Claudio Ciofi degli Atti

SHORT-RANGE CORRELATIONS IN NUCLEI: theoretical predictions and experimental evidences

WORKSHOP

on

Nuclear Structure and Dynamics at Short Distances

February 11 - 22, 2013, INT, Seattle, USA
1. Introduction: why Short Range Correlations?

2. Ab initio solutions of the non relativistic many-body problem and theoretical predictions of SRC.

3. Experimental observations of SRC.

4. Impact of SRC on various fields of physics.

5. Conclusions.
INTRODUCTION: WHY SHORT RANGE CORRELATIONS (SRC)?
Many properties of nuclei measured at low $Q^2$ and generated by the average and collective motions of point-like nucleons can be successfully described in terms of the nuclear Mean Field (Shell Model).

Nowadays it is possible to investigate nuclei at high $Q^2$, probing distances of the order of the nucleon radius ($\approx 1\, fm$), and the following longstanding questions arise:

1. Do nucleon and meson d.o.f. play still a role at short distance, or quark and gluon d.o.f. are the relevant ones?

2. Is the two-nucleon short-range behavior strongly affected by the surrounding nucleons?

3. Does the short-range behavior of nuclei affect cold matter at high densities, e.g. neutron stars?

4. Does the short-range structure of nuclei affect high energy scattering, e.g. pA and AA?

Answering these questions implies the study of Short-Range Correlations (SRC). To this end, one needs dedicated experiments and a well-defined theoretical framework to interpret them.
AB INITIO SOLUTIONS OF THE NUCLEAR MANY-BODY PROBLEM AND THEORETICAL PREDICTIONS OF SRC
THE STANDARD MODEL OF NUCLEI

QCD $\Rightarrow$ Nuclei- non perturbative regime $\Rightarrow$ too difficult
Many-body systems $\Rightarrow$ single out the leading effective d.o.f.
Effective d.o.f. in Nuclei $\Rightarrow$ nucleons and gauge bosons.
Reduction of a field theoretical description to an instantaneous potential description (Schroedinger equation) $\Rightarrow$ two-body, three-body,........,A-body potentials are generated.
PrimaKoff, Holstein 1944

$$(m\text{-body potential}) \approx \left(\frac{v_{Nc}}{c}\right)^{(m-2)} \times (\text{two-body potential})$$

$$\left[-\frac{\hbar^2}{2m_N} \sum_i \hat{\nabla}_i^2 + \sum_{i<j} \hat{v}_2(i,j) + \sum_{i<j<k} \hat{v}_3(i,j,k)\right] \Psi_o(1\ldots A) = E_o \Psi_0$$

$$\Psi_o \equiv \Psi_o(1\ldots A) \quad i \equiv x_i \equiv \{\sigma_i, \tau_i, r_i\} \quad \sum_{i=1}^{A} r_i = 0$$
Theoretical framework: Solve *ab initio* the standard model with realistic interactions $\implies$ compare with experimental data (energy, form factors, transition matrix elements, etc); if agreement is found $\implies$ OK; if not $\implies$ look for new d.o.f.

Modern bare two-nucleon interactions ($\simeq 2000$ phase shifts)

$$\hat{v}_2(x_i,x_j) = \sum_{n=1}^{18} v^{(n)}(r_{ij}) \hat{O}^{(n)}_{ij} \quad r_{ij} \equiv |r_i - r_j|$$

- $\hat{O}^{(1)}_{ij} = 1$
- $\hat{O}^{(2)}_{ij} = \sigma_i \cdot \sigma_j$
- $\hat{O}^{(3)}_{ij} = \tau_i \cdot \tau_j$
- $\hat{O}^{(4)}_{ij} = (\sigma_i \cdot \sigma_j)(\tau_i \cdot \tau_j)$
- $\hat{O}^{(5)}_{ij} = \hat{S}_{ij}$
- $\hat{O}^{(6)}_{ij} = \hat{S}_{ij} \tau_i \cdot \tau_j$

$$\hat{S}_{ij} = 3(\hat{r}_{ij} \cdot \sigma_i)(\hat{r}_{ij} \cdot \sigma_j) - \sigma_i \cdot \sigma_j$$

