FINAL-STATE INTERACTIONS IN QUASIELASTIC ELECTRON AND NEUTRINO-NUCLEUS SCATTERING: THE RELATIVISTIC GREEN’S FUNCTION MODEL

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Neutrino-Nucleus Interactions for Current and Next Generation Neutrino Oscillation Experiment

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nuclear response to the electroweak probe

\[ \omega \approx \frac{q^2}{2m} \]
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QE-peak dominated by one-nucleon knockout
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QE e-nucleus scattering

\[ e + A \rightarrow e' + N + (A - 1) \]
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- both \( e' \) and \( N \) detected one-nucleon-knockout \((e,e'p)\)
- \((A-1)\) is a discrete eigenstate in exclusive \((e,e'p)\)
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- only \( e' \) detected inclusive \((e,e')\)
**QE e-nucleus scattering**

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- both \( e' \) and \( N \) detected **one-nucleon knockout** \( (e,e'p) \)
- \( (A-1) \) is a discrete eigenstate **exclusive** \( (e,e'p) \)
- only \( e' \) detected **inclusive** \( (e,e') \)

**QE \( \nu \)-nucleus scattering**

\[ \nu_l(\bar{\nu}_l) + A \rightarrow R \nu_l(\bar{\nu}_l) + N + (A - 1) \]  

\[ \nu_l(\bar{\nu}_l) + A \rightarrow l^{-}(l^{+}) + N + (A - 1) \]

**NC**

**CC**
QE e-nucleus scattering

\[ e + A \rightarrow e' + N + (A - 1) \]

- both \( e' \) and \( N \) detected one-nucleon knockout \((e,e'p)\)
- \((A-1)\) is a discrete eigenstate n exclusive \((e,e'p)\)
- only \( e' \) detected inclusive \((e,e')\)

QE \( \nu \)-nucleus scattering

\[ \nu_l(\bar{\nu}_l) + A \rightarrow \nu_l(\bar{\nu}_l) + N + (A - 1) \quad \text{NC} \]

\[ \nu_l(\bar{\nu}_l) + A \rightarrow l^-(l^+) + N + (A - 1) \quad \text{CC} \]

- only \( N \) detected semi-inclusive NC and CC
**QE e-nucleus scattering**

\[ e + A \rightarrow e' + N + (A - 1) \]

- both e' and N detected one-nucleon knockout (e,e'p)
- (A-1) is a discrete eigenstate of exclusive (e,e'p)
- only e' detected inclusive (e,e')

**QE ν-nucleus scattering**

\[ \nu_l(\bar{\nu}_l) + A \rightarrow \nu_l(\bar{\nu}_l) + N + (A - 1) \]  
\[ \nu_l(\bar{\nu}_l) + A \rightarrow l^- (l^+) + N + (A - 1) \]  

- only N detected semi-inclusive NC and CC
- only final lepton detected inclusive CC
one-boson exchange

electron scattering

PVES

neutrino scattering

NC

CC
one-boson exchange

\[ \sigma = K L^{\mu\nu} W_{\mu\nu} \]
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kin factor
The lepton tensor contains lepton kinematics.

$$\sigma = K L_{\mu \nu} W_{\mu \nu}$$
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\[ W^{\mu\nu} = \sum_{i,f} J^{\mu}(q) J^{\nu*}(q) \delta(E_i + \omega - E_f) \]

\[ J^{\mu}(q) = \langle f | \hat{J}^{\mu}(q) | i \rangle \]
\[ \sigma = K L^{\mu \nu} W_{\mu \nu} \]

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Direct knockout DWIA \((e,e'p)\)

- **exclusive reaction**: \(n\)
- **DKO mechanism**: the probe interacts through a one-body current with one nucleon which is then emitted, the remaining nucleons are spectators.

\[
|i\rangle \rightarrow |\omega, q\rangle \rightarrow |p\rangle, |n\rangle \rightarrow |f\rangle
\]
Direct knockout DWIA \((e,e'p)\)

