Inclusive Electron- and Neutrino-Nucleus Scattering: Correlations and Currents

• Motivation
• Interactions and currents
• Review of electron scattering
  • Sum rules
  • Euclidean Response
• Electron/Neutrino Scattering from the Deuteron
• Sum Rules for A=12
• Near Future

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NUCLEI
Nuclear Computational Low-Energy Initiative
Response Functions probes of structure and dynamics

Extraction of condensate fraction in liquid He

Sears, et al, PRL, 1982

The 1994 Nobel Prize – Shull & Brockhouse
more examples:
High Tc
Cold Atomic Gases, ...

S(Q) and g(r) for Simple Liquids

Fig. 6. The energy spectrum of excitations. Curve A is the spectrum $E_s(k)$ computed from Eq. (61). Curve B is the spectrum $E_s(k)$ computed with the simpler wave function (5). Curve C is the Landau-type spectrum used by de-Klerk et al. to fit the second sound and specific heat data. Curve D is a Landau-type spectrum with $p_0$ taken the same as in A, and $\mu$ and $\Delta$ chosen to fit the specific heat data. For small $k$, all curves are asymptotic to the line $E = \hbar \omega$.}

phonon-roton spectrum in liquid He

Feynman PRL, 1952

Fig. 5.1 The structure factor $S(Q)$ for $^3$Ar at 85 K. The curve through the experimental points is obtained from a molecular dynamics calculation of Verlet based on a Lennard-Jones potential. (After Yarnell et al., 1973.)

Fig. 5.2 The pair-distribution function $g(r)$ obtained from the experimental results in Fig. 5.1. The mean number density is $\rho = 2.13 \times 10^{22}$ atoms m$^{-3}$. (After Yarnell et al., 1978.)
Scaling variables:
\[ \psi' \approx y/k \]
\[ F_{L,T} \text{ and } f_{L,T} \]

Data at variance with PWIA expectation that \[ f_L \approx f_T \]

Excess strength, especially for \( ^4\text{He} \), in transverse response

Single nucleon couplings factored out

Moments of order inverse internucleon spacing:

Large enhancement of transverse over longitudinal response

Requires beyond single nucleon physics
Longitudinal/Transverse separation in $^{12}$C

![Graph showing longitudinal and transverse responses with different $q$ values](image)

- $q=400$ MeV/c
- $q=500$ MeV/c
- $q=600$ MeV/c

from Benhar, Day, Sick, RMP 2008

data Finn, et al 1984
Nuclear Interactions:

AV18: excellent fit of NN data
  pion exchange plus phenomenology
TNI: Two-pion exchange plus three-pion-exchange
  plus phenomenological short-range repulsion

Chiral Interactions: LO, NLO, N2LO, N3LO
  increasing order results in better fits to data
  uncertainty estimates
  Consistency of two plus three nucleon interactions
  New local interactions at LO..N2LO

~ Gezerlis, et al.,
PRL 2013
**QMC methods**

Basic Idea: project specific low-lying states from initial guess (or source)

\[ \Psi_0 = \exp \left[ -H \tau \right] \Psi_T \]

Use Feynman path integrals to compute propagator

\[ \exp \left[ -H \tau \right] = \prod \exp \left[ -H \delta \tau \right] \]

\[ \exp \left[ -H \delta \tau \right] \approx \exp \left[ -T \delta \tau \right] \exp \left[ -V \delta \tau \right] \]

diffusion branching

Applications: condensed matter (Helium, electronic systems, ...)
nuclear physics (light nuclei, neutron matter, SMMC...)
atomic physics (cold atoms,...)

Various formulations: DMC/GFMC, AFMC, AFDMC, Lattice
**GFMC Algorithm:**

Branching random walk in 3A (36 for $^{12}$C) dimensions

Asynchronous Dynamic Load Balancing (ADLB) Library

Each step moves A particles and updates

\[ 2^A \times \left( \frac{A}{Z} \right) \text{ complex amplitudes (2 GB for $^{12}$C gs)} \]

significant linear algebra for each step
tuned by physicists and math/CS staff at ANL

Similar branching random walks with linear algebra
used in condensed matter physics (lattice calculations)

up to

≈2M threads

Other methods: NCSM, Coupled Cluster, ...
Spectra of Light Nuclei

Spectra must be correct to describe low-energy transitions, reactions, etc.
Carbon-12

