Short-range correlations studied with unitarily transformed interactions and operators

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Motivation

Short-range correlations and high-momentum components in wave functions
Wiringa, Schiavilla, Pieper, Carlson, PRC 85, 021001(R) (2008)

Interaction dependence – AV18 versus N3LO

SRC, high-momenta and unitary transformations
Anderson, Bogner, Furnstahl, Perry, PRC 82, 054001 (2010)
Short-range and tensor correlations

- strong repulsive core: nucleons can not get closer than \( \approx 0.5 \text{ fm} \) → central correlations
- strong dependence on the orientation of the spins due to the tensor force → tensor correlations
- the nuclear force will induce strong short-range correlations in the nuclear wave function
One-body densities for $A=2,3,4$ nuclei

\[
\rho^{(1)}(r_1) = \langle \psi | \sum_{i=1}^{A} \delta^3(\hat{r}_i - r_1) | \psi \rangle
\]

\[
n^{(1)}(k_1) = \langle \psi | \sum_{i=1}^{A} \delta^3(\hat{k}_i - k_1) | \psi \rangle
\]

- one-body densities calculated from **exact wave functions** (Correlated Gaussian Method) for AV8' interaction
- coordinate space densities reflect different sizes and densities of $^2$H, $^3$H, $^3$He, $^4$He and the $0^+_2$ state in $^4$He
- similar high-momentum tails in the one-body momentum distributions

Feldmeier, Horiuchi, Neff, Suzuki, PRC 84, 054003 (2011)
Definition: Two-body densities

# of pairs in given spin-, isospin channels

\[ \rho_{SM, TM}^{(2)}(r_1, r_2) = \langle \psi | \sum_{i<j} A \hat{p}_{ij}^{SM} \hat{p}_{ij}^{TM} \delta^3(\hat{r}_i - r_1) \delta^3(\hat{r}_j - r_2) | \psi \rangle \]

\[ n_{SM, TM}^{(2)}(k_1, k_2) = \langle \psi | \sum_{i<j} A \hat{p}_{ij}^{SM} \hat{p}_{ij}^{TM} \delta^3(\hat{k}_i - k_1) \delta^3(\hat{k}_j - k_2) | \psi \rangle \]

integrated over center-of-mass position \( R = \frac{1}{2}(r_1 + r_2) \) or the total momentum of the nucleon pair \( K = k_1 + k_2 \) of the nucleon:

\[ \rho_{SM, TM}^{\text{rel}}(r) = \langle \psi | \sum_{i<j} A \hat{p}_{ij}^{SM} \hat{p}_{ij}^{TM} \delta^3(\hat{r}_i - \hat{r}_j - r) | \psi \rangle \]

\[ n_{SM, TM}^{\text{rel}}(k) = \langle \psi | \sum_{i<j} A \hat{p}_{ij}^{SM} \hat{p}_{ij}^{TM} \delta^3(\frac{1}{2}(\hat{k}_i - \hat{k}_j) - k) | \psi \rangle \]
Two-body densities in coordinate space for $A=2,3,4$

- two-body densities calculated from exact wave functions (Correlated Gaussian Method) for AV8' interaction
- coordinate space two-body densities reflect correlation hole and tensor correlations
  - → normalize two-body density in coordinate space at $r=1.0$ fm
  - → normalized two-body densities in coordinate space are identical at short distances for all nuclei
  - also true for angular dependence in the tensor channel
Two-body densities in momentum space for $A=2,3,4$

**S=0,T=1**

- use **normalization factors fixed in coordinate space**
- $\rightarrow$ two-body densities in momentum space identical for very high momenta $k > 3\text{fm}^{-1}$
- moderate nucleus dependence in high momentum region $1.5\text{fm}^{-1} < k < 3\text{fm}^{-1}$

Feldmeier, Horiuchi, Neff, Suzuki, PRC **84**, 054003 (2011)
Two-body densities reflect many-body correlations

count the number of pairs in the (ST) channels.