- **short-range repulsion** (common to many systems)
- **intermediate- to long-range tensor character** (unique to nuclei)
THE MEAN FIELD APPROXIMATION

\[ \sum_{i<j} \hat{v}_2(i, j) + \sum_{i<j<k} \hat{v}_3(i, j, k) \Rightarrow \sum_i V_i(i). \]

\[ \downarrow \]

\[ \left[ -\frac{\hbar^2}{2m_N} \sum_i \hat{\nabla}_i^2 + \sum_i V(r_i) \right] \Phi_0(1, \ldots, A) = \epsilon_0 \Phi_0(1, \ldots, A) \]

Mean-field (shell model) wave function

\[ \Phi_0(1, 2, \ldots, A) = \hat{A} \prod_i \phi_{\alpha_i}(x_i) \equiv \Phi_{0p0h} \quad \alpha_i > \alpha_F, \phi_{\alpha_i} = 0 \]

Exact correlated wave function

\[ \Psi_0(1, 2, \ldots, A) = C_{0p0h} \phi_{0p0h} + C_{1p1h} \phi_{1p1h} + C_{2p2h} \phi_{2p2h} + \ldots \]

\[ SRC \rightarrow \sum_{n=1}^{\infty} C_{npnh} \Phi_{npnh} \]
• Direct solution for few-body systems (Faddeev, Fadeev-Yakubowsky): Gloeckle & co.
• Expansion in complete set of basis functions: Suzuki & co.
• No Core Shell Model Vary & Co.
• Introduction of correlations into the mean field wave function by proper correlation operators: Roth, Neff & co.
• SRG: Furnstahal, Schwenk & co.
• Correlated basis functions with Green Function Monte Carlo: Schiavilla, Wiringa & co.
• Correlated basis functions and cluster expansion: Pisa & Perugia Groups
OUR APPROACH

\[ \Psi_o = \hat{F} \Phi_o \]

\[ \hat{F} = \hat{S} \prod_{i<j} \hat{f}_{ij} = \hat{S} \prod_{i<j} \left[ \sum_n f^{(n)}(r_{ij}) \hat{\Omega}_{ij}^{(n)} \right] \]
THE RELEVANT QUANTITY: DENSITY MATRICES

Diagonal one-body density matrix (1BDM) (matter distribution):
\[ \rho_{(1)}(r_1) = \int |\Psi_0(r_1, r_2 \ldots, r_A)|^2 \prod_{i=2}^{A} dr_i \]

Non diagonal (1BDM) (One-body density fluctuations):
\[ \rho_{(1)}(r_1, r'_1) = \int \Psi_0^*(r_1, r_2 \ldots, r_A) r_i \Psi_0(r'_1, r_2 \ldots, r_A) \prod_{i=2}^{A} dr_i \]

Non diagonal 2-body density matrix (2BDM) (two body density fluctuations):
\[ \rho_{(2)}(r_1, r_2; r'_1, r'_2) = \int \Psi_0^*(r_1, r_2 \ldots, r_A) \Psi_0(r'_1, r'_2 \ldots, r_A) \prod_{i=3}^{A} dr_i \]

Diagonal 2BDM:
\[ \rho_{(2)}(r_1, r_2) = \int |\Psi_0(r_1, r_2 \ldots, r_A)|^2 \prod_{i=3}^{A} dr_i \]
The relative (rel) and center-of-mass (CM) density matrices

\[ r = r_1 - r_2 \quad R = (r_1 + r_2)/2 \]

\[ \rho_{(2)}(r, R) = \int |\Psi_0(R + \frac{r}{2}, R - \frac{r}{2}, r_3 \ldots, r_A)|^2 \prod_{i=3}^{A} dr_i \]

\[ \rho_{CM}(R) = \int \rho_{(2)}(r, R) dr \]

\[ \rho_{rel}(r) = \int \rho_{(2)}(r, R) dR \]

The relative 2BDM has been calculated by different groups within different many-body approaches and realistic bare NN interactions.
The RELATIVE 2BDM and the CORRELATION HOLE in FEW-NUCLEON SYSTEMS


Figure 1: The two-body relative distribution in $^3$He and $^4$He (After Ref. [?])
The 2BDM $\rho_{(2)}$ in few-nucleon systems in $(ST)=(10)$ and $(01)$ states


At $r < 1.5\, fm$ the 2BDM exhibits A-independence

UNIVERSALITY of SRC
At $r < 1.5 \text{fm}$ the 2BDM exhibits A-independence in complex nuclei as well.

UNIVERSALITY of SRC
The Correlated 2BDM versus the Mean-Field 2BDM

Figure 2: The two body density distribution within realistic and mean-field approaches for $^{16}O$
• SRC create the *correlation hole*, generated by the cooperation of the *short-range repulsive interaction* and the *intermediate-range tensor attraction*. The basic features of the correlation hole are independent of the mass $A \implies$ universality of SRC.

