Short Distance Studies of the Deuteron

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Introduction: Role of the Deuteron

- Key system to investigate the (repulsive) core of the NN interaction.
- Basis for SRC (structure) studies
- Prime nucleus to test NN models
- Structure needs to be understood in detail at all length scales
virtually no experimental d(e, ep)n data exist for $p_m > 0.5$ GeV/c without large contributions of FSI, MEC and IC

several low $Q^2$ data available

mostly unexplored important for SRC studies
Problems

- **Reaction dynamics:**
  - how does the photon interact with a deeply bound nucleon?
  - what is the EM current structure?

- **Final State Interactions**
  - high $Q^2$: eikonal approximations valid?

- **Deuteron wave function**
  - can one probe NN wave function at small distances?
  - can one find evidence for new degrees of freedom?
  - important for the interpretation of DIS data

All these problems are interconnected. New, high $Q^2$ data are necessary for progress!
\[ \frac{d\sigma}{d\Omega} = \sigma_{Mott} \left[ A(Q^2) + B(Q^2) \tan^2(\theta/2) \right] \]

\[ A = G_C^2 + \frac{2}{3} \eta G_M^2 + \frac{8}{9} \eta^2 G_Q^2 \]

\[ B = \frac{4}{3} \eta (1 + \eta) G_M^2 \]

\[ T_{20} = -\frac{\frac{8}{9} \eta^2 G_Q^2 + \frac{8}{3} \eta G_C G_Q + \frac{2}{3} \eta G_M^2 \left[ \frac{1}{2} + (1 + \eta) \tan^2(\theta/2) \right]}{\sqrt{2} \left[ A + B \tan^2(\theta/2) \right]} \]

\[ \eta = \frac{Q^2}{4M_D^2} \]

**Review Articles:**
Rel. Calculations in Hamiltonian dynamics:

\( \text{IMII and IM+EII} \)

Y. Huang and W. N. Polyzou
PRC 80 (2009) 025503

Rel. Calculations in propagator dynamics:

\( \rho \pi \gamma \phi / \gamma = 0 \)
\( \rho \pi \gamma \phi / \gamma = 6.1 \)

D. R. Phillips et al. PRC 72 (2005) 014006

no MEC contributions

with MEC

2/20/2013 Jefferson Lab Users Group Meeting, June 2012
D(e,e’) summary:

- NR models cannot describe the form factors up to the highest $Q^2$ (RC are very important)
- Indications of dimensional scaling exist.
- Relativistic models successfully describe Deuteron form factors
- MEC contributions are very important
- $\rho\pi\gamma$ exchange current important and not well constrained
Experimental Goal:

Obtain data closely related to the deuteron wave function (momentum distribution) with a minimum of "other contributions" such as FSI, MEC, IC etc.

Ideally ‘measure’ the momentum distribution ⇒ study the $d(e,e'p)$ reaction
D(e,e’p) Reaction Mechanisms

**PWIA**

\[ \frac{d\sigma}{d\omega d\Omega_e d\Omega_N} = k\sigma_{en} S(E_m, P_m) \]

\[ \vec{p}_m = -\vec{p}_i \]

**FSI**

\[ \frac{d\sigma}{d\omega d\Omega_e d\Omega_N} = k\sigma_{en} D(E_m, P_f, P_m) \]

\[ \vec{p}_m \neq -\vec{p}_i \]

**MEC**

expected to be small at large \( Q^2 \)

**IC**

suppressed for \( x > 1 \)

2/20/2013

INT Workshop INT-13, UWA
Missing Momentum Dependences

MAMI \( Q^2 = 0.33 \text{ (GeV/c)}^2 \)
Blomqvist et al. PLB 429 (1998)

JLAB \( Q^2 = 0.67 \text{ (GeV/c)}^2 \)
Ulmer et al. PRL 89 (2002) 062301-1

large FSI included

large IC+MEC

large FSI

This Experiment
Jeschonnek PWBA/AV18
Jeschonnek PWBA/Bonn
Arenhoevel PWBA/Bonn
Arenhoevel DWBA/Bonn
Arenhoevel FULL/Bonn

\( \frac{d^5 \sigma}{dp_\pi d^3 Q_e d^3 \omega} \text{ [fm}^3/\text{MeV s}^2] \)

\( p_\pi \text{ [MeV/c]} \)
cross sections averaged over CLAS acceptance!

