Neutrino-interaction in the resonance region (transition)

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★ Introduction  ν interaction in resonance region

★ Dynamical coupled-channels (DCC) model for

\[ \gamma N, \pi N \rightarrow \pi N, \pi \pi N, \eta N, K\Lambda, K\Sigma \]

extended to  \[ \nu N \rightarrow \pi N, \pi \pi N, \eta N, K\Lambda, K\Sigma \]

★ Future plan

matching with DIS  cf. quark-hadron duality

\[ \nu - \text{nucleus interaction} \]
\( \nu \)-interaction (kinematics & characteristics)

Collaboration at J-PARC Branch of KEK Theory Center

http://j-parc-th.kek.jp/html/English/e-index.html
\(\nu\)-interaction (kinematics & characteristics)

Wide kinematical region with different characteristic

\(\rightarrow\) Combination of different expertise is necessary
Many nucleon resonances in 2\textsuperscript{nd} and 3\textsuperscript{rd} resonance region

Not only $1\pi$ production but also ...
Multi-channel reaction

- $2\pi$ production is comparable to $1\pi$
- $\eta$, $K$ productions (background of proton decay exp.)
Theoretical approach to $\nu$-interaction in resonance region

- **PCAC** (partially conserved axial current)
  
  at $Q^2=0$, axial coupling $\rightarrow$ pion coupling
  
  Paschos et al. (2011); Kamano et al. (2012)

- **Microscopic model**
  
  resonant and non-resonant hadronic interactions
  
  Rein et al. (1981), (1987); Lalalulich et al. (2005), (2006);
  Hernandez et al. (2007), (2010); Lalakulich et al. (2010);
  Sato et al. (2003), (2005); ...
Microscopic models for $\nu$-induced $1\pi$ production

resonant only

$\sum_i N^*_i$

Rein et al. (1981), (1987); Lalulich et al. (2005), (2006)

+ non-resonant (tree-level)

$\sum_i N^*_i$

Hernandez et al. (2007), (2010); Lalakulich et al. (2010)

+ rescattering ($\pi N$ unitarity)

Sato, Lee (2003), (2005)
Dealing with multi-channel reaction

Problems

- **Unitarity** is missing in previous models
- Important $2\pi$ production model is missing
- Previous models for $K$ and $\eta$ production are not well tested by data

To overcome the problems...

We develop **Unitary coupled-channel** model

★ Dynamical coupled-channels (DCC) model for $\gamma N, \pi N \rightarrow \pi N, \pi\pi N, \eta N, K\Lambda, K\Sigma$

★ Extension to $\nu N \rightarrow \pi N, \pi\pi N, \eta N, K\Lambda, K\Sigma$
DCC model for

\[ \gamma N, \pi N \rightarrow \pi N, \pi \pi N, \eta N, K\Lambda, K\Sigma \]

Coupled-channel, 2&3-body unitarity is taken into account
DCC (Dynamical Coupled-Channel) model

Coupled-channel Lippmann-Schwinger equation

\[ T_{ab} = V_{ab} + \sum_c V_{ac} G_c T_{cb} \]

DCC analysis of meson production data


Fully combined analysis of $\gamma N, \pi N \rightarrow \pi N, \eta N, K\Lambda, K\Sigma$

(W $\leq$ 2.1 GeV)

~380 parameters ($N^*$ mass, $N^* \rightarrow MB$ couplings, cutoffs)

to fit $\sim$ 20,000 data points
Partial wave amplitudes of $\pi N$ scattering

Real part

$$I = \frac{3}{2}$$

Imaginary part

Previous model
(fitted to $\pi N \rightarrow \pi N$ data only)

Kamano, Nakamura, Lee, Sato, 2013

[PRC76 065201 (2007)]
Vector current ($Q^2=0$) for $1\pi$
Production is well-tested by data
Eta production reactions

$\pi^- p \rightarrow \eta n$

$\frac{d\sigma}{d\Omega}$ (mb/sr)