• SRC modify the spin-isospin content of the wave function.

• How can we investigate the existence and the properties of the correlation hole? To this end we have to shift to momentum space. What do we expect? We expect: (i) an increase of nucleon high momentum components, and (ii) peculiar momentum configurations in the nuclear wave function.
THE NUMBER OF PAIRS IN SPIN-ISOSPIN STATES.

Pauli Principle: \( l+S+T \)-odd

Shell Model (IPM):

\( A \leq 4 \): \( l - \) even, \((10),(01)\) – \( A > 4 \): \( l - \) even, \((10),(01)\); \( l = \) odd, \((00),(11)\)

\( NN \) interaction creates states \((00)\) and \((11)\) also in \( A \leq 4 \) nuclei

The pair \((ST)\) probabilities:

p-n pair: \( \frac{3}{4} [(10)+(00)] + \frac{1}{4}[(01)+(11)] \)

p-p (n-n) pair: \( (01) +(11) \)
The number of pairs in various \((ST)\) states is then given by

\[
N^{N_1N_2}_{(ST)} = \int d\mathbf{r}_1\, d\mathbf{r}_2 \rho^{N_1N_2}_{ST}(\mathbf{r}_1 = \mathbf{r}_1'; \mathbf{r}_2 = \mathbf{r}_2')
\]
The number of NN pairs in various spin-isospin (ST) states

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>(ST)</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(10)</td>
<td>(01)</td>
<td>(00)</td>
<td>(11)</td>
</tr>
<tr>
<td>²H</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>³He</td>
<td>1.50</td>
<td>1.50</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>1.488</td>
<td>1.360</td>
<td>0.013</td>
<td>0.139</td>
</tr>
<tr>
<td></td>
<td>1.50</td>
<td>1.350</td>
<td>0.01</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>1.489</td>
<td>1.361</td>
<td>0.011</td>
<td>0.139</td>
</tr>
<tr>
<td>⁴He</td>
<td>3</td>
<td>3</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>2.99</td>
<td>2.57</td>
<td>0.01</td>
<td>0.43</td>
</tr>
<tr>
<td></td>
<td>3.02</td>
<td>2.5</td>
<td>0.01</td>
<td>0.47</td>
</tr>
<tr>
<td></td>
<td>2.992</td>
<td>2.572</td>
<td>0.08</td>
<td>0.428</td>
</tr>
<tr>
<td>³⁰O</td>
<td>30</td>
<td>30</td>
<td>6</td>
<td>54</td>
</tr>
<tr>
<td></td>
<td>29.8</td>
<td>27.5</td>
<td>6.075</td>
<td>56.7</td>
</tr>
<tr>
<td></td>
<td>30.05</td>
<td>28.4</td>
<td>6.05</td>
<td>55.5</td>
</tr>
<tr>
<td>⁴⁰Ca</td>
<td>165</td>
<td>165</td>
<td>45</td>
<td>405</td>
</tr>
<tr>
<td></td>
<td>165.18</td>
<td>159.39</td>
<td>45.10</td>
<td>410.34</td>
</tr>
</tbody>
</table>

- NN interaction doesn’t practically affect the state (10) but appreciably reduces the state (01) giving rise to a "visible" content of the (11) state; this is due to a three-body mechanism originating from the tensor force.
THE THREE-BODY MECHANISM

H. Feldemeier, W. Horiuchi, T. Neff, Y. Suzuki

\[ S=0, \, T=1, \, L=0 \quad \text{uncorrelated} \]

\[ S=1, \, T=0, \, L=0 \]

\[ S=1, \, T=1, \, L=0 \quad \text{correlated} \]

\[ S=1, \, T=0, \, L=2 \]

**IPM**: only \( L=0 \) \((10), \,(01)\) states are possible

**Correlated particles**: tensor interaction in the p-n pair in \( L=2 \) can induce a spin flip in the p-p pair with creation of a state \( L=1, \,(11) \) of the pair. Three-body effect.
(i) increase of the high momentum content of the wave function

Mean-field (shell model) wave function

$$\Phi_0(1, 2, \ldots, A) = \hat{A} \prod_{i} \phi_{\alpha_i}(x_i) \equiv \Phi_{0p0h} \quad \alpha_i > \alpha_F, \phi_{\alpha_i} = 0$$

