Theory of the EMC effect and GPDs in Nuclei

Simonetta Liuti
University of Virginia

Nuclear Structure and Dynamics at Short Distances

Institute for Nuclear Theory, University of Washington

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Can the many-body-effects appearing in the interaction current be separated from those appearing in the wave function?

Can conventional nuclear theory provide calculations of the observables measured in coincidence experiments?

What is the relation between two-nucleon correlations and the EMC effect?

What is the role of relativistic effects in the present context?

What experiments can determine the role of three nucleon correlations?

What is the role of quark, as opposed to nucleon or meson, effects in understanding the plateau and the EMC effect?

Which other reactions can be used to elucidate the effects of short-ranged correlations?

How can the EMC effect be studied in semi-inclusive DIS (and exclusive DVCS)?

How do hadronization effects reveal themselves in semi-inclusive DIS?
Outline

✓ Some theoretical issues on the EMC effect

✓ Introduction to Generalized Parton Distributions (GPDs) and Deeply Virtual Compton Scattering (DVCS): Kinematics and Definitions

✓ What we can learn from GPDs and DVCS from Nuclei

✓ QCD analysis of DVCS from Nuclei
... ab initio....

Binding: Akulinichev, Kulagin, Vagradov

Q² –rescaling, “quick-fix”

SRC

S. Liuti, Phys. Rev. C47 (1993): “results will depend on whether similar kinematical regions in the spectral function are integrated over”

Numerator and denominator calculated at shifted $x_{Bj}$ so as to cover the same areas in momentum and removal energy:

$$R = \frac{\sigma_A(y_A = y_D)}{\sigma_D(y_A = y_D)},$$

Numerator and denominator calculated at same $x_{Bj}$
A BRIEF HISTORY...

- The idea of using nuclei as “laboratories for QCD” is introduced in the ‘80s by Brodsky, Frankfurt, Ioffe, Kopeliovich, Miller, A. Mueller, Nikolaev, Pire, Ralston, Strikman....

- Experiments are performed: EMC, NMC @ CERN, E665 @ Fermilab, DY and J/ψ production @ Fermilab, etc...

- Many intricacies and controversies appear: no clear-cut interpretation of the “EMC-effect”, of the onset of shadowing and anti-shadowing (are sum rules satisfied in nuclei? are parton distributions probabilities?), Color Transparency....

- TODAY: Deeply Virtual Exclusive Experiments add a whole new dimension where to explore nuclear medium modifications. One can observe previously inaccessible spatial d.o.f.

Images from Univ. of Frankfurt + G. Miller websites
The EMC Effect: Kinematics and Definitions

\[ q \]

\[ k^+ = x P_A^+ = (x/z)p^+ \]

\[ p^+ = z P_A^+ \]

\[ P_A^+ \]

Nucleon Correlator

\[
W^{\gamma^+}_{\Lambda}(p, k) = \int \frac{d^4 z}{(2\pi)^4} e^{ikz} \langle p, \Lambda | \overline{\Psi}(0)\gamma^+ \mathcal{W} \Psi(z) | p, \Lambda \rangle
\]

\[
W^{\gamma^+}_{\Lambda}(p, x) = \int dk^- d^2 \vec{k}_T W^{\gamma^+}_{\Lambda}(p, k)
\]

\[
= \int \frac{dz^-}{2\pi} e^{ikz} \langle p, \Lambda | \overline{\Psi}(0)\gamma^+ \mathcal{W} \Psi(z) | p, \Lambda \rangle \big|_{z^+=0, z_T=0}
\]
Probabilistic interpretation:

\[ W^\gamma_\Lambda^+ (p, x) = \int dz^- e^{ikz} \langle p, \Lambda | \bar{\phi}(0) \phi(z) | p, \Lambda \rangle \big|_{z^+=0, z_T=0} = f_1(x) \bar{u}(p, \Lambda) \gamma^+ u(p, \Lambda) \]

where \( \phi = (1/2) \gamma^- \gamma^+ \). Then

\[ F_2(x) = \sum_{\Lambda} W^\gamma_\Lambda^+ (p, x) \]

\[ F_2(x) = \sum_X \delta(p^+ - xp^+ - p_X^+) \left| \langle X | \phi(0) | p, \Lambda \rangle \right|^2 \]