Ground and Hoyle State

AV18 + IL7 interaction

Energy

RMS radius

$^{12}\text{C}(0^+)\text{; GFMC with AV18+IL7}$

$\tau$ (MeV$^{-1}$)

$E(\tau)$ (MeV)

$\langle r_p^2 \rangle^{1/2}$ (fm)

$^{12}\text{C} - 0^+$ states – AV18+IL7 – RMS $r(p)$ – 16 Jun 2011

$0^+$ excited state near triple-alpha threshold postulated by Fred Hoyle to explain nuclear abundances

Pieper, Carlson, Lusk, ... see also papers by D. Lee, Meissner, et al
$^{12}$C Electromagnetic Charge Form Factor

Results - Longitudinal form factor
- Experimental data are well reproduced by theory over the whole range of momentum transfers;
- Two-body terms become appreciable only for $q > 3$ fm$^{-1}$, where they interfere destructively with the one-body contributions bringing theory into closer agreement with experiment.

Small role for two-nucleon currents
Excellent agreement with data

Lovato, Gandolfi, Butler, Carlson, Lusk, Pieper, Schiavilla
PRL 2013
Ground State - Hoyle State
Transition form factor

\[ f_{pt}(k) \]

- \[ 12C - M(E0) - AV18+IL7 - \text{one-way orthog.} - f_{pt}(k) - 9 \text{ May 2013} \]

- Data from M. Chernykh  

- Right panel \[ f_{tr}(k)/k^2 \] proportional to \[ M(E_0) \] at \( k = 0 \)

- Large errors at small \( k \) due to large Monte Carlo errors

- Can get better value at \( k = 0 \) by computing \[ \rho_{dd} r^2 \rightarrow f_{tr}(r) \]

- Results with best 0\(^2\)+2\(^2\) wave function in good agreement with data
**Nuclear Electromagnetic Currents**

**LO**: \( eQ^{-2} \)

**NLO**: \( eQ^{-1} \)

**N^2LO**: \( eQ^0 \)

**N3LO** (eQ)

Coupling constants adjusted to
\( \mu(D) \), \( \mu_s(A=3) \): isoscalar
np capture, \( \mu_v(A=3) \): isovector


A ≤ 10 Magnetic Moments with Chiral EFT currents

Hybrid calculations using $A$ V18+IL7 wave functions and $\chi$EFT exchange currents developed in:

Pastore, Schiavilla, & Goity, PRC 78, 064002 (2008); Pastore, et al., PRC 80, 034004 (2009)

$\mu_n/\mu_N$ (n) n

$\mu_{n}$ (n)

-3 -2 -1 0 1 2 3 4

$p$ $^3_2$H $^6_3$Li $^6_2$Li* $^7_3$Li $^7_4$Be $^8_3$Li $^8_4$B $^9_3$Li $^9_4$Be $^9_5$B $^9_3$C $^{10}_3$B $^{10}_4$B*

GFMC(IA) GFMC(FULL) EXPT

\[ A \leq 9 \ M1 \ \text{TRANSITIONS \ with/} \ \chi\text{EFT \ exchange \ currents} \]

- dominant contribution is from OPE
- five LECs at N3LO
- \(d^V_2\) and \(d^V_1\) are fixed assuming \(\Delta\) resonance saturation
- \(d^S\) and \(c^S\) are fit to experimental \(\mu_c\) and \(\mu_S(3\text{H}/3\text{He})\)
- \(c^V\) is fit to experimental \(\mu_V(3\text{H}/3\text{He})\)
- \(\Lambda = 600\ \text{MeV}\)

Pastore, Pieper, Schiavilla & Wiringa

PRC 87, 035503 (2013)

Two-nucleon currents critical to understand low-energy transitions
Higher resolution: Momentum Distributions

higher momentum components dominated by two-nucleon physics

strength at ~ 2 fm\(^{-1}\) due to tensor correlations
Back-to-back pairs: pn vs pp,nn in $^{12}$C

JLAB, BNL
back-to-back pairs in $^{12}$C

np pairs dominate over nn and pp


http://www.phy.anl.gov/theory/research/momenta2/
Neutron-Proton pairs

Most pairs at high q have low CM Q

Proton-Proton pairs

np pairs dominate for $^4$He, $^{12}$C

$$\rho(p, q) = \langle 0 | \exp[iq \cdot (r - r')] \exp[iQ \cdot (R + R')] | 0 \rangle$$
pair momenta vs Q: pn vs pp, nn in $^4$He
Inclusive Scattering and Response Functions

\[ R_{L,T} (q, \omega) = \sum_{f} \delta(\omega + E_0 + E_f) \left| \left\langle f \mid \mathcal{O}_{L,T} \mid 0 \right\rangle \right|^2 \]

Knowledge of response \( \Leftrightarrow \) inclusive cross-sections requires knowledge of all final states.