Correlated wave function

$$\Psi_0(1, 2, \ldots, A) = C_{0p0h} \phi_{0p0h} + C_{1p1h} \phi_{1p1h} + C_{2p2h} \phi_{2p2h} + \ldots$$

$$SRC \quad \Rightarrow \sum_{n=2}^{\infty} C_{nph} \phi_{nph}$$

Thus:

SRC populate states (n particle-n hole) with momentum much higher than the Fermi momentum $$k_F \simeq 1.4 fm^{-1}$$!!!
Momentum conservation

\[ \sum_{1}^{A} \vec{k}_i = 0 \]

Consider a nucleon with high momentum \( \vec{k}_1 \)

In a mean-field configuration

\[ \vec{k}_1 \simeq - \sum_{2}^{A} \vec{k}_i \quad \vec{k}_i \simeq \frac{\vec{k}_1}{A} \]

In a two-nucleon correlation configuration

\[ \vec{k}_1 \simeq - \vec{k}_2 \quad \vec{K}_{A-2} = \sum_{3}^{A} \vec{k}_i \simeq 0 \quad \vec{k}_{rel} \simeq \vec{k}_1 \quad \vec{K}_{CM} = - \vec{K}_{A-2} \simeq 0 \]

\textbf{SRC :} \textcolor{red}{\text{HIGH}} relative and \textcolor{blue}{\text{LOW}} CM momenta of a pair.

THE HIGH MOMENTUM COMPONENTS IN THE ONE-BODY MOMENTUM DISTRIBUTION

\[
\rho(r_1, r'_1) = \int \Psi_0^*(r_1, r_2 \ldots, r_A) \Psi_0(r'_1, r_2 \ldots, r_A) \prod_{i=2}^{A} dr_i
\]

\[
n(k) = \int e^{-i \mathbf{k} \cdot (r_1 - r'_1)} \rho(r_1, r'_1) dr_1 dr'_1
\]

\[
n_A(k_1) = \sum_{ST} n_A^{(ST)}(k_1) = \\
= \int d\mathbf{r}_1 d\mathbf{r}'_1 e^{i \mathbf{k}_1 \cdot (\mathbf{r}_1 - \mathbf{r}'_1)} \sum_{ST} \int d\mathbf{r}_2 \rho_{ST}^{N_1N_2}(\mathbf{r}_1, \mathbf{r}'_1; \mathbf{r}_2)
\]

Alvioli, CdA, Kaptari, Mezzetti, Morita, 
The ratio $n_A(k)/n_D(k)$ according to recent calculations.

The increase of the ratio with $k$ originates from the spin-isospin dependence of the momentum distributions and from the CM motion of the pair in the nucleus.
A proton is correlated with one p-n and one p-p pair; a neutron with two n-p pair $\rightarrow$ Tensor dominance in neutron (proton) distributions in $^3He$ ($^3H$) and in neutron-rich nuclei.
TWO-BODY MOMENTUM DISTRIBUTIONS

\[ k_{\text{rel}} \equiv k = \frac{1}{2} (k_1 - k_2) \quad K_{CM} \equiv K = k_1 + k_2 \]

1. \[ n(k_1, k_2) = n(k_{\text{rel}}, K_{CM}) = n(k_{\text{rel}}, K_{CM}, \theta) = \]
\[ = \frac{1}{(2\pi)^6} \int \! \! dr \, dr' \, dR \, dR' \, e^{-i K \cdot (R-R')} \, e^{-i \mathbf{k} \cdot (r-r')} \rho^{(2)}(r, r'; R, R') \]

2. \[ n(k_{\text{rel}}, K_{CM} = 0) \]
\[ K_{CM} = 0 \implies k_2 = -k_1, \text{ back-to-back nucleons, like in the deuteron} \]

3. \[ n_{\text{rel}}(k) = \frac{1}{(2\pi)^3} \int \! \! n(k, K) \, dK \]

4. \[ n_{CM}(K) = \frac{1}{(2\pi)^3} \int \! \! n(k, K) \, dk \]
$n(k_{rel}), n(K_{CM})$ in FEW-NUCLEON SYSTEMS

Schiavilla et al Phys. Rev. Lett. 98(2007)132501  \quad q \equiv k_{rel} \quad Q \equiv K_{CM}$

TENSOR DOMINANCE
$n(k_{\text{rel}}, K_{CM} = 0)$ in COMPLEX NUCLEI

SPIN-ISOSPIN DEPENDENCE of $n_{rel}(k_{rel})$ in FEW-NUCLEON SYSTEMS


UNIVERSALITY: $n_{rel}^A(k_{rel}) \simeq C_A n_D(k)$ in (10) state
THE 3D PICTURE OF $n(k_{rel}, K_{CM}) = n(k_{rel}, K_{CM}, \Theta)$

! VERY IMPORTANT !