<table>
<thead>
<tr>
<th>state \ (ST)</th>
<th>(10)</th>
<th>(01)</th>
<th>(11)</th>
<th>(00)</th>
</tr>
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<tbody>
<tr>
<td>d</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>t</td>
<td>1.490</td>
<td>1.361</td>
<td>0.139</td>
<td>0.010</td>
</tr>
<tr>
<td>h</td>
<td>1.489</td>
<td>1.361</td>
<td>0.139</td>
<td>0.011</td>
</tr>
<tr>
<td>α</td>
<td>2.992</td>
<td>2.572</td>
<td>0.428</td>
<td>0.008</td>
</tr>
<tr>
<td>α*</td>
<td>2.966</td>
<td>2.714</td>
<td>0.286</td>
<td>0.034</td>
</tr>
</tbody>
</table>

- occupation in (ST)=(10) almost exactly as in IPM
- (ST)=(01) significantly depopulated in favor of (ST)=(11)
- three-body correlations induced by the two-body tensor force: depopulation of (ST)=(01) channel is the price one has to pay for getting the full binding from the tensor force
Unitary Correlation Operator Method

\[ n_{S,T}^{rel}(r) \quad S = 1, M_S = 1, T = 0 \]

**central correlator** \( \hat{C}_r \) shifts density out of the repulsive core

**tensor correlator** \( \hat{C}_\Omega \) aligns density with spin orientation

both central and tensor correlations are essential for binding

Roth, Neff, Feldmeier, Prog. Part. Nucl. Phys. 65, 51 (2010)
Flow equation

\[ \frac{d\hat{H}_\alpha}{d\alpha} = [\hat{\eta}_\alpha, \hat{H}_\alpha]_\_ \]

Unitary transformation of Hamiltonian and other operators

\[ \hat{H}_\alpha = \hat{U}_\alpha^\dagger \hat{H} \hat{U}_\alpha, \quad \hat{B}_\alpha = \hat{U}_\alpha^\dagger \hat{B} \hat{U}_\alpha \]

Flow equation for \( \hat{U}_\alpha \)

\[ \frac{d\hat{U}_\alpha}{d\alpha} = -\hat{U}_\alpha \hat{\eta}_\alpha \]

Metagenerator

\[ \hat{\eta}_\alpha = (2\mu)^2 \left[ \hat{\tau}_{\text{int}}, \hat{H}_\alpha \right]_\_ = 2\mu \left[ \hat{K}^2, \hat{H}_\alpha \right]_\_ \]
simultaneous SRG evolution for transformed Hamiltonian and transformation matrix on the two-body level

\[
\frac{d\hat{H}_\alpha}{d\alpha} = \left[ \hat{n}_\alpha, \hat{H}_\alpha \right]_-, \quad \frac{d\hat{U}_\alpha}{d\alpha} = -\hat{U}_\alpha \hat{n}_\alpha
\]

Solve many-body problem with SRG transformed Hamiltonian in the NCSM

\[
\hat{H}_\alpha |\psi_\alpha\rangle = E_\alpha |\psi_\alpha\rangle
\]

Calculate expectation values of “bare” and “effective” two-body density operators

\[
\rho_{\text{bare}} = \langle \psi_\alpha | \hat{\rho} | \psi_\alpha \rangle, \quad \rho_{\text{effective}} = \langle \psi_\alpha | \hat{U}_\alpha^\dagger \hat{\rho} \hat{U}_\alpha | \psi_\alpha \rangle
\]

→ Check for convergence of NCSM calculations and \(\alpha\)-dependence
Similarity Renormalization Group
Implementation Details

- SRG evolution for $\hat{H}_\alpha$ and $\hat{U}_\alpha$ in momentum space $k_{\text{max}} = 15\text{fm}^{-1}$
- Operators only depend on relative coordinates and not on the center-of-mass of the pairs
- (SRG transformed) momentum space matrix elements are expanded in HO basis
- $jj$-coupled matrix elements are calculated using the Talmi-Moshinski procedure
- a slight modification is needed if we look at the two-body densities also as a function of pair momentum
Similarity Renormalization Group
Hamiltonian Flow

$AV8'$

$N3LO$

$\alpha = 0.00$ (bare)
**Similarity Renormalization Group**

Hamiltonian Flow

\[ \alpha = 0.01 \text{fm}^4 \]
Introduction

Unitary Transformations

4 He Results

4 He, 6 Li, 10 B, 12 C Results

Summary

Similarity Renormalization Group

Hamiltonian Flow

$\alpha = 0.04 \text{fm}^4$
**Similarity Renormalization Group**

**Hamiltonian Flow**

$AV8'$

$\alpha = 0.20\text{fm}^4$

$\text{SRG drives the Hamiltonian towards a band-diagonal structure}$
4 He Results