$W = 1498$ MeV
$W = 1586$ MeV
$W = 1699$ MeV

$W = 1805$ MeV
$W = 1897$ MeV

$\theta$ (deg)
Vector current \((Q^2=0)\) for \(\eta\)

Production is well-tested by data

\[ \gamma p \rightarrow \eta p \]
KY production reactions

\[ d\sigma/d\Omega \ (\mu b/sr) \]

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Mass (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi^+ p \rightarrow K^+\Sigma^+ )</td>
<td>1732, 1845, 1985, 2031</td>
</tr>
<tr>
<td>( \pi^- p \rightarrow K^0\Lambda^0 )</td>
<td>1757, 1879, 1966, 2059</td>
</tr>
<tr>
<td>( \pi^- p \rightarrow K^0\Sigma^0 )</td>
<td>1792, 1879, 1966, 2059</td>
</tr>
</tbody>
</table>

Kamano, Nakamura, Lee, Sato, 2012
Vector current ($Q^2=0$) for $K$

Production is well-tested by data
\( \pi^0 N \rightarrow \pi \pi N \) (parameters had been fitted to \( \pi^0 N \rightarrow \pi N \))

Kamano, Julia-Diaz, Lee, Matsuyama, Sato, PRC79 025206 (2009)
\(\gamma N \rightarrow \pi \pi N\)

(parameters had been fitted to \(\pi N, \gamma N \rightarrow \pi N\))

Kamano, Julia-Diaz, Lee, Matsuyama, Sato, PRC80 065203 (2009)

\(\gamma p \rightarrow \pi^+ \pi^- p\)

\(\gamma p \rightarrow \pi^0 \pi^0 p\)

\(\gamma p \rightarrow \pi^+ \pi^0 n\)

\[\sigma \text{ (\mu b)}\]

\[W \text{ (MeV)}\]

- Good description near threshold
- Good shape of invariant mass distribution
- Total cross sections overestimate data for \(W \geq 1.5\) GeV
Short Summary

• $\nu N$ scattering in resonance region is multi-channel reaction

• Unitary coupled-channels model is ideal

• DCC model for $\gamma N, \pi N \rightarrow \pi N, \pi\pi N, \eta N, K\Lambda, K\Sigma$ is developed

• Model is extensively tested by $\gamma N, \pi N \rightarrow \pi N, \pi\pi N, \eta N, K\Lambda, K\Sigma$ data $\rightarrow$ reliable vector current to be applied to $\nu$-scattering
Extended DCC model for

\[ \nu N \rightarrow \pi N, \pi\pi N, \eta N, K\Lambda, K\Sigma \]

1. PCAC-based calculation \((Q^2=0)\)  Kamano, Nakamura, Lee, Sato,  PRD 86, 097503 (2012)

2. Dynamical axial current  (strategy)

3. Photon emission in NC process  (some comments)
PCAC-based calculation for forward scattering of

\[ \nu N \rightarrow \pi N, \pi\pi N, \eta N, K\Lambda, K\Sigma \]

Kamano et al., PRD 86, 097503 (2012)

Cf. Paschos et al., arXiv:1212.4662

Objectives

- Set a starting point for full dynamical model
- Relative importance of different channels
- Comparison with Rein-Sehgal model
Formalism

Cross section for $\nu N \rightarrow X$ \quad ($X = \pi N, \pi\pi N, \eta N, KA, KS$)

$$\theta \rightarrow 0 \quad \frac{d\sigma}{dE_\ell d\Omega_\ell} = \frac{G_F^2}{2\pi^2} E_\ell^2 \left(2W_1 \sin^2 \frac{\theta}{2} + W_2 \cos^2 \frac{\theta}{2} \pm W_3 \frac{E_\nu + E_\ell}{m_N} \sin^2 \frac{\theta}{2}\right)$$

$$Q^2 \rightarrow 0 \quad W_2 = \frac{Q^2}{q^2} \sum \left[ \frac{1}{2} \left(|\langle J^x \rangle|^2 + |\langle J^y \rangle|^2\right) + \frac{Q^2}{q_c^2} \left|\langle J^0 + \frac{\omega_c}{Q^2} q \cdot J \rangle\right|^2 \right]$$