- **exclusive reaction**: \(n\)
- **DKO mechanism**: the probe interacts through a one-body current with one nucleon which is then emitted; the remaining nucleons are spectators

\[
\langle f \mid J^\mu(q) \mid i \rangle \rightarrow \lambda_n^{1/2} \langle \chi_{p}^{(-)} \mid j^\mu(q) \mid \phi_n \rangle
\]
Direct knockout DWIA (e,e′p)

\[ \lambda_n^{1/2} \langle \chi^{(-)} | j^\mu | \phi_n \rangle \]

- \( j^\mu \) one-body nuclear current
- \( \chi^{(-)} \) s.p. scattering w.f. \( H^{+}(\omega+E_m) \)
- \( \phi_n \) s.p. bound state overlap function \( H(-E_m) \)
- \( \lambda_n \) spectroscopic factor
- \( \chi^{(-)} \) and \( \phi \) consistently derived as eigenfunctions of a Feshbach optical model Hamiltonian

\[ \mathcal{H}(E) = PHP + PHQ \frac{1}{E - QHQ + i\eta} QHP \]
Direct knockout DWIA \((e,e'p)\)

in the calculations

- phenomenological ingredients usually adopted
- \(\chi^{(-)}\) phenomenological optical potential
- \(\phi_n\) phenomenological s.p. wave functions
- \(\lambda_n\) extracted in comparison with data: reduction factor applied to the calculated c.s. to reproduce the magnitude of the experimental c.s.
Direct knockout DWIA (e,e’p)

in the calculations

- phenomenological ingredients usually adopted
- $\chi^{(-)}$ phenomenological optical potential
- $\phi_n$ phenomenological s.p. wave functions
- $\lambda_n$ extracted in comparison with data: reduction factor applied to the calculated c.s. to reproduce the magnitude of the experimental c.s.

both DWIA and RDWIA give an excellent description of (e,e’p) data in a wide range of nuclei and in different kinematics
only scattered electron detected
all final nuclear states are included
in the QE region the main contribution is given by the interaction on single nucleons and direct one-nucleon emission
**INCLUSIVE SCATTERING : IMPULSE APPROXIMATION**

- **IA**: c.s given by the sum of integrated direct one-nucleon emission over all the nucleons

- **IPSM**: $\sum_n$ over all occupied states in the SM,
INCLUSIVE SCATTERING : IMPULSE APPROXIMATION

- IA : c.s given by the sum of integrated direct one-nucleon emission over all the nucleons

- IPSM : \( \sum_n \) over all occupied states in the SM, (simple and conceptually clear model to include all the nucleons)
**INCLUSIVE SCATTERING : IMPULSE APPROXIMATION**

- **IA**: c.s given by the sum of integrated direct one-nucleon emission over all the nucleons

- **IPSM**: $\sum_n$ over all occupied states in the SM, (simple and conceptually clear model to include all the nucleons)

- **FSI**...?
INCLUSIVE SCATTERING: FSI

RDWIA: sum of 1NKO where FSI are described by a complex OP with an imaginary absorptive part does not conserve the flux.
INCLUSIVE SCATTERING: FSI

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RPWIA: FSI neglected
INCLUSIVE SCATTERING: FSI

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RPWIA
FSI neglected

REAL POTENTIAL
INCLUSIVE SCATTERING: FSI

RDWIA

sum of 1NKO where FSI are described by a complex OP with an imaginary absorptive part does not conserve the flux

RPWIA

FSI neglected

REAL POTENTIAL

rROP

only the real part of the OP: conserves the flux but it is conceptually wrong
INCLUSIVE SCATTERING: FSI

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RMF

RELATIVISTIC MEAN FIELD: same real energy-independent potential of bound states

Orthogonalization, fulfills dispersion relations and maintains the continuity equation
INCLUSIVE SCATTERING: FSI