In a nucleus:

\[ W^\gamma_\Lambda^+ (P_A, k) = \int d^4p \ W(P_A, p) W^\gamma_\Lambda^+ (P, k) \]
Naïve Convolution Formula

\[ F_2 (x) = \int_x^A dz \ f_A (z) F_2^N (x/z) \]

\[ f_A (z) = 2\pi M \int dE \int_{k_{min}(z,E)} dkk \ P_A (k,E) \]


\[ \langle z \rangle \approx 1 - \frac{\langle E \rangle}{M} \]

x=0

3He

No SRC

x=0.6
But.....
LC extension of the Hugenholtz VanHove theorem

G. Miller
Because of Koltun Sum Rule

\[ \langle E \rangle = 2 \epsilon_A + \frac{A - 2}{A - 1} \langle T \rangle - \langle V_3 \rangle \]

There is a tension between E and T, the two slopes are related, no extra d.o.f. for modeling...
Beyond naïve convolution formula In nuclei:

\[ T^A_{\mu\nu}(P_A, \Delta) = \int \frac{d^4P}{(2\pi)^4} T^N_{\mu\nu}(k, P, \Delta) \mathcal{M}^A(P, P_A, \Delta), \]

where:

\[ \mathcal{M}^A_{ij}(P, P_A, \Delta) = \int d^4y e^{iP\cdot y} \langle P'_A|\overline{\Psi}_{A,j}(-y/2)\Psi_{A,i}(y/2)|P_A\rangle. \]

The punch line: factorization is broken

\[ F^A(X, \zeta, t) = \int \frac{d^4P}{(2\pi)^4} F^N_{OFF}(X_N, \zeta_N, P^2, t) \mathcal{M}^A(P, P_A, \Delta), \]

We cannot factor out the transverse degrees of freedom!

to $m^2 \approx 0.02 \text{ GeV}$. Whether the distribution of quarks in a pion so far off shell is the same as the distribution on shell is anybody’s guess. In any case, advocates of pion and other convolution based models ignore any $p^2$ dependence of the constituents’ quark distributions.

The assumptions leading to a convolution model are arguable at best.

R. Jaffe, 1985
A clear case where FSI dominates exists at low $x_{Bj}$

Brodsky, Schmidt, Yang
Brodsky, Hoyer, Peigne’, Sannino

“model FSI at low x in terms of Pomeron, Reggeon, Odderon exchanges” (see S.Brodsky’s talk)
Brodsky: the Glauber-Gribov picture involves interference between rescattering amplitudes.
Key argument (Hoyer and Vanttinen): at low $x$ the LC time is long enough to allow for coherent effects
Parton off-shellness for PDFs

\[ k^2 (\text{GeV}^2) \]

\[ k_T = 0 \]

\[ k_T = 0.1 \text{ GeV} \]

\[ k_T = 0.3 \text{ GeV} \]

\[ k_T = 0.5 \text{ GeV} \]
Liu and Taneja (2005)

Effect is related to transverse motion of quarks
Next using GPDs formalism, I will argue that at large $x$ parton reinteractions in nuclei are also leading effects because of the enlarged parton offshellness.
Nuclear GPDs: motivations

- Nuclear GPDs, by providing spatial distributions of partonic configurations in hadrons allow us to discern among different proposed mechanisms for the nuclear EMC effect \( \Rightarrow \) interesting connection with TMDs (role of FSI)

- Nuclear GPDs shed light on the role of OAM in hadrons for spin \( \neq 1/2 \) (\( S=1 \), deuteron), (\( S=0, ^4\text{He} \)).