• If $n(k_{rel}, K_{CM}, \Theta)$ is $\Theta$ independent, it means that $n(k_{rel}, K_{CM}) = n(k_{rel}) n(K_{CM})$ i.e. the relative and CM motions factorize.
$n(k_{rel}, K_{CM}, \theta)$ symbols-$\Theta = 90^\circ$, dashes-$\Theta = 180^\circ$, full-$2H$.

at large values of $k_{rel}$ and small values of $K_{CM}$ we have:

$$n^{pn}(k_{rel}, K_{CM}) \rightarrow n^{pn}(k_{rel}, K_{CM}) \approx n^D(k_{rel}) n_{CM}(K_{CM})$$

Factorization is proved by a rigorous many-body calculation.
We demonstrated that in the region $k_{\text{rel}} \geq k_{\text{rel}}^-(K_{CM})$ factorization occurs.

\[ k_1 + k_2 - K_{CM} = 0, \quad k_{\text{rel}} = (k_1 - k_2)/2, \quad k_2 = -k_1 + K_{CM}, \quad k_{\text{rel}} = k_1 - K_{CM}/2 \]

\[
n_{pn}(k_{rel}, K_{CM}) \simeq n_D(k_{rel}) n_{CM}(K_{CM}) = n_D(|k_1 - \frac{K_{CM}}{2}|) n_{CM}(K_{CM})
\]

which means

\[
n^N(k_1) \simeq \int n_D(|k_1 - \frac{K_{CM}}{2}|) n_{CM}^N(K_{CM}) \, dK_{CM} = \int P^N(k_1, E_{A-1}^*) \, dE_{A-1}^*
\]

where $P^N(k_1, E_{A-1}^*)$ is the NUCLEON SPECTRAL FUNCTION

\[
P^N(k_1, E_{A-1}^*) = \int n_D(|k_1 - \frac{K_{CM}^N}{2}|) n_{CM}^N(K_{CM}) \, dK_{CM} \times
\]

\[
\times \delta \left( E_{A-1}^* - \frac{A - 2}{2m_N(A - 1)} \left[ k_1 - \frac{A - 1}{A - 2} K_{CM} \right]^2 \right)
\]
Points: numerical calculation of the spectral functions of $^3$He (Ciofi degli Atti, Pace, Salmè, PRC 21 (1980)805) and NM (Benhar, Fabrocini, Fantoni, Nucl. Phys. A550(1992)201)

Curves: 2N correlation model

\[ P_i^A(k, E) = \int d^3k_{cm} n_{rel}^A(\sqrt{k - k_{cm} / 2}) n_{cm}^A(\sqrt{k_{cm}}) \]

\[ S \left[ E - E_{th}^{(2)} - \frac{(A-2)}{2M(A-1)} \left( \frac{k - (A-1)k_{cm}}{(A-2)} \right)^2 \right] \]

Recently (Massimiliano’s talk)

\[ n_{cm}^A(k_{cm}) \quad n_{rel}^A(k_{rel}) \]

have been calculated by many-body approach

\[ \rightarrow \text{no free parameters!} \]

CdA, Simula Nucl. Phys. 1996

Claudio Ciofi degli Atti
The CM distribution of pn and pp pairs

\[ n_{CM}^{pN}(K_{CM}) = \int d\mathbf{k}_{rel} \, n_{CM}^{pN}(\mathbf{k}_{rel}, K_{CM}) \]

The low momentum part of \( n_{CM}(K_{CM}) \) for \( A > 4 \) agrees with a Gaussian \( e^{-\alpha K_{CM}^2} \), in agreement with the convolution model where

\[ \alpha = \left[ \frac{3(A - 1)}{4(A - 2)} \right] \cdot \left[ \frac{1}{m_N < T_{SM}>} \right] \]