RDWIA
sum of 1NKO where FSI are described by a complex OP with an imaginary absorptive part does not conserve the flux

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RELATIVISTIC MEAN FIELD: same real energy-independent potential of bound states
Orthogonalization, fulfills dispersion relations and maintains the continuity equation

RGF
GREEN’S FUNCTION complex OP conserves the flux consistent description of FSI in exclusive and inclusive QE electron scattering
FSI for the inclusive scattering: Green's Function Model

\((e,e')\) nonrelativistic


\((e,e')\) relativistic


CC relativistic


A. Meucci, J.A. Caballero, C. Giusti, J.M. Udias PRC (2011) 83 064614 (RGF-RMF)

A. Meucci, C. Giusti, M. Vorabbi, PRD 88 (2013) 013006

comparison with MiniBooNE data


A. Meucci, C. Giusti, F.D. Pacati PRD (2011) 84 113003

A. Meucci, C. Giusti, PRD (2012) 85 093002

the components of the inclusive response are expressed in terms of the Green's function the full A-body propagator

with suitable approximations can be written in terms of the s.p. optical model Green's function

the explicit calculation of the s.p. Green's function can be avoided by its spectral representation which is based on a biorthogonal expansion in terms of the eigenfunctions of the non Herm optical potential V and V+ matrix elements similar to RDWIA

scattering states eigenfunctions of V and V+ (absorption and gain of flux): the imaginary part redistributes the flux and the total flux is conserved

consistent treatment of FSI in the exclusive and in the inclusive scattering
FSI for the inclusive scattering: Green’s Function Model

\[ W^{\mu\nu}(\omega, q) = \sum_n \left[ \text{Re} T_{n}^{\mu\nu}(E_f - \varepsilon_n, E_f - \varepsilon_n) - \frac{1}{\pi} \mathcal{P} \int_{\mathcal{M}}^{\infty} d\varepsilon E_f - \varepsilon_n - \varepsilon \text{Im} T_{n}^{\mu\nu}(\varepsilon, E_f - \varepsilon_n) \right] \]
FSI for the inclusive scattering: Green's Function Model

\[ W_{\mu\mu}(\omega, q) = \sum_n \left[ \text{Re} T_{n}^{\mu\mu}(E_f - \varepsilon_n, E_f - \varepsilon_n) - \frac{1}{\pi} \mathcal{P} \int_{M}^{\infty} \frac{d\xi}{E_f - \varepsilon_n - \xi} \text{Im} T_{n}^{\mu\mu}(\xi, E_f - \varepsilon_n) \right] \]

\[ T_{n}^{\mu\mu}(\xi, E) = \lambda_n \langle \varphi_n | j^{\dagger}(q) \sqrt{1 - V'(E)} | \chi_{\xi}^{(-)}(E) \rangle \langle \chi_{\xi}^{(-)}(E) | \sqrt{1 - V'(E)} j^{\mu}(q) | \varphi_n \rangle \]
FSI for the inclusive scattering: Green’s Function Model

\[ W_{\mu\mu}(\omega, q) = \sum_n \left[ \text{Re} T_{n}^{\mu\mu}(E_f - \varepsilon_n, E_f - \varepsilon_n) - \frac{1}{\pi} \mathcal{P} \int_{M}^{\infty} d\varepsilon \frac{1}{E_f - \varepsilon_n - \varepsilon} \text{Im} T_{n}^{\mu\mu}(\varepsilon, E_f - \varepsilon_n) \right] \]

\[ T_{n}^{\mu\mu}(\varepsilon, E) = \lambda_n \langle \varphi_n | j^{\mu\dagger}(q) \sqrt{1 - \mathcal{V}(E)} \chi_{\varepsilon}^{(-)}(E) \rangle \langle \chi_{\varepsilon}^{(-)}(E) | \sqrt{1 - \mathcal{V}(E)} j^{\mu}(q) | \varphi_n \rangle \]
FSI for the inclusive scattering: Green's Function Model