CVC & PCAC \quad \langle q \cdot J \rangle = \langle q \cdot V \rangle - \langle q \cdot A \rangle = i f_\pi m_\pi^2 \langle \hat{\pi} \rangle$

LSZ & smoothness \quad $\langle X|\hat{\pi}|N\rangle = \frac{\sqrt{2} \omega_c}{m_\pi^2} \mathcal{T}_{\pi N \rightarrow X}(0) \sim \frac{\sqrt{2} \omega_c}{m_\pi^2} \mathcal{T}_{\pi N \rightarrow X}(m_\pi^2)$

Finally \quad $F_2 \equiv \omega W_2 = \frac{2f_\pi^2}{\pi} \sigma_{\pi N \rightarrow X}$ \quad $\sigma_{\pi N \rightarrow X}$ is from our DCC model
Results

- Prediction based on model well tested by data
- $\pi N$ dominates for $W \leq 1.5$ GeV
- $\pi\pi N$ becomes comparable to $\pi N$ for $W \geq 1.5$ GeV
- Smaller contribution from $\eta N$ and $KY \sim O(10^{-1}) - O(10^{-2})$
- Agreement with SL (no PCAC) in $\Delta$ region
Comparison with Rein-Sehgal model

- Lower $\Delta$ peak of RS model
- RS overestimate in higher energy regions (DCC model is tested by data)
Comparison with Rein-Sehgal model

\[ \nu_e + p \rightarrow e^- + p + \pi^+ \]

Similar findings by Leitner et al., PoS NUFAC08 (2008) 009
Comparison with Rein-Sehgal model

\[ \nu_e + n \rightarrow e^- + p + \pi^0 \]

\[ \nu_e + n \rightarrow e^- + n + \pi^+ \]

Comparison in whole kinematical region will be done after axial current model is developed
DCC model for forward $\nu N \rightarrow \pi N, \pi\pi N, \eta N, K\Lambda, K\Sigma$ via PCAC

- Prediction based on model well tested by data
- $\pi\pi N$ comparable to $\pi N$ for $W \geq 1.5$ GeV (first $\nu N \rightarrow \pi\pi N$)
- First $\nu N \rightarrow \eta N, K\Upsilon$ based on data

Comparison with Rein-Sehgal model:

- RS has Lower $\Delta$ peak
- RS overestimates cross section at higher energies
Full DCC model for $\nu N \rightarrow \pi N, \pi\pi N, \eta N, K\Lambda, K\Sigma$

Kamano, Nakamura, Lee, Sato, in progress

Vector current

$Q^2=0$

$\gamma p \rightarrow MB$ analysis has been done

$\gamma n \rightarrow \pi N, \eta n$ analysis is ongoing $\rightarrow$ isospin separation necessary for calculating $\nu$-interaction

$Q^2 \neq 0$ (electromagnetic form factors for $VNN^*$ couplings)

obtainable from $(e,e'\pi)$ data analysis (will be done soon)
Previous analysis of $1\pi$ electroproduction data

Juliz-Diaz et al., PRC 80, 025207 (2009)

Fit to structure functions from CLAS for $p (e,e'\pi) p$

$$\frac{d\sigma^5}{dE_{e'}d\Omega_{e'}d\Omega_{\pi}} = \Gamma_\gamma \left[ \sigma_T + \epsilon\sigma_L + \sqrt{2\epsilon(1+\epsilon)}\sigma_{LT} \cos\phi_\pi^* + \epsilon\sigma_{TT} \cos 2\phi_\pi^* + h_e \sqrt{2\epsilon(1-\epsilon)}\sigma_{LT'} \sin\phi_\pi^* \right].$$

