, 425 (1979)

$E_{3}^{(1)}=1.27E_2$

$E_{3}^{(0)}=16.522E_2$

My calculation

$E_{3}^{(1)}=1.007E_2$

$E_{3}^{(0)}=4.511E_2$

K.Helrich and H-W. Hammer, PRA83,052703(2011)

They pointed out to need effective range correction. $E_{3}^{(1)}=1.145E_2,$

$E_{3}^{(0)}=10.578E_2$

SAPT96

$^4\text{He}+^4\text{He}+^4\text{He}$

$0\text{ mK}$

$^4\text{He}+^4\text{He}$

$-40.8\text{ mK}$

$(^4\text{He}+^4\text{He})+^4\text{He}$

$-41.098\text{ mK}$

$0^+$

$-184.06\text{ mK}$

$0^+$

$-183\text{ mK}$

Our result

Kilic et al.
We have no second bound state.

Our result

0+ -73.88 mK
Kilic et al.

-74.3 mK

We have no second bound state.
Tetramer system

\[ ^4\text{He} + ^4\text{He} + ^4\text{He} + ^4\text{He} \rightarrow -184.6 \text{ mK} \]

\[ ( ^4\text{He} + ^4\text{He} + ^4\text{He} ) + ^4\text{He} \]

\[ ^0\text{+} -184.06 \text{ mK} \]

In progress. We do not have the converged result.

\[ ^3\text{He} + ^3\text{He} \]

\[ ^0\text{+} -433 \text{ mK} \]

\[ ^0\text{+} -435 \text{ mK} \]

Kilic et al.

Not yet

Future plan
The calculation of trimer and tetramer systems is still in progress.

When the calculation of 2D is finished, I want to compare results in 2D and those in 3D.

For example,
Correlation between the 3N (\(^3\)H) and 4N (\(^4\)He) binding energies for different NN potentials is approximately linear. This line is called Tjon line (well known in nuclear few-body physics). How is this type of correlation in \(^4\)He–atom tetramer?

The slope is universal?
B_{\alpha\tau} correlation

Nuclear Tjon line

A. Nogga et al., PRL 85 (2000)

\[ B_{3}^{(0)} - B_{4}^{(0)} \] correlation

Atomic Tjon line

E. Hiyama & M. K, PRA 85 (2012)

**2-body realistic forces**

**3-body** $B_{t}$

**4-body $B_{2}$**

Slope = 4.8

Slope = 4.778

Least square fit
3D world
Atomic Tjon line

E. Hiyama & M. K, PRA 85 (2012)

$B_3^{(0)} - B_4^{(0)}$

Slope = 2.9

Further study is in progress.

2D world

Various realistic $^4$He-$^4$He potentials

Slope = 3.9

J. A. Tjon, PRA21, 1334 (1980)
Slope = 2.9
Summary

In 3-dimension and 2-dimension world

- LM2M2 potential
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- TTY potential
- HFD-B3-FCI1 potential
- CCSAPT07 potential
- PCKJS potential
- HFD_B potential
Thank you!
in 2 dimensions. We follow the conventions of Ref. [169] in which the scattering length $a$ and the effective range $r_s$ are defined by

$$\frac{1}{2} \pi \cot \delta_0(k) = \gamma + \ln(\frac{1}{2}ka) + \frac{1}{4}r_s^2k^2 + O(k^4),$$  \hspace{1cm} (414)$$

where $\gamma \simeq 0.577216$ is Euler's constant. The binding energy of the shallow dimer in the scaling limit is given by
Gaussian Expansion Method (GEM)

\[ H = -\frac{\hbar^2}{2\mu_{r_c}} \nabla_{r_c}^2 - \frac{\hbar^2}{2\mu_R} \nabla_R^2 + V^{(1)}(r_1) + V^{(2)}(r_2) + V^{(3)}(r_3) \cdot \]

\[ [H - E] \Psi_{JM} = 0 \]

\[ \Psi_{JM} = \Phi_{JM}^{(1)}(r_1, R_1) + \Phi_{JM}^{(2)}(r_2, R_2) + \Phi_{JM}^{(3)}(r_3, R_3) \]
Basis functions of each Jacobi coordinate
\( c = 1 - 3 \)

\[
\Psi_{JM} = \Phi_{JM}^{(1)}(r_1, R_1) + \Phi_{JM}^{(2)}(r_2, R_2) + \Phi_{JM}^{(3)}(r_3, R_3)
\]

\[
\phi_{nl}^{(c)}(r_c) Y_{lm}(\hat{r}_c), \quad \psi_{NL}^{(c)}(R_c) Y_{LM}(\hat{R}_c)
\]

\[
\Phi_{JM}^{(c)}(r_c, R_c) = \sum_{n, N L} C_{NL, lm} \phi_{nl}^{(c)}(r_c) \psi_{NL}^{(c)}(R_c) [Y_l(\hat{r}_c) \otimes Y_L(\hat{r}_c)]_{JM}
\]

Determined by diagonalizing H
Radial part: Gaussian function

\[ \phi_{nl}(r) = r^l e^{-(r/r_n)^2} \]

\[ \psi_{NL}(R) = R^L e^{-(R/R_N)^2} \]

Gaussian ranges in geometric progression

\[ r_n = r_1 a^{n-1} \quad (n = 1 - n_{max}) \]

\[ R_N = R_1 A^{N-1} \quad (N = 1 - N_{max}) \]

Both the short-range correlations and the exponentially-damped tail are simultaneously reproduced accurately.
Next, by solving eigenstate problem, we get eigenenergy $E$ and unknown coefficients $C_n$.

$$\begin{pmatrix} (H_{in}) - E (N_{in}) \end{pmatrix} \begin{pmatrix} C_n \end{pmatrix} = 0$$