\[ W^{\mu\mu}(\omega, q) = \sum_n \left[ \text{Re} T^{\mu\mu}_n(E_f - \varepsilon_n, E_f - \varepsilon_n) - \frac{1}{\pi} \mathcal{P} \int_M^{\infty} d\varepsilon \frac{1}{E_f - \varepsilon_n - \varepsilon} \text{Im} T^{\mu\mu}_n(\varepsilon, E_f - \varepsilon_n) \right] \]

\[ T^{\mu\mu}_n(\varepsilon, E) = \lambda_n \langle \varphi_n | j^{\mu\dagger}(q) \sqrt{1 - \mathcal{V}'(E)} \chi_{E}^{(-)}(E) \rangle \langle \chi_{E}^{(-)}(E) | \sqrt{1 - \mathcal{V}'(E)} j^\mu(q) | \varphi_n \rangle \]

interference between different channels
FSI for the inclusive scattering: Green's Function Model

\[ W^{\mu\nu}(\omega, q) = \sum_n \left[ \text{Re} T^{\mu\nu}_n(E_f - \varepsilon_n, E_f - \varepsilon_n) - \frac{1}{\pi} \mathcal{P} \int_M^\infty d\varepsilon \frac{1}{E_f - \varepsilon_n - \varepsilon} \text{Im} T^{\mu\nu}_n(\varepsilon, E_f - \varepsilon_n) \right] \]

\[ T^{\mu\nu}_n(\varepsilon, E) = \lambda_n \langle \varphi_n | j^{\mu\dagger}(q) \sqrt{1 - V'(E)} \mid \chi^{(-)}_\varepsilon(E) \rangle \langle \chi^{(-)}_\varepsilon(E) \mid \sqrt{1 - V'(E)} j^\mu(q) \mid \varphi_n \rangle \]
FSI for the inclusive scattering: Green’s Function Model

\[ W^{\mu\mu}(\omega, q) = \sum_n \left[ \text{Re} T^{\mu\mu}_n(E_f - \varepsilon_n, E_f - \varepsilon_n) - \frac{1}{\pi} \mathcal{P} \int_M d\varepsilon \frac{1}{E_f - \varepsilon_n - \varepsilon} \text{Im} T^{\mu\mu}_n(\varepsilon, E_f - \varepsilon_n) \right] \]

\[ T^{\mu\mu}_n(\varepsilon, E) = \lambda_n \langle \varphi_n | j^{\mu\dagger}(q) \sqrt{1 - \mathcal{V}'(E)} | \chi^{(-)}_\varepsilon(E) \rangle \langle \chi^{(-)}_\varepsilon(E) | \sqrt{1 - \mathcal{V}'(E)} j^{\mu}(q) | \varphi_n \rangle \]

eigenfunctions of \( V \) and \( V^+ \)
FSI for the inclusive scattering: Green’s Function Model

\[
W^{\mu\nu}(\omega, q) = \sum_n \left[ \text{Re} T_n^{\mu\nu}(E_f - \varepsilon_n, E_f - \varepsilon_n) - \frac{1}{\pi} \mathcal{P} \int_M^{\infty} d\mathcal{E} \frac{1}{E_f - \varepsilon_n - \mathcal{E}} \text{Im} T_n^{\mu\nu}(\mathcal{E}, E_f - \varepsilon_n) \right]
\]

\[
T_n^{\mu\nu}(\mathcal{E}, E) = \lambda_n \langle \varphi_n | j^{\mu\dagger}(q) \sqrt{1 - \mathcal{V}'(E)} | \chi^{(-)}_\mathcal{E}(E) \rangle \langle \chi^{(-)}_\mathcal{E}(E) | \sqrt{1 - \mathcal{V}'(E)} j^{\mu}(q) | \varphi_n \rangle
\]