- Moments: quarks and gluons angular momentum

- Finally, nuclear GPDs allow one to validate the onset of Color Transparency phenomena by monitoring directly in coordinate space the dominance of partonic small size configurations
Off-forward EMC effect: Longitudinal Convolution Formula

Nuclear Hadronic Tensor

\[ T^A_{\mu\nu}(P_A, \Delta) = \int \frac{d^4 P}{(2\pi)^4} T^N_{\mu\nu}(k, P, \Delta) \mathcal{M}^A(P, P_A, \Delta), \]

Nuclear Correlator

\[ \mathcal{M}^A_{ij}(P, P_A, \Delta) = \int d^4 y e^{iP\cdot y} \langle P'_A | \bar{\Psi}_{A,j}(-y/2) \Psi_{A,i}(y/2) | P_A \rangle. \]
Non-forward kinematics

Longitudinal momentum fractions

\[
Z = A \frac{P^+}{P_A^+} \quad \zeta = A \frac{\Delta^+/P_A^+}{P^+} = \frac{\Delta^+/P^+}{P_A^+}
\]

\[
X = A \frac{k^+/P_A^+}{P_A^+} \quad X-\zeta = A \frac{k^+/P_A^+}{P_A^+}
\]

\[
X/Z = \frac{k^+/P^+}{P_A^+} \quad Z-\zeta = A \frac{P^+/P_A^+}{P_A^+}
\]
Use e.g. a spectator model (with a spin 0 diquark) to take a closer look to the nucleon correlator $\mathcal{M}^N_{ij}$. 

\[
\mathcal{M}^N_{ij} = \bar{U}_i(P', S)\Gamma_{i\alpha}(k', P) \frac{(k' + m)_{\alpha\beta}}{k'^2 - m^2} \frac{(k + m)_{\beta\gamma}}{k^2 - m^2} \Gamma_{\gamma j}(k, P)U_j(P, S)
\]
From $M^N$ to $M^A$ (Spin 0)

$$M^A_{i,j} = U_{A-1}(P'_A, S) \Gamma_A(P', P'_A) \frac{(P' + M)}{P'^2 - M^2} \frac{(P + M)}{P^2 - M^2} \Gamma_A(P, P_A) U_{A-1}(P_A, S)$$

$U_{A-1} \rightarrow$ spectator $A - 1$ nucleons with mass $M^*_{A-1}$

$\Gamma_A \rightarrow$ nuclear vertex function

$$M^A_{i,j} = N_A \left( \sum_S U_i(P, S) \bar{U}_j(P', S) \right) \rho_A(P^2, P'^2)$$

Non-forward nuclear spectral function

$$\rho_A(P^2, P'^2) \approx S_A(|P|, |P'|, E)$$

$$= \sum_f \Phi_f(|P|) \Phi_f^*(|P'|) \delta \left( E - (E^f_{A-1} - E_A) \right)$$

With LC variables $\rho_A(Y, \zeta, t, P^2)$
Meaning of model beyond longitudinal convolution

\[ \Delta t = \Delta^2 \]

\[ q' = q + \Delta \]

\[ k' = k - \Delta \]

\[ P' = P - \Delta \]

\[ P_A' = P_A - \Delta \]

FSI are present, they affect how one goes “off the LC” in the transverse direction, this effect is larger in nuclei.

Nuclei are a unique handle to test/highlight role of partons multi-correlations, ISI and FSI!
Off-shellness

$t = -0.1 \text{ GeV}^2$

\[ k^2 (\text{GeV}^2) \]

\[ x \]

\[ x \]
Explanation of Result

- Why larger dip?