4 He advantages
- exact two-body densities available for AV8’ interaction
- “bare” N3LO can be converged in NCSM

Objectives
- Compare AV8’ and N3LO results
- Check for NCSM convergence
- Check flow dependence $\alpha = 0.01, 0.04, 0.20\text{fm}^4$ ($\Lambda = 3.16, 2.24, 1.50\text{fm}^{-1}$)
- Can we see many-body effects?
**Convergence with the model space size**

$AV8' - \alpha = 0.04\text{fm}^4$

Two-body Density in Coordinate Space

$\hbar \Omega = 20\text{MeV}$ corresponds to roughly the size of $^4\text{He}$
Convergence with the model space size

$$\text{AV8'} - \alpha = 0.04 \text{fm}^4$$

Two-body Density in Coordinate Space

$\hbar \Omega = 20 \text{MeV}$ corresponds to roughly the size of $^4\text{He}$
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Convergence with the model space size

$AV8' - \alpha = 0.04 \text{fm}^4$

Two-body Density in Coordinate Space

\[ \bar{\rho}(r) = \frac{\rho(r)}{\rho(0)} \]

$\hbar\Omega = 20\text{MeV}$ corresponds to roughly the size of $^4\text{He}$
Convergence with the model space size

\[ AV8' - \alpha = 0.04 \text{fm}^4 \]

Two-body Density in Coordinate Space

\[ \rho_{\text{rel}}(r) \text{ [fm}^{-3}] \]

\[ \rho_{\text{eff}}(r) \text{ [fm}^{-3}] \]

\[ r \text{ [fm]} \]

\[ \hbar \Omega = 20 \text{MeV} \] corresponds to roughly the size of \(^4\text{He}\)
Convergence with the model space size

$AV8' - \alpha = 0.04\text{fm}^4$

Two-body Density in Coordinate Space

$\hbar\Omega = 20\text{MeV}$ corresponds to roughly the size of $^4\text{He}$
Convergence with the model space size

\( AV8' - \alpha = 0.04 \text{fm}^4 \)

Two-body Density in Coordinate Space

\[ \rho_{\text{rel}}(r) \text{ [fm}^{-3}] \]

- \( S=0, T=0 \)
- \( N=12 \)
- \( N=10 \)

\[ \rho_{\text{eff}}(r) \text{ [fm}^{-3}] \]

- \( S=0, T=0 \)
- \( N=12 \)
- \( N=10 \)

\[ \hbar \Omega = 20 \text{MeV} \] corresponds to roughly the size of \( ^4\text{He} \)
Convergence with the model space size

AV8’ – \( \alpha = 0.04 \text{fm}^4 \)

Two-body Density in Coordinate Space

\[ \rho_{\text{rel}}(r) \text{ [fm}^{-3}] \]

\( \rho_{\text{bare}}^{\text{op}} \) \( \times 100 \)

\( \rho_{\text{eff}}^{\text{op}} \) \( \times 100 \)

\( S=0,T=0 \)

\( N=14 \) \( \text{red} \)

\( N=12 \) \( \text{orange} \)

\( S=0,T=1 \)

\( N=14 \) \( \text{red} \)

\( N=12 \) \( \text{orange} \)

\( S=1,T=0 \)

\( N=14 \) \( \text{blue} \)

\( N=12 \) \( \text{dashed blue} \)

\( S=1,T=1 \)

\( N=14 \) \( \text{purple} \)

\( N=12 \) \( \text{dashed purple} \)

\[ \hbar \Omega = 20 \text{MeV} \] corresponds to roughly the size of \(^4\text{He}\)
Convergence with the model space size
$AV8' - \alpha = 0.04\text{fm}^4$

Two-body Density in Coordinate Space

$\hbar \Omega = 20\text{MeV}$ corresponds to roughly the size of $^4\text{He}$
Convergence with the model space size

\( AV8' - \alpha = 0.04 \text{fm}^4 \)

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Convergence with the model space size

$AV8' - \alpha = 0.04\text{fm}^4$

Two-body Density in Momentum Space

$\hbar\Omega = 20\text{MeV}$ corresponds to roughly the size of $^4\text{He}$
Convergence with the model space size