(Theor. Prediction for \( A = 12; \sigma = 139 \text{ MeV/c} \) (Nucl. Phys. 1966); Exp. value from \( ^{12}C(p,ppn)X; \sigma = 143 \pm 17 \text{ MeV/c} \) (PRL 2003).)
THE CONVOLUTION STRUCTURE OF $P(k, E)$ IS A GENERAL FEATURE OF THE SPECTRAL FUNCTION, RESULTING FROM SOME GENERAL PROPERTIES OF THE MANY-BODY WAVE FUNCTIONS IN MOMENTUM SPACE.
Towards the 2NC region, where $|K_{cm}| \ll |k_{rel}|$, if the factorization, and therefore the convolution model, holds, one should have

$$R = \frac{|M(K_{cm}, k_{rel})|^2}{|M(0, k_{rel})|^2} \sim \frac{n_{cm}(|K_{cm}|)}{n_{cm}(|K_{cm}| = 0)} = \text{constant} \cdot n_{cm}(|K_{cm}|)$$

This behavior is found indeed: a clear signature of factorization in momentum space.

The high momentum and energy behaviour of the nucleon spectral function of nuclear matter within the Brueckner–Bethe–Goldstone approach

M. Baldo\textsuperscript{a}, M. Borromeo\textsuperscript{b,c}, C. Ciofi degli Atti\textsuperscript{b,c}

\textsuperscript{a} INFN, Sezione di Catania, 57 Corso Italia, I-95129 Catania, Italy
\textsuperscript{b} Dipartimento di Fisica, Università di Perugia, Via A. Pascoli, I-06100 Perugia, Italy
\textsuperscript{c} INFN, Sezione di Perugia, Via A. Pascoli, I-06100 Perugia, Italy

Received 14 February 1996

Abstract

The nuclear single-particle spectral function is considered in the region of high momentum and high removal energy. For these kinematical conditions, far away from the quasi-particle peak, the spectral function is expected to be dominated by nucleon–nucleon correlations. It has been previously argued that the spectral function can be written as a convolution between the two-body relative momentum distribution and the corresponding centre-of-mass distribution of the correlated pairs which characterize the structure of the ground state in this energy–momentum region. It is shown that the convolution model can be microscopically derived from the Brueckner–Bethe–Goldstone (BBG) expansion. At the same time, this result also allows us to establish a direct link between the spectral function and the defect function of the BBG theory. From a numerical comparison with the microscopic spectral function the convolution model turns out to be highly accurate in the relevant momentum and energy range.
3. The spectral function of nuclear matter within the BBG theory

In NM the spectral function corresponding to the nucleon self-energy \( M(k, E) = V(k, E) + iW(k, E) \), is given by the well-known result [6]

\[
P(k, E) = -\frac{1}{\pi} \text{Im} \, G(k, E) = \frac{1}{\pi} \frac{W(k, E)}{(-E - k^2/2m - V(k, E))^2 + W(k, E)^2},
\]

where \( G(k, E) \) is the single-particle Green function

\[
G(k, E) = \frac{1}{-E - k^2/2m - V(k, E) - iW(k, E)}.
\]

It has to be noticed that the real, \( V(k, E) \), and imaginary parts, \( W(k, E) \), of the self-energy are highly off-shell in the considered energy and momentum ranges. We are interested in the region where \( E \) is much greater than the Fermi energy \( E_F \). For high \( k \) and \( E \), one finds

\[
E + \frac{k^2}{2m} \gg |V(k, E)|, |W(k, E)|,
\]

as can be seen from the results shown in Ref. [7], and the spectral function can thus

\[
P(k, E) = \frac{\pi^2 \rho^2}{16} \int \frac{d^3P}{(2\pi)^3} n_{\text{rel}}(|k - \frac{1}{2}P|) n_{\text{cm}}^{\text{FG}}(P) \times \delta \left( E - E_{\text{thr}}^{(2)} - E^* - \frac{1}{2m} (P - k)^2 \right),
\]
Fig. 4. Comparison between the SF obtained from the convolution model (dashed lines) and the one obtained from BBG theory (diamonds) for different values of the nucleon momentum $k$. 
3 EXPERIMENTAL EVIDENCE OF SRC
3.1 The momentum distributions from inclusive $A(e,e')X$ processes

- Errors very large
- At high $k$: errors much less than the difference between Mean-Field and correlated distributions
- Experimental data exist only for a limited range of $A$ and low values of momenta.