Using LC approx.: \[ H_A(X, t) \approx H_N\left(\frac{X}{1 - \langle E(t) \rangle / M}\right) \]

\[ \langle E(t) \rangle \approx \langle E(t = 0) \rangle \rightarrow \text{no sensible difference} \]

Using Active-\(k_\perp\): \[ H_A(X, t) \approx H_N\left(\frac{X}{\langle Y(P^2, t) \rangle}\right) \]

\[ \langle Y(P^2, t) \rangle \neq \langle Y(P^2, t = 0) \rangle \]

- Similarly for \(k_\perp\)-dependent mechanism giving anti-shadowing

Effect due to “non-trivial” \(t\) dependence of higher moments in nuclei

GPDs trigger on \(k_\perp\) dependent effects!!
Extracting GPDs from Cross Sections and Beam Spin Asymmetries
The diagram represents the process $(ep \rightarrow e'p'\gamma)$, which is a reaction in particle physics involving the scattering of an electron-proton system leading to the production of another electron, a proton, and a gamma ray. The equations shown are part of the theoretical framework to calculate the cross-sections for this process.

The first equation expresses the differential cross-section in terms of the Born and DVCS amplitudes as:

$$\frac{d^5\sigma(\lambda, \pm e)}{d^5\Phi} = \frac{d\sigma_0}{dQ^2dx_B} \left| T^{BH}(\lambda) \pm T^{DVCS}(\lambda) \right|^2 / |e|^6$$

$$= \frac{d\sigma_0}{dQ^2dx_B} \left[ |T^{BH}(\lambda)|^2 + |T^{DVCS}(\lambda)|^2 \mp \mathcal{I}(\lambda) \right] \frac{1}{e^6}$$

The second part of the document involves the sum over the mass and the sum over the momentum transfer for the process, represented as:

$$\frac{d^4\Sigma}{dQ^2dx_Bdtd\phi} \equiv \frac{d^4\sigma^+}{dQ^2dx_Bdtd\phi} - \frac{d^4\sigma^-}{dQ^2dx_Bdtd\phi}$$

$$\frac{d^4\sigma}{dQ^2dx_Bdtd\phi} \equiv \frac{d^4\sigma^+}{dQ^2dx_Bdtd\phi} + \frac{d^4\sigma^-}{dQ^2dx_Bdtd\phi}$$

The function $\mathcal{F}(\zeta, t)$ is defined as:

$$\mathcal{F}(\zeta, t) = -i\pi \sum_q e_q^2 [F^q(\zeta, \zeta, t) - F^q(-\zeta, \zeta, t)] + \mathcal{P} \int_{1-\zeta}^1 dX \left( \frac{1}{X - \zeta} + \frac{1}{X} \right) F^q(X, \zeta, t).$$

Here, $F^q$ is a function of the momentum transfer $X$, $\zeta$, and $t$, and $\mathcal{P}$ denotes the principal value of the integral. The notation $F_q \equiv H_q, E_q$ is used to indicate the relationship between the functions and the variables.
BH, DVCS and Interference contributions azimuthal dependence written explicitly (Belitsky, Muller, Kirchner)

\[
T_{BH}^2 = \frac{e^6(1+e^2)^{-2}}{x_A y^2 t \mathcal{P}_1(\varphi)\mathcal{P}_2(\varphi)} \sum_{n=0}^{n=2} c_n^{BH} \cos(n\varphi),
\]

\[
|T_{DVCS}^\lambda|^2 = \frac{e^6}{y^2 Q^2} \sum_{n=0}^{n=2} \left\{ c_n^{DVCS} \cos(n\varphi) + \lambda s_n^{DVCS} \sin(n\varphi) \right\},
\]

\[
T^\lambda = \frac{e^6}{x_A y^3 t \mathcal{P}_1(\varphi)\mathcal{P}_2(\varphi)} \sum_{n=0}^{n=3} \left\{ c_n^T \cos(n\varphi) + \lambda s_n^T \sin(n\varphi) \right\}.
\]

Coefficients correspond to the L,T,LT,TT,LT', ... terms in the x-sec.
4He: Spin 0