$AV8' - \alpha = 0.04 \text{fm}^4$

Two-body Density in Momentum Space

\[ \hbar \Omega = 20 \text{MeV} \] corresponds to roughly the size of $^4\text{He}$
Convergence with the model space size

$AV^8' - \alpha = 0.04\text{fm}^4$

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**Convergence with the model space size**

\( \text{AV8'} - \alpha = 0.04 \text{fm}^4 \)

**Two-body Density in Momentum Space**

\[ \hbar \Omega = 20 \text{MeV} \] corresponds to roughly the size of \(^4\text{He}\)
Convergence with the model space size

$AV8' - \alpha = 0.04 \text{fm}^4$

Two-body Density in Momentum Space

\( \hbar \Omega = 20 \text{MeV} \) corresponds to roughly the size of \(^4\text{He}\)
Convergence with the model space size

\[ AV8' - \alpha = 0.04\text{fm}^4 \]

Two-body Density in Momentum Space

\[ \bar{n}(k) \text{ [fm}^{-3}\text{]} \]

\hbar \Omega = 20\text{MeV} \text{ corresponds to roughly the size of } ^4\text{He}
Flow dependence

AV8’ Interaction

Two-body Density in Coordinate Space

- $\rho_{\text{rel}}^{\text{eff}}(r)$ in bare and effective densities
- $S=0, T=0$ vs $S=0, T=1$ vs $S=1, T=0$ vs $S=1, T=1$
- $\alpha=0.01$, $\alpha=0.04$, $\alpha=0.20$
- $\times 100$ vs $\times 10$
Flow dependence
AV8' Interaction

Two-body Density in Coordinate Space

\[ \rho_{rel}(r) [\text{fm}^{-3}] \]

- **bare**: red
- **\( \alpha = 0.01 \)**: cyan
- **\( \alpha = 0.04 \)**: green
- **\( \alpha = 0.20 \)**: orange

\( S=0, T=0 \)

\( S=0, T=1 \)

\( S=1, T=0 \)

\( S=1, T=1 \)

\( \times 100 \)

\( \times 10 \)

strong \( \alpha \)-dependence
weak \( \alpha \)-dependence
Flow dependence
N3LO Interaction

Two-body Density in Coordinate Space

\[ \rho^{rel}(r) \text{ [fm}^{-3}\text{]} \]

\[ \rho^{eff}(r) \text{ [fm}^{-3}\text{]} \]

\( r \) [fm]

\( S=0, T=0 \)
- bare
- \( \alpha=0.01 \)
- \( \alpha=0.04 \)
- \( \alpha=0.20 \)

\( S=1, T=0 \)
- bare
- \( \alpha=0.01 \)
- \( \alpha=0.04 \)
- \( \alpha=0.20 \)

\( S=1, T=1 \)
- bare
- \( \alpha=0.01 \)
- \( \alpha=0.04 \)
- \( \alpha=0.20 \)
Two-body Density in Coordinate Space

- Bare density
  - $S=0, T=0$
  - $S=0, T=1$
  - $S=1, T=0$
  - $S=1, T=1$

- Effective density
  - $S=0, T=0$
  - $S=0, T=1$
  - $S=1, T=0$
  - $S=1, T=1$

- Parameters:
  - $\alpha = 0.01$
  - $\alpha = 0.04$
  - $\alpha = 0.20$

- Scale factors:
  - $\times 10$
  - $\times 100$

Weaker repulsive core
Flow dependence

AV8’ Interaction

Two-body Density in Momentum Space

- $S=0,T=0$
- $S=0,T=1$
- $S=1,T=0$
- $S=1,T=1$

- bare
- $\alpha=0.01$
- $\alpha=0.04$
- $\alpha=0.20$

$k [fm^{-1}]$

$\rho^{\text{el}}(k) [fm^3]$
Flow dependence
AV8' Interaction

Two-body Density in Momentum Space

$\alpha$-dependence

strong $\alpha$-dependence

weak $\alpha$-dependence

strong $\alpha$-dependence

S=0, T=0
bare
$\alpha=0.01$
$\alpha=0.04$
$\alpha=0.20$

S=0, T=1
bare
$\alpha=0.01$
$\alpha=0.04$
$\alpha=0.20$
Two-body Density in Momentum Space