$W < 1.6$ GeV

$Q^2 < 1.5$ (GeV/c)$^2$
Full DCC model for $\nu N \rightarrow \pi N, \pi\pi N, \eta N, K\Lambda, K\Sigma$

Kamano, Nakamura, Lee, Sato, in progress

Axial current

$Q^2 = 0$

non-resonant mechanisms

resonant mechanisms

$Q^2 \neq 0$ (axial form factors of $ANN^*$)

experimental information is necessary to fix this
How to fix axial form factors of $^{\text{ANN}}$*

Neutrino data on deuteron from ANL and BNL: $\nu_\mu d \rightarrow \mu NN \pi$

Elementary amplitude

$W, Z \rightarrow \pi$

Fermi motion of deuteron
Hernandez et al., PRD 81 (2010)

Fermi motion of deuteron + rescattering effects
Jiajun and Lee in progress
How to fix axial form factors of $ANN^*$

Neutrino data

Discrepancy of ANL and BNL data $\Rightarrow$ new deuterium data are highly hoped

$$C_5^A(0) = 1.00 \pm 0.11$$

$$M_{A\Delta} = 0.93 \pm 0.07 \text{ (GeV)}$$

Hernandez et al., PRD 76 (2006); 81 (2010)
How to fix axial form factors of $\text{ANN}^*$

Electron data: parity violation asymmetry

$$A = \frac{d\sigma_R - d\sigma_L}{d\sigma_R + d\sigma_L}$$

$$G^A_{N\Delta}(Q^2) = \frac{1}{2} [M^2 - M^2_\Delta + Q^2] C_4^A(Q^2)$$

$$- M^2 C_5^A(Q^2)$$

$$G^A_{N\Delta}(Q^2) = -0.05 \pm (0.35)_{\text{stat}} \pm (0.34)_{\text{sys}} \pm (0.06)_{\text{th}}$$

Theory

$$G^A_{N\Delta}(Q^2) = -0.196$$
How to fix axial form factors of $\textit{ANN}^*$

Quark-hadron duality : coming back to this later
Photon emission in NC

Δ−Region : Hill (2010), Zhang et al. (2012)
Δ+2nd resonance : Wang et al. (2013)

Relevant to T2K and MiniBooNE

With our DCC model

Photon emission in higher N* region relevant to future experiments ??
Matching with DIS

Purposes:

• Make sure smooth transition from RES to DIS (electron scattering)
• Fix axial form factors of ANN* (neutrino scattering)
Matching with DIS

\[ Q^2 (\text{GeV/c})^2 \]

\[ \nu (\text{GeV}) \]

\[ W (\text{GeV}) \]

- QE region
- RES region
- DIS region

0.94, 1.23, 2.0
Matching with DIS

\[ \frac{d\sigma}{d\Omega dE'} = \frac{4\alpha^2 E'^2}{Q^4} \left[ 2W_1 \sin^2 \frac{\theta}{2} + W_2 \cos^2 \frac{\theta}{2} \right] \]

\[ \frac{d\sigma^{\nu, \bar{\nu}}}{dE'd\Omega'} = \frac{G_F^2 |V_{ud}|^2}{2\pi^2} E'^2 \left[ 2W_1 \sin^2 \frac{\theta}{2} + W_2 \cos^2 \frac{\theta}{2} \pm W_3 \frac{E + E'}{m_N} \sin^2 \frac{\theta}{2} \right] \]

In terms of matrix elements of **hadronic currents**

\[ W_1 = \frac{1}{2} \sum \left[ \left| \langle f | J^x_\alpha | i \rangle \right|^2 + \left| \langle f | J^y_\alpha | i \rangle \right|^2 \right] , \]

\[ W_2 = \frac{Q^2}{q^2} \sum \left[ \frac{1}{2} \left| \langle f | J^x_\alpha | i \rangle \right|^2 + \left| \langle f | J^y_\alpha | i \rangle \right|^2 \right] + \frac{Q^2}{q^2_c} \left| \langle f | \bar{J}_\alpha | i \rangle \right|^2 \]