Bethe-Heitler

\[
c_0^{BH} = \left\{ (2 - y)^2 + y^2(1 + \epsilon^2)^2 \right\} \left\{ \frac{\epsilon^2 Q^2}{t} + 4(1 - x_A) + (4x_A + \epsilon^2) \frac{t}{Q^2} \right\} \\
+ 2\epsilon^2 \left\{ 4(1 - y)(3 + 2\epsilon^2) + y^2(2 - \epsilon^4) \right\} - 4x_A^2(2 - y)^2(2 + \epsilon^2) \frac{t}{Q^2} \\
+ 8K^2\frac{\epsilon^2 Q^2}{t} F_A^2,
\]

(24)

\[
c_1^{BH} = -8(2 - y)K \left\{ 2x_A + \epsilon^2 - \frac{\epsilon^2 Q^2}{t} \right\} F_A^2,
\]

(25)

\[
c_2^{BH} = 8K^2\frac{\epsilon^2 Q^2}{t} F_A^2,
\]

(26)

DVCS

\[
c_0^{DVCS} = 2(2 - 2y + y^2) \mathcal{H}_A \mathcal{H}_A^*.
\]

Interference

\[
c_0^I = -8(2 - y) \frac{t}{Q^2} F_A \text{Re}\{\mathcal{H}_A\} \\
\times \left\{ (2 - x_A)(1 - y) - (1 - x_A)(2 - y)^2 \left( 1 - \frac{t_{\text{min}}}{Q^2} \right) \right\},
\]

\[
c_1^I = 8K(2y - y^2 - 2) F_A \text{Re}\{\mathcal{H}_A\},
\]

\[
s_1^I = 8Ky(2 - y) F_A \text{Im}\{\mathcal{H}_A\}.
\]
Interference between BH and DVCS from Nuclear Beam Spin Asymmetry

Nuclear Beam Spin Asymmetry

\[ A^{(A)}_{LU} = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} \approx \frac{s_1^T}{c_o^{BH}} \sin \phi \]

\[ s_1^T \propto \Im \mathcal{H}_A F_A(t) \]

\[ c_o^{BH} \propto [F_A(t)]^2 \]

\[ \Im \mathcal{H}_A(X, \zeta, t) = -\pi \sum_q e_q^2 \left[ H^q_A(\zeta, \zeta, t) + H^q_A(\zeta, \zeta, t) \right] \]

(Kirchner and Mueller, 2004)
Coherent vs. Incoherent processes

$q$  

$\Delta q = q' - q$

$DVCS$

Non-forward nucleon

$P^+_A$

Forward nucleon

$q'$

$q' = q + \Delta$
\( \mathcal{I}_{coh}(\zeta, t) = \mathcal{K} H^A(\zeta, t) \times Z^2 F^A(t) \)

\[ H^A(\zeta, t) = \int \frac{d^2 P_1}{2(2\pi)^3} N \rho^A(Y, P^2; \zeta, t) H^N \left( \frac{\zeta}{Y}, \frac{\zeta}{Y}, t; P^2; \right) \]

↑ off-forward EMC-effect ↑

\( \mathcal{I}_{inc}(\zeta, t) = \mathcal{K} H_0^A(\zeta, t) \times Z F_1^N(t) \)

\[ H_0^A(\zeta, t) = \int \frac{d^2 P_1}{2(2\pi)^3} N \rho_0^A(Y, P^2) H^N \left( \frac{\zeta}{Y}, \frac{\zeta}{Y}, t; P^2 \right) \]
Hermes ⇒ first data  

\[
R_{LU}^{\sin \phi} (A/p) = 0.91 \pm 0.19 \text{ coherent}
\]
\[
R_{LU}^{\sin \phi} (A/p) = 0.93 \pm 0.23 \text{ incoherent}
\]
Hermes data

\[ R_{LU}^{(A)}(\zeta, t) = \frac{A_{LU}^A}{A_{LU}^p} = \frac{Z^2 I_{coh}^A + ZT_{incoh}^A}{F_D^{p} I_{DVCS}(\zeta, t) F_1(t)} \times \frac{F_1^2(t)}{Z^2 F_A^2(t) + ZF_1^2(t)} \]

\[ I_{coh}^A = F_{DVCS}(\zeta, t) F_A(t) \quad I_{incoh}^A = F_{DVCS, 0}(\zeta, t) F_1(t) \]
Jlab/Hall B analysis (K. Hafidi et al.) in progress (talk by H. Egiyan at DIS 2010)
The candidate experimental phase space for coherent DVCS is limited and statistics does not allow for a multi-dimensional analysis as was performed in the CLAS p-DVCS experiment.