loss of flux
FSI for the inclusive scattering: Green's Function Model

\[ W^{\mu\mu}(\omega, q) = \sum_n \left[ \text{Re} T_n^{\mu\mu}(E_f - \varepsilon_n, E_f - \varepsilon_n) - \frac{1}{\pi} \mathcal{P} \int_M^{\infty} d\mathcal{E} \frac{1}{E_f - \varepsilon_n - \mathcal{E}} \text{Im} T_n^{\mu\mu}(\mathcal{E}, E_f - \varepsilon_n) \right] \]

\[ T_n^{\mu\mu}(\mathcal{E}, E) = \lambda_n \langle \varphi_n | j^{\mu\dagger}(q) \sqrt{1 - \mathcal{V}'(E)} | \chi^{(-)}_{\mathcal{E}}(E) \rangle \langle \chi^{(-)}_{\mathcal{E}}(E) | \sqrt{1 - \mathcal{V}'(E)} j^{\mu}(q) | \varphi_n \rangle \]

gain of flux

loss of flux
FSI for the inclusive scattering: Green’s Function Model

\[ W^{\mu\mu}(\omega, q) = \sum_n \left[ \text{Re} T_n^{\mu\mu}(E_f - \varepsilon_n, E_f - \varepsilon_n) - \frac{1}{\pi} \mathcal{P} \int_M^{\infty} d\varepsilon \frac{1}{E_f - \varepsilon_n - \varepsilon} \text{Im} T_n^{\mu\mu}(\varepsilon, E_f - \varepsilon_n) \right] \]

\[ T_n^{\mu\mu}(\varepsilon, E) = \lambda_n \langle \varphi_n | j^{\mu\dagger}(q) \sqrt{1 - \mathcal{V}'(E)} | \chi_\varepsilon^-(E) \rangle \langle \chi_\varepsilon^-(E) | \sqrt{1 - \mathcal{V}'(E)} j^{\mu}(q) | \varphi_n \rangle \]

*gain of flux*  
*loss of flux*

Flux redistributed and conserved

The imaginary part of the optical potential is responsible for the redistribution of the flux among the different channels.
FSI for the inclusive scattering: 
Green's Function Model

\[ W^{\mu\nu}(\omega, q) = \sum_n \left[ \text{Re} T^{\mu\nu}_n(E_f - \varepsilon_n, E_f - \varepsilon_n) - \frac{1}{\pi} \operatorname{P} \int_M^\infty d\varepsilon \frac{1}{E_f - \varepsilon_n - \varepsilon} \text{Im} T^{\mu\nu}_n(\varepsilon, E_f - \varepsilon_n) \right] \]

\[ T^{\mu\nu}_n(\varepsilon, E) = \lambda_n \langle \varphi_n | j^{\mu\dagger}(q) \sqrt{1 - V'(E)} | \chi_\varepsilon^(-)(E) \rangle \langle \chi_\varepsilon^(-)(E) | \sqrt{1 - V'(E)} j^{\nu}(q) | \varphi_n \rangle \]

Gain of flux
Loss of flux

For a real optical potential \( V=V^* \) the second term vanishes and the nuclear response is given by the sum of all the integrated one-nucleon knockout processes (without absorption).
\[ ^{16}\text{O}(e,e') \]

Data from Frascati NPA 602 405 (1996)
\[ ^{16}\text{O}(e,e') \]

Data from Frascati NPA 602 405 (1996)
(e, $e'$)

$E_0 = 1080 \text{ MeV} \quad \vartheta = 32^\circ$

$E_0 = 841 \text{ MeV} \quad \vartheta = 45.5^\circ$

$E_0 = 2020 \text{ MeV} \quad \vartheta = 20^\circ$
Comparison of relativistic models

$^{12}$C(e,e')

$E_0 = 1$ GeV

q = 500 MeV/c

$q = 1000$ MeV/c

FSI

RPWIA

rROP

RGF1

RGF2

RMF

\( ^{12}\text{C}(e,e') \)