\[ W_3 = -\frac{2m_N}{|q|} \sum \text{Im} \left[ \langle f | J^x_\alpha | i \rangle \langle f | J^y_\alpha | i \rangle^* \right] , \]
Matching with DIS

\[ N(e, e') \]
\[ \frac{d\sigma}{d\Omega dE'} = \frac{4\alpha^2 E'^2}{Q^4} \left[ 2W_1 \sin^2 \frac{\theta}{2} + W_2 \cos^2 \frac{\theta}{2} \right] \]

\[ N(\nu, l') \]
\[ \frac{d\sigma_{\nu,\bar{\nu}}}{dE'd\Omega'} = \frac{G_F^2}{2\pi^2} |V_{ud}|^2 E'^2 \left[ 2W_1 \sin^2 \frac{\theta}{2} + W_2 \cos^2 \frac{\theta}{2} \pm W_3 \frac{E + E'}{m_N} \sin^2 \frac{\theta}{2} \right] \]

\[ F_1(x, Q^2) = m_N W_1(x, Q^2) \]
\[ F_2(x, Q^2) = \omega W_2(x, Q^2), \]
\[ F_3(x, Q^2) = \omega W_3(x, Q^2), \]

In terms of \textit{parton distribution functions (PDF)}

\textbf{For } N(e, e') \quad \begin{align*}
F_2(x, Q^2) &= x \left[ \frac{4}{9} (u(x, Q^2) + \bar{u}(x, Q^2)) + \frac{1}{9} (d(x, Q^2) + \bar{d}(x, Q^2)) \right] \\
F_1(x, Q^2) &= F_2(x, Q^2)/2x, \end{align*}

\textbf{For } CC \nu \quad \begin{align*}
F_2(x, Q^2) &= 2x F_1(x, Q^2) = 2x (d(x, Q^2) + \bar{u}(x, Q^2)) \\
F_3(x, Q^2) &= 2(d(x, Q^2) - \bar{u}(x, Q^2)). \end{align*}
Matching with DIS

\[ N(e, e') \]
\[
\frac{d\sigma}{d\Omega dE'} = \frac{4\alpha^2 E'^2}{Q^4} [2W_1 \sin^2 \frac{\theta}{2} + W_2 \cos^2 \frac{\theta}{2}]
\]

\[ N(\nu, l') \]
\[
\frac{d\sigma^{\nu,\bar{\nu}}}{dE'd\Omega'} = \frac{G_F^2 |V_{ud}|^2}{2\pi^2} E'^2 [2W_1 \sin^2 \frac{\theta}{2} + W_2 \cos^2 \frac{\theta}{2} \pm W_3 \frac{E + E'}{m_N} \sin^2 \frac{\theta}{2}]
\]

\[ F_1(x, Q^2) = m_N W_1(x, Q^2) \]
\[ F_2(x, Q^2) = \omega W_2(x, Q^2) \]
\[ F_3(x, Q^2) = \omega W_3(x, Q^2) \]

Matching \( F_i^{DCC} \leftrightarrow F_i^{PDF} \) at \( W \approx 2 \text{ GeV}, \ Q^2 > 1 \ (\text{GeV/c})^2 \) (overlapping region)
Matching with DIS

- Make sure smooth transition for electron scattering

- Some axial form factors of $ANN^*$ can be fixed for neutrino scattering

- *Unitarity, coupled-channels* are essential for matching only *inclusive* structure functions can be matched

  $2\pi$ contribution is large
Matching with low-$Q^2$ region

Model for low-$Q^2$ region

Kulagin and Petti, PRD 76, 094023 (2007)  PCAC + extrapolation to finite $Q^2$ from 0
Bodek and Yang, arXiv:1011.6592  PDF extrapolated to lower $Q^2$ from DIS region
Matching on the boundary
Quark-hadron duality
Quark-hadron duality