See also a recent review: Arrington et al, Progr. Part. Nucl. Phys. 2012
3.2 The inclusive cross section ratio (a very useful quantity)


\[ \sigma_A(x_B, Q^2) \approx \frac{A}{2} a_2(A) \sigma_2(1.5 < x_B < 2, Q^2) + \frac{A}{3} a_3(A) \sigma_3(2 < x_B < 3, Q^2) + \ldots \]
Linked cluster expansion for the calculation of the semi-inclusive $A(e,e'p)X$ processes using correlated Glauber wave functions

Claudio Ciofi degli Atti  
*Department of Physics, University of Perugia, and Istituto Nazionale di Fisica Nucleare, Sezione di Perugia, Via A. Pascoli, I-06100 Perugia, Italy*

Daniele Treleani  
*Department of Theoretical Physics, University of Trieste, and Istituto Nazionale di Fisica Nucleare, Sezione di Trieste, and ICTP, Strada Costiera 11, I-34014, Trieste, Italy*  
(Received 2 February 1999; published 25 June 1999)

The distorted one-body mixed density matrix, which is the basic nuclear quantity appearing in the definition of the cross section for the semi-inclusive $A(e,e'p)X$ processes, is calculated within a linked cluster expansion based upon correlated wave functions and the Glauber multiple scattering theory to take into account the final state interaction of the ejected nucleon. The nuclear transparency for $^{16}\text{O}$ and $^{40}\text{Ca}$ is calculated using realistic central and noncentral correlations and the important role played by the latter is illustrated.

[S0556-2813(99)00208-3]
the cross section \(1\) becomes directly proportional to the distorted momentum distributions \(8\), i.e.,

\[
n_D(k_m) = (2\pi)^{-3} \int e^{ik_m(r-r')} \rho_D(r,r') \, d\mathbf{r} \, d\mathbf{r}', \tag{12}
\]

where

\[
\rho_D(r,r') = \frac{\langle \Psi_A S_G^* \hat{O}(r,r') S_G \Psi_A' \rangle}{\langle \Psi_A \Psi_A' \rangle} \tag{13}
\]

is the one-body mixed density matrix, and

\[
\hat{O}(r,r') = \sum_i \delta(r_i - r) \delta(r_i' - r') \prod_{j \neq i} \delta(r_j - r_j') \tag{14}
\]

the one-body density operator. In Eq. \(13\) and in the rest of the paper, the primed quantities have to be evaluated at \(r'\) with \(i = 1 \cdots A\). By integrating \(\langle \hat{O} \rangle\) the nucleus trans

\[
S_G(r_1 \cdots r_A) = \prod_{j=2}^A G(r_1, r_j) = \prod_{j=2}^A [1 - \theta(z_j - z_1) \Gamma(b_1 - b_j)], \tag{7}
\]
3.3 Exclusive one-body knock-out reactions $A(a,a'N)X a=(e,N)$

The $(e,e'p)$ process on mean field and correlated nucleons.

**Mean Field:**

$k_1 + k_{A-1} = 0$

**Correlations:**

$k_1 + k_2 + k_3 = 0$

**Simple model:**

$$\begin{cases} k_2 \simeq -k_1 & k_3 \simeq 0 \\ E^*_{A-2} = 0 \\ E^*_{A-1} \simeq \frac{A-2}{A-1} \frac{k_1^2}{2m_N} \end{cases}$$

**Realistic model:**

$$\begin{cases} k_3 \neq 0 & E^*_{A-2} \neq 0 \\ E^*_{A-1} = \frac{A-2}{2m_N(A-1)} \left[k_1 + \frac{A-1}{A-2} k_3\right]^2 + \bar{E}_{A-2} \end{cases}$$

C. Ciofi degli Atti

Valparaiso, February 2009
One important caveat

\[
\begin{align*}
\text{WISH} & \quad \text{REALITY} \\
P(k_1, E) & \quad p = k_1 + q \\
\text{Impulse Approximation} & \quad \text{Final State Interaction}
\end{align*}
\]

\[ e + ^3He \rightarrow e' + p + ^2H \quad e + ^3He \rightarrow e' + p + (pn) \]


FSI under control. SRC peak observed. Agreement with other groups.
3.4 Exclusive two-body knock-out reactions \( A(a,a'2N)X \) \( a=(e,N) \Rightarrow \) two-body nucleon spectral function.

By detecting 2 Nucleons in the final state the initial pair correlation can be studied.