$S=0, T=0$

bare

$\alpha=0.01$

$\alpha=0.04$

$\alpha=0.20$

$S=0, T=1$

bare

$\alpha=0.01$

$\alpha=0.04$

$\alpha=0.20$

$S=1, T=0$

bare

$\alpha=0.01$

$\alpha=0.04$

$\alpha=0.20$

$S=1, T=1$

bare

$\alpha=0.01$

$\alpha=0.04$

$\alpha=0.20$

$S=0, T=0$

bare

$\alpha=0.01$

$\alpha=0.04$

$\alpha=0.20$

$S=0, T=1$

bare

$\alpha=0.01$

$\alpha=0.04$

$\alpha=0.20$

$S=1, T=0$

bare

$\alpha=0.01$

$\alpha=0.04$

$\alpha=0.20$

$S=1, T=1$

bare

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$\alpha=0.04$

$\alpha=0.20$

$S=0, T=0$

bare

$\alpha=0.01$

$\alpha=0.04$

$\alpha=0.20$

$S=0, T=1$

bare

$\alpha=0.01$

$\alpha=0.04$

$\alpha=0.20$

$S=1, T=0$

bare

$\alpha=0.01$

$\alpha=0.04$

$\alpha=0.20$

$S=1, T=1$

bare

$\alpha=0.01$

$\alpha=0.04$

$\alpha=0.20$
Contributions from different angular momenta

N3LO Interaction

Two-body Density in Momentum Space $S = 1, T = 0$
Contributions from different angular momenta

N3LO Interaction

Two-body Density in Momentum Space $S = 1, T = 0$

$L = 2$ pairs dominate momentum distributions above the Fermi surface
Contributions from different angular momenta
N3LO Interaction

Two-body Density in Momentum Space $S = 0, T = 1$
Contributions from different angular momenta

N3LO Interaction

Two-body Density in Momentum Space $S = 0$, $T = 1$

$L = 0$ pairs dominate momentum distributions above the Fermi surface
Relative contributions of ST channels

**AV8⁺**

![Graphs showing relative contributions of ST channels for AV8⁺](image1)

**N3LO**

![Graphs showing relative contributions of ST channels for N3LO](image2)
Relative contributions of ST channels

**AV8’**

**N3LO**

strong $\alpha$-dependence

# $pp$-pairs depends on many-body correlations
Two-body Density in Momentum Space
Two-body Density in Momentum Space

- $S=0, T=0$
- $S=0, T=1$
- $S=1, T=0$
- $S=1, T=1$

**Pair momentum $\approx 0$**

N3LO Interaction

- $n_{rel}^{\text{rel}}(\bm{k} \equiv \bm{0})$
- $n_{eff}^{\text{eff}}(\bm{k} \equiv \bm{0})$

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Two-body Density in Momentum Space $S = 1, T = 0$
Two-body Density in Momentum Space $S = 0, T = 1$
Relative contributions of ST channels

N3LO interaction

all pair momenta

pair momenta ≈ 0

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Relative contributions of ST channels

N3LO interaction

all pair momenta

pair momenta ≈ 0

α-dependence significantly weaker

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**Calculation**

- “bare” AV18 and N3LO can not be converged
- NCSM convergence only for larger flow parameters

**Objectives**

- Compare AV18 and N3LO results
- Check for NCSM convergence
- Check flow dependence $\alpha = 0.04, 0.20\text{fm}^4$ ($\Lambda = 2.24, 1.50\text{fm}^{-1}$)
- What is different from $^4\text{He}$?
AV18, $\alpha = 0.04\text{fm}^4$

Two-body Density in Momentum Space

4\text{He} (N=16)

$^6\text{Li} (N=12)$

$^{10}\text{B} (N=8)$

$^{12}\text{C} (N=8)$

$\rho^{\text{eff}}(k) [\text{fm}^3]$
AV18, $\alpha = 0.04 \text{fm}^4$

Two-body Density in Momentum Space

$^4\text{He}$ Results

$^6\text{Li}$ Results

$^{10}\text{B}$ Results

$^{12}\text{C}$ Results

$^4\text{He}$, $^6\text{Li}$, $^{10}\text{B}$, $^{12}\text{C}$ Results
**AV18, \( \alpha = 0.20 \text{fm}^4 \)**