Global duality in electron-nucleon scattering

Ex. PDF from $W \approx 3$ GeV, $Q^2 \approx 10$ (GeV/c)$^2$
Quark-hadron duality

Local duality in Neutrino-nucleon scattering in $\Delta$ region

$Q^2 = 0.4$ (GeV/c)$^2$

Matsui, Sato, Lee, PRC 72 (2005)

CTEQ06 vs. SL model

Nachtmann variable:

$$\xi = \frac{2x}{1 + (1 + 4x^2 m_N^2 / Q^2)^{1/2}}$$

QH duality holds for isoscalar target
Quark-hadron duality

Electron-nucleus scattering

Lalakulich et al, PRC 79 (2009)

Broadening of resonance peaks ➞ Fermi motion, many-body correlation, FSI
Quark-hadron duality

**HOPE**: Utilize QH duality practically

- PDF ↔ Structure functions in resonance region

  - **Electron scattering**: Res SF → PDF (in kinematics where data are scarce)
  
  - **Neutrino scattering**: PDF → axial form factor of $\mathcal{A}_N$*

  - $1\pi \& 2\pi$ model is necessary → DCC model can do

**Quantitative (theoretical) understanding of QH duality is prerequisite**
Sometimes, QH duality could do a very bad job

Comparison of SL model and Wandzura-Wilczek formula for electron scattering

\[ Q^2 = 0.21 \text{ (GeV/c)}^2 \]

Matsui, Sato, Lee, PRC 72 (2005)
Quark-hadron duality

**HOPE**: Utilize QH duality practically

PDF ↔ Structure functions in resonance region

- **Electron scattering**: Res SF → PDF (in kinematics where data are scarce)
- **Neutrino scattering**: PDF → axial form factor of $ANN^*$

$1\pi \& 2\pi$ model is necessary → DCC model can do

*Quantitative (theoretical) understanding of QH duality is prerequisite*

Can be (has been ?) studied in electron scattering

critical comments are welcome
Neutrino-nucleus interaction
Neutrino-nucleus interaction

To be done with DCC model

Our previous experience with Sato-Lee model PRC 54 2660 (1996)

- $\pi N$ channel only
- $\Delta$ only

✔ Incoherent 1 $\pi$ production
✔ Coherent 1 $\pi$ production

Difficult remaining issues

- Final state interactions
- Medium effects (width broadening of resonances)
Sato-Lee (SL) model

- $\pi N$ channel only
- $\Delta$ only

Sato and Lee, PRC 54 2660 (1996)

ANL data
Sato-Lee (SL) model

- $\pi N$ channel only
- $\Delta$ only

Sato and Lee, PRC 54 2660 (1996)

ANL data
Inclusive $\nu$-A scattering in $\Delta$ region

- Quasi-elastic (QE)
- $\Delta$-excitation ($1\pi$)

\[
\frac{d\sigma}{dE_\ell d\Omega_\ell} = \frac{p_\ell}{p_\nu} \frac{G_F^2 \cos \theta_C^2}{8\pi^2} L_{\mu\nu} W^{\mu\nu}
\]

$L_{\mu\nu}$ : Leptonic tensor

$W^{\mu\nu}$ : Hadronic tensor
\[ W^{\mu \nu} \sim \int d\vec{p}' \, d\vec{k} \, d\vec{p} \, \theta(p_F - |\vec{p}'|) \, \theta(|\vec{p}| - p_F) \]

\[ \times \quad \Lambda^{\mu \nu'} \langle \pi N(p') | j_{\mu'} | N(p) \rangle_{\pi N - cm} \] \leftarrow \text{Fermi Gas}

\[ \times \quad \Lambda^{\nu \nu'} \langle \pi N(p') | j_{\nu'} | N(p) \rangle^*_{\pi N - cm} \] \leftarrow \text{SL}