Triple coincidence experiment \( A(a,\alpha NN)X \) \( a = \{p, e\} \)

BNL and JLAB EXPERIMENTS

(Watson, Gilad, Piasetzky, & coworkers, this Workshop)
$^{12}C(p, p'pN)X$ AGK BNL (2003); Piasetzky talk

Experiment

Analysis
Piasetzky, Sargsian, Frankfurt, Strikman, Watson
PRL 162504 (2006)
IMPACT OF SRC ON VARIOUS FIELDS OF PHYSICS
3.1. Transition from hadron to quark gluon descriptions of nuclei

Nucleon radius \( \langle r^2 \rangle^{1/2} \approx 0.8 \text{fm}^{-1} \) ⇒ Nucleon overlap.

3.2 Formation of cold dense nuclear matter in the laboratory and the structure of neutron stars

Implications for Neutron Stars

- At the core of neutron stars, most accepted models assume \( \sim 95\% \) neutrons, \( \sim 5\% \) protons
- Neglecting the np-SRC interactions, one can assume two separate Fermi gases
- Since np interaction is large compared to nn, n gas heats the p gas
- This could effect the upper limit on mass of neutron and allow the neutrons in the star decay

3.3 High energy hadron-Nucleus and Nucleus-Nucleus scattering


Glauber \ + \ Gribov Inelastic shadowing \ + \ SRC

(Glauber) \hspace{1cm} (Inelastic Shadowing)
The exact expansion of $|\Psi_0|^2$ (Glauber, Foldy & Walecka):

$$|\Psi_0(r_1, \ldots, r_A)|^2 = \prod_{j=1}^{A} \rho(r_j) + \sum_{i<j=1}^{A} \Delta(r_i, r_j) \prod_{k \neq (il)}^{A-2} \rho(r_k) +$$

$$+ \sum_{(i<j) \neq (k<l)}^{A-4} \Delta(r_i, r_j) \Delta(r_k, r_l) \prod_{m \neq (i,j,k,l)}^{A-4} \rho_1(r_m) + \ldots$$

$$\Delta(r_i, r_j) = \rho^{(2)}(r_i, r_j) - \rho^{(1)}(r_i) \rho^{(1)}(r_j);$$

$$\rho^{(1)}(r_1) = \int |\Psi_0(r_1, \ldots, r_A)|^2 \prod_{i=2}^{A} dr_i; \quad \rho^{(2)}(r_1, r_2) = \int |\Psi_0(r_1, \ldots, r_A)|^2 \prod_{i=3}^{A} dr_i$$

$$\int dr_j \rho^{(2)}(r_i, r_j) = \rho^{(1)}(r_i); \quad \int dr_j \Delta(r_i, r_j) = 0$$

$$\rho^{(1)}(r) = \rho(r)$$
R. J. Glauber, *High Energy Collision Theory, 1971*

"Various types of correlations in positions and spin may exist between nucleons of an actual nucleus ... If the system being considered is spatially uniform an idea of the magnitude and nature of the effects due to pair correlations may be obtained by assuming that the range of NN force $a$ is smaller than the range of correlations $l_c$ and the nuclear radius $R$

$$l_c \gg a \text{ and } R \gg a$$

Because $R$ is not vastly larger than $a$, and the correlation length $l_c$ is not too different in magnitude from the force range, the approximations that follow from these conditions should only be used for rough estimates".

C. Ciofi degli Atti
Claudio Ciofi degli Atti

Otranto, September 10-15, 2010

February 11-22, 2013 INT, Seattle, USA
**The total neutron – Nucleus cross section at high energies:**


- No free parameters!!
- Full SRC.
- Gribov inelastic shadowing at lowest order.
- Main result: SRC increase the opacity, Gribov IS decreases it, the two effects being of about the same order in this energy range.
- What about higher order Gribov corrections?.

C. Ciofi degli Atti
Another recent calculation of the effects of SRC in high energy scattering processes:

”A Monte Carlo generator of nucleon configurations in complex nuclei including Nucleon-Nucleon correlations”

M. Alvioli, H.J. Drescher and M. Strikman,
4. CONCLUSIONS
• NN SRC can be calculated \textit{ab initio} with realistic NN interactions. They exhibit several universal (\textit{independent of }A) features.

• robust evidence on the effects of NN SRC have been collected in the last few years both in few-nucleon systems and $^{12}C$.

• NN SRC can provide basic information on the nature of the NN force. The experimental information so far obtained is in agreement with the current picture of phenomenological realistic NN interactions.

• SRC can have relevant effects on the structure of cold dense hadronic matter and high energy $h-A$ and $A-A$ scattering processes.

• The successful experimental study of NN SRC is a relatively new field of research that has to be continued, extending it to an increasing number of nuclei and to the investigation of the 3D structure of SRC (JLab, JPARC(?)).