$20^\circ \leq \theta_{nq} \leq 160^\circ$

$Q^2 = 2 \pm 0.25 \ (GeV/c)^2$

$Q^2 = 3 \pm 0.5 \ (GeV/c)^2$

- PWIA + FSI + N\Delta
- PWIA + FSI
- PWIA

FSI n-p dominate

black: Paris potential
red: V18 potential

Egyian et al. (CLAS) PRL 98 (2007)
At high $Q^2$ FSI as Rescattering

IA Amplitude (real):

$A_I$

Rescattering Amplitude (at high energy mostly imaginary):

$A_R$

Total scattering amplitude:  $A = A_I + i A_R$

Cross Section:

$|A|^2 \sim \sigma = |A_I|^2 + 2 |A_I||A_R| + |A_R|^2$

$R = \frac{\sigma}{\sigma_I} = 1 - 2 \frac{|A_I||A_R|}{|A_I|^2} + \frac{|A_R|^2}{|A_I|^2}$
CLAS

- Simultaneous measurement of kinematics
- Focus on $Q^2$ dependence
- e6 running period
- $Q^2 = 2, 3, 4, 5$ (GeV/c)$^2$
- Further analysis possible: Data Mining

Hall A

- $Q^2 = 0.8, 2.1$ and $3.5$ (GeV/c)$^2$: constant for each set
- $p_{\text{miss}} = 0.2, 0.4$ and $0.5$ GeV/c: angular distribution
- $20^\circ \leq \theta_{pq} \leq 140^\circ$
- Angular range for each $p_{\text{miss}}$ dependent on kinematics
CLAS

Data: Egyian et al. (CLAS) PRL 98 (2007)
Calculation M. Sargsian

\[ p_m = 250 \pm 50 \text{ MeV/c} \]

\[ p_m = 500 \pm 100 \text{ MeV/c} \]
Hall A \hspace{1cm} Q^2 = 3.5 (GeV/c)^2 \hspace{1cm} \Delta p_m \pm 20 MeV/c

\[ p_m = 50 \text{ MeV/c} \]

\[ p_m = 100 \text{ MeV/c} \]

\[ p_m = 200 \text{ MeV/c} \]

\[ p_m = 400 \text{ MeV/c} \]

\[ p_m = 500 \text{ MeV/c} \]

Calculations: M. Sargsian

\[ \phi = 180^\circ \]

\[ \phi = 0^\circ \]

\[ \phi = 0^\circ \]

\[ \phi = 180^\circ \]
for recoil angles around 40° FSI seem to be minimal and independent of $p_m$
Summary of angular distributions
At $Q^2 = 3.5 \, (\text{GeV/c})^2$

$$R = \frac{\sigma}{\sigma_{PWIA}}$$

$\sigma$ is experimental or calculated cross section

WB et al. PRL 107 (2011) 262501
small FSI difference

large WF difference

large FSI

small WF difference
each angle offset by 0.1

‘yellow’ n(p) Paris

‘cyan’ n(p) CD Bonn
Lower $Q^2$

Thesis H. Khanal

$R = \frac{\sigma_{\text{EXP}}}{\sigma_{\text{PWIA}}}$

No FSI ‘crossing’
Eikonal regime not yet reached

Crossing
Eikonal regime reached

= 2.1
Double Ratios

Preliminary

$= 2.1$

$Q^2 = 3.5$

Re-scattering increasing with $Q^2$
Extraction of $\rho(\alpha, p_t)$

• attempt to extract $\rho(\alpha, P_T)$ from experimental data
• Theoretical foundation:

Relativistic Description of the Deuteron, L.L Frankfurt and M. Strikman, Nuclear Physics B148 (1979) 107

Advantages of working on LC:

- at high $Q^2$, FSI is mostly transverse $\alpha$ is approx. conserved by FSI
- $\rho(\alpha)$ is very little affected by re-scattering
- at high energies: $\bar{NN}$ become important but unimportant on LC (photon energy is 0)
- $\rho(\alpha)$ necessary for interpretation of DIS data of nuclei

$$F_{2d}(x) = \sum N \int x^2 F_{2N}(\frac{x}{\alpha}) \rho(\alpha) \frac{d\alpha}{\alpha}$$
Light Cone Variables

Light cone variables for experimentalists:

4-vector: \( \mathbf{V} = (V^0, \vec{V}) \)  
light cone: \( \mathbf{V} = (V^+, V^-, \vec{V}_T) \)

\[ V^\pm = V^0 \pm V_z \]

Lorentz Transformation along z-axis:

\[ V'^\pm = e^\psi V^\pm \quad \psi = \frac{1}{2} \ln \left( \frac{1 + \beta}{1 - \beta} \right) \]

= scalar multiplication

Important property \( \frac{V'^\pm}{V^\pm} \) boost invariant
Deuteron Momentum Distributions on the Light Cone (LC)

LC momentum

\[ p^- = E - p_z \]

LC momentum fraction

\[ \alpha = A \frac{p_i^-}{P_A^-} \]

analogous to “x” for quark distributions

\[ \alpha \text{ is frame independent for boosts along the z-axis} \]
LC cross section

$$\frac{d\sigma}{dE'_e d\Omega_e d\Omega_p} = K \sigma_{eN}^{LC} (\alpha, p_t) \rho(\alpha, p_t)$$

Spectator (neutron) momentum fraction

$$\alpha_s = 2 \frac{E_s - p^z_s}{M_D}$$

Remember in lab:

$$P_D^- = M_D$$ and $$A = 2$$

Proton momentum fraction

$$\alpha = 2 - \alpha_s$$
LC momentum distribution

$$\rho(\alpha, p_t) = \frac{|\Psi_d(k)|^2 E_k}{2 - \alpha}$$

$$k = \sqrt{\frac{M_N^2 + p_t^2}{\alpha_s(2 - \alpha_s)} - M_N^2}$$

$$E_k = \sqrt{M_n^2 + p_t^2}$$

$k$ relative nucleon momentum in np system in light cone

Normalization:

$$\int \rho(\alpha) \frac{d\alpha}{\alpha} 2\pi p_t dp_t = 1$$

LC Momentum sum rule

$$\int \alpha \rho(\alpha) \frac{d\alpha}{\alpha} 2\pi p_t dp_t = 1$$
α conservation as function of nucleon momenta

\[ \alpha \text{ change} \]

\[ \Delta \alpha \]

\[ <k^2_\perp > = 0.5 \text{ GeV}^2/c^2 \]

transverse momentum transfer in re-scattering (FSI)

typical proton momenta
Contours of $k = \text{const}$

$P_T \text{(GeV/c)}$

$s$
Experimental $\rho(\alpha, p_t)$ distributions

- Determine $d(e,e'p)n$ cross section for each $\alpha_s, p_t$ bin
- Divide by $K\sigma_{eN}^{LC}$
- Problem: phase space acceptance
- Results should be as independent as possible of phase space cut
- Missing information due to cuts
Phase Space Coverage at $Q^2 = 3.5$

20% cut

2.5% cut
\[
\int \rho(\alpha, P_T) 2\pi P_T dP_T \approx \sum \rho(\alpha, P_T) 2\pi P_T \Delta P_T
\]
Interpolating missing data

Fit function: $\rho(\alpha) = \gamma \rho_{LC}(\alpha^*) e^{-\left(\delta_{s,l}(\alpha - A)\right)^2}$

$\alpha^* = 1 + \beta(\alpha - A)$

Parameters: $\alpha, \beta, \gamma, \delta_{s,l}, A$

use $\delta_s$ for $\alpha < A$

use $\delta_l$ for $\alpha > A$

Calculated using model: $\rho_{LC}(\alpha)$
(e.g. Paris WF)
20% cut

2.5% cut

$P_t = 0.51$

$P_t = 0.51$
\( \rho(\alpha) \) using fit interpolation

\[ \int \rho(\alpha) \, d\alpha = 0.97 \pm 0.005 \]

\[ \int \rho(\alpha) \, d\alpha = 0.97 \pm 0.005 \]

\[ \int \rho(\alpha) \, d\alpha = 0.94 \pm 0.002 \]

\[ \int \rho(\alpha) \, d\alpha = 0.94 \pm 0.002 \]