* Fermi Gas to Spectral Function

Benhar et al. NPA 579 493 (1994)

\[ \frac{3}{4\pi p_F^3} \int d\vec{p} \, \theta(p_F - |\vec{p}|) \rightarrow \int d\vec{p} \, dE \, P(\vec{p}, E) \]
Nuclear Effect for $1\pi$ in $\Delta$-region

$$\nu_e + ^{12}\text{C} \rightarrow e^- + \pi + X$$

$E_\nu = 1\ \text{GeV}$

![Graph showing $A^{-1}d^2\sigma/dQdE$ for different models.](image-url)
Non-resonant effect for $\pi$

$$\nu_e + ^{12}\text{C} \rightarrow e^- + \pi + X$$

$E_\nu = 1 \text{ GeV}$
Confronting with Data (Inclusive)

\[ \nu_e + {}^{12}\text{C} \rightarrow e^- + X \quad (E_{\nu} = 1 \text{ GeV}) \]

\[ e^- + {}^{12}\text{C} \rightarrow e^- + X \quad (E_e = 1.1 \text{ GeV}) \]

* Dip region: Beyond impulse approximation, another nuclear correlation

* Higher \( W \): Higher mass resonances  
  (To be done with DCC model)
Another big issue

- Final state interaction
  - transport model
  - optical potential

- Medium effects on resonance properties (shift of mass, width)
  - pion-nucleus scattering data
Coherent $\pi$ production in $\nu$-A scattering in $\Delta$ region

Nakamura et al., PRC 81 035502 (2010)

Motivation

- NC $\pi^0$ can fake $(\nu_\mu \rightarrow \nu_e)$ event
- Puzzling experimental situation

No evidence for CC K2K (2005), SciBooNE (2008)

Signature for NC MiniBooNE

Naive expectation from isospin matrix element: $\sigma_{CC} \sim 2 \sigma_{NC}$
Theoretical approaches to coherent $\pi$ production

* **PCAC** (Partially Conserved Axial Current)-based model
  - Rein, Sehgal, NPB 223 (1983)
  - Kartavtsev et al., PRD 74 (2006)
  - Paschos, Schalla, PRD 80 (2009)

* **Dynamical microscopic model**
Dynamical model for coherent production

Nakamura et al, PRC 81 (2010)

Ingredients

* Elementary amplitudes (\(\nu N \rightarrow \mu^- \pi^+ N\), \(\nu N \rightarrow \nu \pi^0 N\)) [SL model]

* Medium effect on \(\Delta\) (mass, width, non-locality) [\(\Delta\)-hole model]

* Final state interaction
Medium effect on $\Delta$

\[
\frac{1}{E - m_\Delta^0 - \Sigma_\Delta} \quad \Rightarrow \quad \frac{1}{E - m_\Delta^0 - \Sigma_\Delta - H_\Delta - \Sigma_{Pauli} - \Sigma_{spr}}
\]

\[
H_\Delta = T_\Delta + V_\Delta + H_{A-1} , \quad T_\Delta \Rightarrow \text{non-local effect}
\]

Spreading potential \[
\Sigma_{spr} = V_C \rho(r) + V_{LS}(r) \vec{L}_\Delta \cdot \vec{S}_\Delta
\]

Parameters (complex) : $V_C, V_{LS} \quad \rightarrow \quad \pi$-nucleus (total & elastic) scattering data
Final State Interaction

\[ \pi N \text{ t-matrix} \quad (\text{SL model}) \]

\[ \pi \rightarrow \pi \quad = \quad \text{non-res} \quad + \quad \text{dressed} \Delta \]

\[ \pi A \text{ potential} \]

\[ \pi \rightarrow \pi \quad = \quad \text{non-res} \quad + \quad \text{dressed} \Delta \]

\[ \pi A \text{ t-matrix} \quad (\text{Lippmann-Schwinger equation}) \]

\[ \pi \rightarrow \pi \quad = \quad \text{non-res} \quad + \quad \text{dressed} \Delta \quad + \ldots \]
Final State Interaction