\[ -t \leq 0.2 \text{ GeV}^2 \quad 0.1 \leq x_B \leq 0.3 \quad 1 \text{ GeV}^2 \leq Q^2 \leq 2.4 \text{ GeV}^2 \]
**DVCS Asymmetries**

- **asym signal bin 0**
  - $0 < -t < 0.15$
  - $0.15 < -t < 0.3$

- **asym signal bin 1**
  - $0.3 < -t < 0.5$
  - $0.5 < -t < 1.0$

Large $t$ data are irrelevant.
Other calculations and observables
Guzey and Siddikov (2006)

Introduce meson d.o.f. "pion excess" model

“conventional” nuclear effects
S.L. and S.Taneja

\[ R_u^{(0,0)} \]
\[ R_d^{(0,0)} \]

S. Scopetta

F. Cano and B. Pire

Deuteron
(data from Hall A, Mazouz et al.)

\[ ^4\text{He} \]

\[ ^3\text{He} \]
Moments: Angular Momentum Sum Rules

$$M_n^A(t) = \left( \int_0^A dy y^{n-1} f_A(y, t) \right) \left( \int_0^1 dx x^{n-2} [x H^N(x, \xi, t)] \right),$$

(e,e') \quad t=0

Off-shell effects

\[ F^A(t) = F^{A,\text{point}}(t) F^N(t) \]

\[ M_2^A(\xi, t) = M_{2,\text{point}}^A(t) M_2^N(t) + M_{0,\text{point}}^A(t) \frac{4}{5} d_1^N(t) \xi^2, \]

The 1\textsuperscript{st} moment: form factors

\[ \langle x(t) \rangle_A = \frac{M_2^A(t)}{F^A(t)} = \frac{M_{2,\text{point}}^A(t)}{F_{A,\text{point}}(t)} \frac{M_2^N(t)}{F^N(t)} = \langle y(t) \rangle_A \langle x(t) \rangle_N, \]

The D-term in a nucleus reads:

\[ d_1^A(t) = M_{0,\text{point}}^A(t) d_1^N(t). \]
\[ d_1^A(0) \approx \frac{1}{1 - \bar{E}/M + 2/3\langle P^2 \rangle/2M^2} A d_1^N(0) \]

\[ \neq \]

\[ d_1^A(0) \propto A^{7/3} \]  

Polyakov – Liquid Drop Model
Nuclear Exclusive: Form Factor in Nuclei

S.L., hep-ph/0601125

\[ F_A(t) = \int_0^A dx H_A(x,t) \]

\[ F_A^{LC}(t) = F_A^{\text{point}}(t) F_N(t) \]

\[ F_A(t) = \int_X^A dY \int dP^2 \rho_A(Y,t;P^2) H_N \left( \frac{X}{Y}, t; P^2 \right) \]

\[ \hat{F}_1^N(t) = \left[ \frac{F_A(t)}{F_A^{LC}(t)} \right] F_1^N(t) \]

↑ Medium Modified Form Factor ↑
Effect of shadowing on in medium form factor!
“If configurations with small radii exist, they can be isolated by performing CT and/or nuclear filtering experiments”

Define filter as

$$
\Pi(b) = \begin{cases} 
1 & b < b_{\text{max}}(A) \\
0 & b \geq b_{\text{max}}(A) 
\end{cases}
$$