Comparison of relativistic models

\( E_0 = 1 \text{ GeV} \)

- \( q=500 \text{ MeV}/c \)
  - \( \cdots \cdots \cdots \cdots \) RPWIA
  - - - - - rROP
  - - - - - - - - RGF1
  - - - - - - - - - RGF2
  - - - - - - - - - - - RMF

- \( q=1000 \text{ MeV}/c \)
  - - - - - - - - - - - EDAD1
  - - - - - - - - - - - - - - EDAD2

A. Meucci, J.A. Caballero, C. Giusti, F.D. Pacati, J.M. Udias

PRC 80 (2009) 024605
Comparison of relativistic models

$^{12}\text{C}(e,e')$

$E_0 = 1 \text{ GeV}$

$q = 500 \text{ MeV/c}$

$q = 1000 \text{ MeV/c}$


comparison of relativistic models

$q = 500$ MeV/c

$q = 800$ MeV/c

$q = 1000$ MeV/c

FSI

RGF1

RGF2

RMF

$^{12}$C(e,e') comparison of relativistic models

FSI

differences increase with q

q=500 MeV/c

q=800 MeV/c

q=1000 MeV/c

RGF1

RGF2

RMF
QE SCALING FUNCTION: RGF, RMF

A. Meucci, J.A. Caballero, C. Giusti, F.D. Pacati, J.M. Udias PRC 83 (2011) 064614
\( ^{12}C(\nu_\mu, \mu^-) \)

Comparison of relativistic models

FSI

- RPWIA
- rROP
- RGF1
- RGF-EDAI
- RMF

Electron and neutrino scattering: different results

A. Meucci, J.A. Caballero, C. Giusti, F.D. Pacati, J.M. Udias PRC 83 (2011) 064614
$^{12}C(\nu_\mu, \mu^-)$

Comparison of relativistic models

$E_\nu = 1000$ MeV
$q = 500$ MeV/c

GF1
GF-EDAI
RMF
rROP
RPWIA

$E_\nu = 1000$ MeV
$q = 1000$ MeV/c

EDAI $A$-independent for $^{12}C$
DIFFERENT DESCRIPTIONS OF FSI

RMF

- real energy-independent MF reproduces nuclear saturation properties, purely nucleonic contribution, no information from scattering reactions explicitly incorporated

RGF

- complex energy-dependent phen. ROP fitted to elastic p-A scattering, incorporates information from scattering reactions
- the imaginary part includes the overall effect of inelastic channels not included in other models based on the IA, (multinucleon, rescattering, non nucleonic).
- Contributions of inelastic channels not included microscopically but recovered in the model by the Im part of the ROP, not univocally determined only from elastic phenomenology
- different ROP reproduce elastic p-A scatt. can give different predictions for non elastic observables
Comparison RMF-RGF deeper understanding of nuclear effects (FSI) which may play a crucial role in the analysis of MiniBooNE data, which may receive important contributions from non-nucleonic excitations and multi-nucleon processes.
Comparison with MiniBooNe CCQE data


\[ \nu_\mu + ^{12}C \rightarrow \mu^- + X \]
Comparison with MiniBooNe CCQE data


$$\nu_\mu + ^{12}C \rightarrow \mu^- + X$$

Measured cross sections larger than the predictions of the RFG model and of other more sophisticated models.

Unusually large values of the nucleon axial mass must be used to reproduce the data (about 30% larger)
MiniBooNe CCQE data

flux integrated double differential cross section

$M_A = 1.35 \text{ GeV}$

flux unfolded $\nu_\mu$ CCQE cross section per neutron as a function of $E_{\nu}$ compared with predictions of a RFG model

A.A Aguilar-Arevalo et al. PRD PRC 81 (2010) 092005
A larger axial mass may be interpreted as an effective way to include medium effects not taken into account by the RFG model and by other models.