$\pi^\pm - ^{12}\text{C}$ scattering

[Data : NPB 17, 168 (1970), PRC 29, 561 (1984)]

\[\sigma_{\text{tot}} \text{ [mb]}\]

\[d\sigma/d\Omega_{\pi} \text{ [mb/sr]}\]

\[\pi^+ - ^{12}\text{C} (T_\pi = 80 \text{ MeV})\]

\[\pi^- - ^{12}\text{C} (T_\pi = 180 \text{ MeV})\]

\[\pi^- - ^{12}\text{C} (T_\pi = 280 \text{ MeV})\]
Coherent $\pi$ production

$\pi A$ potential

$\gamma A \rightarrow \pi A$ potential

$\gamma A \rightarrow \pi A$ amplitude
Photo coherent $\pi$ production on $^{12}$C

$E_{\gamma}^{\text{CM}} = 290$ MeV

- Parameter-free prediction
- Good testing ground for microscopic models
- Important medium effects

[Data: Krusche et al., PLB526 (2002)]
CC Coherent $\pi$ production on $^{12}$C

- Large medium effects in $\Delta$ region
- Enhancement due to non-resonance (interference with $\Delta$)
  32 (10) % at $E_\nu = 0.5$ (1) GeV
- No contribution from (tree-level) non-resonant mechanism

Amaro et al., PRD 79 (2009); Hernandez et al., PRD 82 (2010)
Non-locality of $\Delta$-propagation

Leitner et al., PRC 79, 057601 (2009)

- 60, 30, 20 % reduction for $E_\nu = 0.5, 1, 1.5$ GeV for free $\Delta$ and Full
- Non-local effect is still important after including medium effects
- All previous microscopic calculations used local approximation
Comparison with Data

Large discrepancy between data and theory in $\frac{\sigma(\text{CC}^{\pi^+})}{\sigma(\text{NC}^{\pi^0})}$

$\frac{\sigma(\text{CC}^{\pi^+})}{\sigma(\text{NC}^{\pi^0})} = 0.14^{+0.30}_{-0.28}$  \hspace{1cm} \text{Kurimoto et al., PRD 81 (2010)}$

$\frac{\sigma(\text{CC}^{\pi^+})}{\sigma(\text{NC}^{\pi^0})} = 1 \sim 2$  \hspace{1cm} \text{all theoretical calculations}$
Data analysis of coherent $\text{NC}_{\pi^0}$ with Rein-Sehgal model

$$\eta \equiv E_\pi (1 - \cos \theta_\pi)$$

- $\eta$ is useful to break degeneracy of several pion productions in data

- Discrepancy between Monte Carlo (RS model) and microscopic models
  Amaro et al., PRD 79 (2009); Hernandez et al., PRD 80 (2009); 82 (2010)
  Nakamura et al, PRC 81 (2010)

$\Rightarrow$ possible overestimation of NC cross section
Summary

★ Dynamical coupled-channels (DCC) model for

\[ \gamma N, \pi N \Rightarrow \pi N, \pi \pi N, \eta N, K\Lambda, K\Sigma \]

extended to \( \nu N \Rightarrow \pi N, \pi \pi N, \eta N, K\Lambda, K\Sigma \) (ongoing)

★ Matching with DIS and low-\( Q^2 \) regions

   cf. quark-hadron duality

★ \( \nu \)–nucleus interaction

   SL model-based calculation for (in)coherent \( \pi \) production
Questions/comments

- DCC model for $\nu N \rightarrow \pi N, \pi \pi N, \eta N, K\Lambda, K\Sigma$ being developed (done for $Q^2=0$) first serious $2\pi$ production model ($1\pi$ and $2\pi$ are comparable in resonance region)
- Rein-Sehgal model needs to be improved (replaced)
- New deuterium experiment is highly hoped
deuteron reaction model being developed
- NC photon emission in higher resonance region relevant?
- Matching with DIS (low-$Q^2$) needs all coupled-channels $\Rightarrow$ DCC model can do this
- Is making use of QH duality a promising direction to fix axial form factors?
- Nuclear effects are another difficult problem