Start with the deuteron, can enumerate all final states. 
Use for test of Monte Carlo codes 
Accurate predictions: could use to make absolute flux measurements
Electron Scattering on Deuterium

A few % increase due to two-body currents at the top of the QE peak in \( R_L \), much larger as \( \omega \) increases.

\[ R_L(q,\omega) = \text{MeV}^{-1} \]

\[ R_T(q,\omega) = \text{MeV}^{-1} \]

- 1-body
- (1+2)-body

\( q=300 \text{ MeV} \)
$\nu$-Deuteron Scattering up to GeV Energy

Shen et al. (2012)

\[ j_{NC}^\mu = -2 \sin^2 \theta_W \ j_{\gamma,S}^\mu + (1 - 2 \sin^2 \theta_W) \ j_{\gamma,z}^\mu + j_{z}^\mu \]

\[ j_{CC}^\mu = j_{\pm}^\mu + j_{\pm}^{5z} \quad j_{\pm} = j_x \pm i j_y \quad [T_a, j_{\gamma,z}^\mu] = i \epsilon_{azb} j_{b}^\mu (1) \]

\[ j_{CC}^\mu \] reproduces well known weak transitions in $A \leq 7$ nuclei and $\mu$-capture rates in $d$ and $^3$He [Schiavilla and Wiringa (2002); Marcucci et al. (2012)]
Deuterons: Neutral Current
Comparison of 1-body PW to isolated p + n and ratio

FIG. 16: (color online) The “model” (P+N) NC cross sections for neutrino and antineutrino are compared with plane-wave one-body (PW 1-body) results, see text for explanation. Inset: ratio of neutrino NC versus antineutrino NC cross section.
FIG. 17: (color online) Same as Fig. 16, but for CC cross sections.

Charged Current on Deuteron
Heavier Nuclei ($A>2$)

Easy to calculate Sum Rules: ground-state observable

$$S(q) = \int d\omega \ R(q, \omega) = \langle 0 | O^\dagger(q) \ O(q) | 0 \rangle$$

Sum Rules are independent of final states (and FSI)

$$E(q) = \int d\omega \ \omega \ R(q, \omega) = \langle 0 | O^\dagger(q) \ H O(q) | 0 \rangle$$

For spin-isospin independent interactions $E(q) = q^2/2m$

For nuclear physics $E(q) > q^2/2m$, not reproduced by spectral function alone
Longitudinal and Transverse Electromagnetic Response in A=3,4, 12

(e, e’) Inclusive Response: Scaling Analysis

Donnelly and Sick (1999)

Transverse / Longitudinal enhancement requires more than single-nucleon physics
Carbon-12 : Electron Scattering
Longitudinal Sum Rule

\[ S_L(q) = \langle 0 | \rho^\dagger(q) \rho(q) |0 \rangle \]

new Jlab experiment soon, also neutrino experiments again small role for two-nucleon currents
Transverse Sum Rule

Two-nucleon currents contribute \( \sim 50\% \) enhancement

Jlab experiments, neutrino experiments

Lovato, Gandolfi, Butler, Carlson, Lusk, Pieper, Schiavilla
PRL 2013
Sum Rules and Euclidean Response

Real-time response

\[ R(q, \omega) = \langle 0 | j^\dagger(q) | f \rangle \langle f | j(q) | 0 \rangle \delta(\omega - (E_f - E_0)) \]

\[ R(q, \omega) = \int dt \langle 0 | j^\dagger(q) \exp[iHt] j(q) | 0 \rangle \exp[i\omega t] \]

Short time ‘t’ : sum rules
Long time: higher energy resolution
No general method for strongly-correlated quantum systems, typically use model final states

Short-time theories well known - operator product expansion, ....
The static structure factor (density-response sum rule) as a function of $q$. The results have been obtained with the VMC, with and without long-range correlations in the Jastrow. Fermi momentum have been precisely determined theoretically and compared to experiment across the BCS-BEC crossover. Other properties related to the contact have also been computed, in particular the static structure factor, and the results are in very good agreement with experimental data. Future directions include dynamic response of the unitary Fermi gas, inhomogeneous systems and the transitions from three- to two-dimensions, and multi-component Fermi gases.