Before drawing conclusions all nuclear effects must be investigated.
Comparison with MiniBooNe CCQE data

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Before drawing conclusions all nuclear effects must be investigated.

FSI
Differences between Electron and Neutrino Scattering

**electron scattering**:
beam energy known, \( \omega \) and \( q \) known. cross section as a function of \( \omega \)

**neutrino scattering**:
axial current
beam energy and \( \omega \) not known

calculations over the energy range relevant for the neutrino flux

the flux-average procedure can include contributions from different kinematic regions where the neutrino flux has significant strength, contributions other than 1-nucleon emission
Comparison with MiniBooNe CCQE data

\[ 0.4 < \cos \theta_\mu < 0.5 \]

\[ 0.7 < \cos \theta_\mu < 0.8 \]

\[ 0.8 < \cos \theta_\mu < 0.9 \]
Comparison with MiniBooNe CCQE data

\[ 0.4 < \cos \theta_\mu < 0.5 \]
Comparison with MiniBooNe CCQE data
CCQE antineutrino-nucleus scattering

The MiniBooNE collaboration has measured CCQE $\bar{\nu}$ events

In the calculations vector-axial response constructive in neutrino scattering destructive in antineutrino scattering with respect to L and T responses

$\bar{\nu}_\mu$ flux smaller and with lower average energy than $\nu_\mu$ flux
CCQE antineutrino scattering

\[ ^{12}C(\bar{\nu}_\mu, \mu^+) \]

- **RPWIA**
- **rROP**
- **RGF EDAI**
- **RGF-EDAD1**

Graphs showing the distribution of \( T_\mu \) with different values of \( \cos \theta_\mu \): 0.85, 0.75, 0.45, and 0.15.
Comparison with MiniBooNE NCE data

Measurement of the flux averaged neutral-current elastic (NCE) differential cross section on CH$_2$ as a function of $Q^2$

PRD 82 092005 (2010)

The NCE cross section presented as scattering from individual nucleons and consists of 3 different processes: scattering of free protons in H, bound protons and neutrons in C
only the outgoing nucleon is detected: semi-inclusive scattering

FSI?

RDWIA: sum of all integrated exclusive 1NKO channels with absorptive imaginary part of the ROP. The imaginary part accounts for the flux lost in each channel towards other inelastic channels. Some of these reaction channels are not included in the experimental cross section when one nucleon is detected. For these channels RDWIA is correct, but there are channels excluded by the RDWIA and included in the experimental c.s.

RGF recovers the flux lost to these channels but can include also contributions of channels not included in the semi-inclusive cross section.

we can expect RDWIA smaller and RGF larger than the experimental cross sections

relevance of contributions neglected in RDWIA and added in RGF depends on kinematics
Comparison with MiniBooNE NCE data
QE $\nu$-nucleus scattering

- Models developed for QE electron-nucleus scattering applied to QE neutrino-nucleus scattering
- RGF description of FSI in the inclusive scattering
- RGF enhances the c.s. and gives results able to reproduce the MiniBooNE data with the standard value of $M_A$
- Enhancement due to the translation to the inclusive strength of the overall effect of inelastic channels (multi-nucleon, non-nucleonic rescattering...)
- Inelastic contributions recovered in the RGF by the imaginary part of the ROP, not included explicitly in the model with a microscopic calculation, the role of different inelastic processes cannot be disentangled and we cannot attribute the enhancement to a particular effect
- Other models including multi-nucleonic excitations reproduce the MiniBooNE data
- Different models indicate... effects beyond IA
more data needed, comparison of the results of different models helpful for a deeper understanding, careful evaluation of all nuclear effects is required

reduce theoretical uncertainties

RGF better determination of the phenomenological ROP which closely fulfills dispersion relations

2-body MEC not included in the model would require a new model (two-particle GF)

everything should be done consistently in the model