20% cut

2.5% cut
Experimental $|\psi(k)|^2$ distributions

- Determine $d(e,e'p)n$ cross section for each $\theta_{nq}, \rho_m$ bin
- Divide by $K\sigma_e^{LC}$
- Calculate $|\Psi_d(k)|^2$

Paris WF

Small FSI

Large FSI

FSI, $\Delta$?
Rotational Invariance
Response Functions

\[
\frac{d^5\sigma}{d\omega d\Omega_e d\Omega_p} = \sigma_M f_{rec} \left( v_L R_L + v_T R_T + v_{LT} R_{LT} \cos(\phi) + v_{TT} R_{TT} \cos(2\phi) \right)
\]

\[
A_{LT} = \frac{\sigma(\phi = 0) - \sigma(\phi = 180)}{\sigma(\phi = 0) + \sigma(\phi = 180)}
\]
At low $Q^2 A_{LT}$ is well understood

Future Experiment at 12 GeV

- Determine cross sections at missing momenta up to 1 GeV/c
- Measure at well defined kinematic settings
- Selected kinematics to minimize contributions from FSI
- Selected kinematics to minimize effects of delta excitation
Measurements in Hall C

Beam:
Energy: 11 GeV
Current: 80 µA

Electron arm fixed at:
SHMS at $p_{\text{cen}} = 9.32$ GeV/c
$\theta_e = 11.68^\circ$
$Q^2 = 4.25 \ (\text{GeV/c})^2$
$x = 1.35$

Vary proton arm to measure:
$p_m = 0.5, 0.6, 0.7, 0.8, 0.9, 1.0 \ \text{GeV/c}$
HMS 1.96 d $p_{\text{cen}}$ d 2.3 GeV/c
Angles: 63.5° e $\theta_p$ e 53.1

Target: 15 cm LHD
FSI Reduction

Reduction of FSI:  \[ \sigma \sim |A_I|^2 - 2|A_I||A_R| + |A_R|^2 \]

- \( b \) determined by nucleon size
- Cancellation due to imaginary rescattering amplitude
- Valid only for high energy (GEA)

\[ \sigma_R = |A_R|^2 \]
\[ \sigma_{\text{int}} = -2|A_I||A_R| \]

Rescattering determined by slope factor:

\[ f_s = e^{-\frac{b k_t^2}{2}} \]
\[ k_t = p_m \sin(\theta_{pm,q}) \]
\[ b \sim 6(GeV/c)^{-2} \]

\( f_s \) relatively flat up to \( k_t \approx 0.5(GeV/c) \) \( \Rightarrow p_m \approx 0.8(GeV/c) \)

Both terms are equal \( \Rightarrow \)
Interference and rescattering cancel

- \( b \) determined by nucleon size
- Cancellation due to imaginary rescattering amplitude
- Valid only for high energy (GEA)
Angular Distributions up to $p_m = 1$ GeV/c

FSI depend weakly on $p_m$

Calculation: M. Sargsian
Expected Results

- Measured cross sections for $p_m$ up to 1 GeV/c
- Errors: dominated by statistics: 7% - 20%
- Estimated systematic error $\pm 5\%$
- Very good theoretical support available
- JLAB uniquely suited for high $p_m$ study
- Good coincidence commissioning experiment
- 21 days of beam time
Summary

• High $Q^2$ $d(e,e'p)n$ can be described using generalized eikonal approximation for $Q^2 > 2$ GeV/c
• There is a window to study the Deuteron momentum distribution, CD Bonn seems OK
• Current analysis of lower $Q^2$ data (H.Khanal thesis) soon complete
• First attempt to extract $\alpha$ distributions, Paris seems OK
• Increase kinematics coverage for $\alpha$ determination
• High $R_{LT}$ at high $P_T$ cannot be reproduced
• 12 GeV: very high missing (up to 1 GeV/c) momenta