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References


Example: Unitary Fermi Gas

Cold Atoms
zero-range interaction
infinite scattering length

strength > 1

peaks at $q^2 / 2m; q^2 / 4m$
latter not reproducible with PWIA or spectral fn
Imaginary-time correlator (Euclidean Response)

\[
R(q, \tau) = \langle 0 | \hat{j}^\dagger(q) \exp[-H\tau] \hat{j}(q) | 0 \rangle
\]

Converts quantum dynamics to statistical mechanics
short time : sum rules (high energy)
long ‘time’ : low energy response (collective modes,...)
Why do FSI add to high-energy response? Longitudinal electron scattering

PWIA (or spectral function):
response tied to charge propagation
charge propagation charged to nucleon propagation
(momentum distribution)

Full Interacting system:
charge can propagate through pion exchange:
faster response (low ‘mass’) adds to high-energy tail
Towards (short-time) Dynamics: Euclidean Response

$^3$He and $^4$He Transverse Euclidean Response Functions

$^3$He Transverse

- Excess strength in quasielastic region ($\tau > 0.01$ MeV$^{-1}$)
- Larger in $A = 4$ than in $A = 3$, as already inferred from $S_T$
Neutrino Scattering:

\[
\left( \frac{d\sigma}{d\epsilon' d\Omega} \right)_{\nu/\bar{\nu}} = \frac{G_F^2}{2\pi^2} k' \epsilon' \cos^2 \frac{\theta}{2} \left[ R_{00} + \frac{\omega^2}{q^2} R_{zz} - \frac{\omega}{q} R_{0z} + \left( \tan^2 \frac{\theta}{2} + \frac{Q^2}{2q^2} \right) R_{xx} \mp \tan \frac{\theta}{2} \sqrt{\tan^2 \frac{\theta}{2} + \frac{Q^2}{q^2}} R_{xy} \right]
\]

\[
R_{00}(q, \omega) = \sum_i \sum_f \delta(\omega + m_A - E_f) \left| \langle f | j^0(q, \omega) | i \rangle \right|^2,
\]

\[
R_{zz}(q, \omega) = \sum_i \sum_f \delta(\omega + m_A - E_f) \left| \langle f | j^z(q, \omega) | i \rangle \right|^2,
\]

\[
R_{0z}(q, \omega) = \sum_i \sum_f \delta(\omega + m_A - E_f) \left[ \langle f | j^0(q, \omega) | i \rangle \right.
\times \langle f | j^z(q, \omega) | i \rangle^* + \text{c.c.} \left. \right],
\]

\[
R_{xx}(q, \omega) = \sum_i \sum_f \delta(\omega + m_A - E_f) \left[ \left| \langle f | j^x(q, \omega) | i \rangle \right|^2 + \left| \langle f | j^y(q, \omega) | i \rangle \right|^2 \right],
\]

\[
R_{xy}(q, \omega) = \sum_i \sum_f \delta(\omega + m_A - E_f) \left[ \langle f | j^x(q, \omega) | i \rangle \right.
\times \langle f | j^y(q, \omega) | i \rangle^* - \text{c.c.} \left. \right],
\]
Neutrino/Anti-neutrino Scattering
5 response functions
Neutral current sum rules for $^{12}$C
Present and Near Future

- Calculations of neutral and charged current scattering on the deuteron (neutrinos and anti-neutrinos) completed

- Codes for neutral current and nearly charged current completed for use in Quantum Monte Carlo calculations

- Calculations of Sum Rules (NC) completed

- Calculations for Euclidean response expected in ~ 1 year

- Studying quasi-analytic approaches to dynamic response in high q, omega region

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ANL devoting ~50-100M core-hours to this project plus staff/postdoc time
INCITE award to NUCLEI project amount largest in country
- neutrino scattering is an important goal
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