**Two-body Density in Momentum Space**

- **\( ^4\text{He} \)**: \( N=16 \)
  - Bare densities
  - Effective densities

- **\( ^6\text{Li} \)**: \( N=12 \)
  - Bare densities
  - Effective densities

- **\( ^{10}\text{B} \)**: \( N=8 \)
  - Bare densities
  - Effective densities

- **\( ^{12}\text{C} \)**: \( N=8 \)
  - Bare densities
  - Effective densities

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Two-body Density in Momentum Space

N3LO, $\alpha = 0.04\text{fm}^4$

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N3LO, $\alpha = 0.20\text{fm}^4$

Two-body Density in Momentum Space

For $^4\text{He}$, $^6\text{Li}$, $^{10}\text{B}$, and $^{12}\text{C}$, the two-body density in momentum space is shown. The plots illustrate the density $n^0(k)$ and $n^{\text{eff}}(k)$ for different channels (pp, pn, nn) as a function of momentum $k$ [fm$^{-1}$].
pp, pn, nn contributions

AV18

N3LO

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pp, pn, nn contributions

**AV18**

4 He Results

\[
^4\text{He} H N = 16 \quad \begin{array}{lll}
\text{pp} & \text{pn} & \text{nn} \\
0 & 1 & 2 & 3 & 4 \\
0.0 & 0.2 & 0.4 & 0.6 & 0.8 & 1.0
\end{array}
\]

\[
^6\text{Li} H N = 12 \quad \begin{array}{lll}
\text{pp} & \text{pn} & \text{nn} \\
0 & 1 & 2 & 3 & 4 \\
0.0 & 0.2 & 0.4 & 0.6 & 0.8 & 1.0
\end{array}
\]

\[
^{10}\text{B} H N = 8 \quad \begin{array}{lll}
\text{pp} & \text{pn} & \text{nn} \\
0 & 1 & 2 & 3 & 4 \\
0.0 & 0.2 & 0.4 & 0.6 & 0.8 & 1.0
\end{array}
\]

\[
^{12}\text{C} H N = 8 \quad \begin{array}{lll}
\text{pp} & \text{pn} & \text{nn} \\
0 & 1 & 2 & 3 & 4 \\
0.0 & 0.2 & 0.4 & 0.6 & 0.8 & 1.0
\end{array}
\]

**N3LO**

\[
^4\text{He} H N = 16 \quad \begin{array}{lll}
\text{pp} & \text{pn} & \text{nn} \\
0 & 1 & 2 & 3 & 4 \\
0.0 & 0.2 & 0.4 & 0.6 & 0.8 & 1.0
\end{array}
\]

\[
^6\text{Li} H N = 12 \quad \begin{array}{lll}
\text{pp} & \text{pn} & \text{nn} \\
0 & 1 & 2 & 3 & 4 \\
0.0 & 0.2 & 0.4 & 0.6 & 0.8 & 1.0
\end{array}
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\[
^{10}\text{B} H N = 8 \quad \begin{array}{lll}
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0 & 1 & 2 & 3 & 4 \\
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\[
^{12}\text{C} H N = 8 \quad \begin{array}{lll}
\text{pp} & \text{pn} & \text{nn} \\
0 & 1 & 2 & 3 & 4 \\
0.0 & 0.2 & 0.4 & 0.6 & 0.8 & 1.0
\end{array}
\]

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**AV18 and N3LO results pretty similar**
Summary

Similarity Renormalization Group
- SRG evolved Hamiltonian and transformation matrix
- “bare” and “effective” density operators

$^4$He Two-body densities
- AV8’ and N3LO interactions
- short-range and high-momentum components described by effective operators
- high-momentum components above the Fermi momentum dominated by $L = 2$ pairs
- weak $\alpha$-dependence in the $S = 1, T = 0$ channel
- strong $\alpha$-dependence in the $S = 0, T = 1$ channel due to many-body correlations
- AV8’ and N3LO interaction results differ mainly in the $S = 0, T = 1$ channel due to different many-body correlations

$^4$He, $^6$Li, $^{10}$B, $^{12}$C Two-body densities
- $T = 1$ pairs with $L = 1$ fill up the momentum distribution above the Fermi momentum
- less sensitivity to many-body correlations
- AV18 and N3LO provide very similar results