This affects the GPD as

$$
H_A(x,Q^2) = \int_0^{b_{\text{max}}(A)} db \, b \, q(x,b) J_0(b\Delta),
$$

Transparency ratio

$$
T_A(Q^2) = \frac{\left[ \int_0^1 dx H_A(x, \Delta) \right]^2}{\left[ \int_0^1 dx H(x, \Delta) \right]^2},
$$

\[ q(x, b) = A(x) \exp[-\alpha(x) b] \]

\[ R = 1 - \exp(-\alpha b_{\text{max}})[\alpha b_{\text{max}} J_0(\Delta b_{\text{max}}) + \cos(\Delta b_{\text{max}})]. \]

\( b_{\text{max}} > b_{\text{GPD}} \)

\( b_{\text{max}} < b_{\text{GPD}} \)

\( \alpha(x) \approx (1-x) \) soft \( k_T \), large nucleon

\( \alpha(x) \approx (1-x)^2 \) hard \( k_T \), small nucleon size
Physical Interpretation of the various deuteron GPDs: Form Factors

\[ \int H_1(x, \xi, t) \, dx = G_1(t) \]
\[ \int H_2(x, \xi, t) \, dx = G_2(t) \]
\[ \int H_3(x, \xi, t) \, dx = G_3(t) \]
\[ \int H_4(x, \xi, t) \, dx = 0 \]
\[ \int H_5(x, \xi, t) \, dx = 0 \]

\[ G_C(t) = G_1(t) + \frac{2}{3} \eta G_Q(t) \]
\[ G_M(t) = G_2(t) \]
\[ G_Q(t) = G_1(t) - G_2(t) + (1 + \eta) G_3(t) \]

\[ G_C(0) = 1 \]
\[ G_M(0) = \frac{M_D}{M_N} \mu_D = 1.714 \]
\[ G_Q(0) = M_D^2 Q_D = 25.83 \]

\[ \eta = \frac{t}{2M_D^2} \]
Physical Interpretation of the various deuteron GPDs: PDFs

\[ H_1(x,0,0) = \frac{1}{3} \left( q^1(x) + q^{-1}(x) + q^0(x) \right) = f_1(x) \]

\[ H_5(x,0,0) = \left( q^0(x) - \frac{q^1(x) + q^{-1}(x)}{2} \right) = b_1(x) \]
Angular Momentum Sum Rule: Taneja et al, PRD 2012

\[
\frac{1}{2} g_{q,g}^5 = \frac{1}{2} \int dx \, x H_2(x, 0, 0) = J_{z}^{q,g}
\]

\[
J_q = \frac{1}{2} \int dx \, x [H_q(x, 0, 0) + E_q(x, 0, 0)], \quad J_q = \frac{1}{2} \int dx \, x H_2^q(x, 0, 0),
\]

\[F_1 + F_2 = G_M\]

\[G_M\]
Nuclear effect much larger than in unpolarized scattering

Needs to be treated systematically...
Other relations

\[ \int dx d\xi [H_1(x, \xi, t) - \frac{1}{3} H_5(x, \xi, t)] = G_1(t) + \xi^2 G_3(t) \] (7)

\[ \int dx d\xi H_2(x, \xi, t) = G_5(t) \] (8)

\[ \int dx d\xi H_3(x, \xi, t) = G_2(t) + \xi^2 G_4(t) \] (9)

\[ \int dx d\xi H_4(x, \xi, t) = \xi G_6(t) \] (10)

\[ \int dx d\xi H_5(x, \xi, t) = G_7(t) \] (11)

Momentum

Angular Momentum

Quadrupole

Connected to \( b_1 \) SR
Conclusions and Outlook

- Exclusive experiments in nuclei provide an even better laboratory to study QCD in coordinate space:
  - vast phenomenology...
  - study short LC distance structure of nuclei at the wave function level
- We have seen more constraints on GPDs from nuclei...
- ...and at the same time new insights on nuclear modifications from GPDs
- Re-interactions are important and emphasize transverse d.o.f.: need to explore connections between $k_T$ and $b$
- Comparison between GPD models and data is indeed possible...GPD extraction is possible!!!
Maybe if I just sit very, very still, nothing will happen.