Fluctuation-Dissipation Dynamics of Fission and Heavy-Ion Fusion Reactions

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Theories for fission

- Transition state method
  - Statistical approach
    - Fission width

- Fluctuation-dissipation dynamics
  - Dynamical approach (Macroscopic)
    - Fission width
    - Saddle-to-scission dynamics

- Time-dependent mean field theory
  - Dynamical approach (Microscopic)
    - Saddle-to-scission dynamics
Transition state method

- **Fission width**

\[ dE \int_0^{E-B_f} \frac{vdE}{2\pi\hbar} \rho^*_A(E - B_f - \varepsilon) = dE \frac{\Gamma_f}{\hbar} \rho_A(E) \]

\[ \Gamma_f = \frac{1}{2\pi\rho_A(E)} \int_0^{E-B_f} d\varepsilon \rho^*_A(E - B_f - \varepsilon) \]

- **Fermi gas level density**

\[ \rho(E^*) \propto \exp\left(2\sqrt{aE^*}\right) \]

\[ \Gamma_f = \frac{T}{2\pi} \exp\left(-\frac{B_f}{T}\right) \]

\[ E^* = aT^2 \]

# of decay per unit time
Key quantities in fission

- Fission rate
  - Height of fission barrier

- Particle emission
  - Neutron emission
    - Pre-scission neutrons
    - Scission neutrons
    - Post-scission neutrons
  - Charged particle emission
  - Gamma emission

- Fission fragments
  - Mass distribution
  - Total kinetic energy distribution
Mass & TKE distribution

- Mass distribution is essentially determined by shell correction energy
  - At saddle: position & height fission barrier
  - Fission valleys and ridges
  - Fragments: magic number

- TKE (total kinetic energy)
  - Mainly Coulomb repulsion
  - Scission configuration
    - Nature of nuclear dissipation

- Dynamical treatment is necessary
Fluctuation-dissipation dynamics

- Brownian picture
  - Macroscopic degree(s) of freedom interacting with microscopic degrees of freedom in thermal motion
    - Dissipation (collective \(\rightarrow\) nucleonic) \(\Leftarrow\) Friction
    - Fluctuation (nucleonic \(\rightarrow\) collective) \(\Leftarrow\) Random force

- Macroscopic degrees of freedom
  - = Nuclear shape
  - (elongation, deformation, neck, mass asymmetry etc.)
Two approaches

Langevin equation

\[
\frac{dq}{dt} = \frac{p}{m} \quad \frac{dp}{dt} = -\frac{\partial U}{\partial q} - \frac{\gamma}{m} p + \sqrt{\gamma T} R(t)
\]

\[\langle R(t) \rangle = 0, \quad \langle R(t_1) R(t_2) \rangle = 2\delta(t_1 - t_2)\]

Eq. of motion of a Brownian particle

Focker-Planck (Kramers) equation

\[
\frac{\partial f}{\partial t} = -\frac{p}{m} \frac{\partial f}{\partial q} + \frac{\partial U}{\partial q} \frac{\partial f}{\partial p} + \frac{\gamma}{m} \frac{\partial}{\partial p} (pf) + \gamma T \frac{\partial^2 f}{\partial p^2}
\]

Distribution of the Brownian particles
Fission width by Kramers theory

- Diffusion process over barrier

\[
\frac{\partial f}{\partial t} = -\frac{p}{m}\frac{\partial f}{\partial q} + \frac{\partial U}{\partial q}\frac{\partial f}{\partial p} + \frac{\gamma}{m}\frac{\partial}{\partial p}(pf) + \gamma T \frac{\partial^2 f}{\partial p^2}
\]

- \(U = \) parabola + inverse parabola
  - Analytic solution

\[
\Gamma_f = K \frac{\hbar \omega}{2\pi} \exp\left(-\frac{B_f}{T}\right)
\]

\[
K = \frac{1}{2\omega_B} \left[ \sqrt{\beta^2 + 4\omega_B^2} - \beta \right], \quad \beta = \frac{\gamma}{m}
\]

Kramers factor

\[
U = \begin{cases} 
\frac{1}{2} m \omega^2 q^2 \\
B_f - \frac{1}{2} m \omega_B^2 (q - q_B)^2
\end{cases}
\]

- cf. BW

\[
\Gamma_f = \frac{T}{2\pi} \exp\left(-\frac{B_f}{T}\right)
\]
Transport coefficients

- Inertia mass tensor
  - Hydrodynamical mass
    - Werner-Wheeler approximation
  - Cranking mass

- Friction tensor
  - One-body friction
    - Wall and Window formula
    - Interaction of nucleons with nuclear surface (wall)
    - Exchanging nucleons through the neck window
  - Two-body viscosity
    - Energy loss by nucleon-nucleon collisions
TKE systematics

- Viola systematics
  (1985, V. E. Viola)
  \[
  \langle E_K \rangle = 0.1071 \frac{Z^2}{A^{1/3}} + 22.2 \text{ MeV}
  \]
  Data before 1966
  \[
  \langle E_K \rangle = 0.1189 \frac{Z^2}{A^{1/3}} + 7.3 \text{ MeV}
  \]
  Data up to 1984
- Main contribution to TKE comes from Coulomb repulsion
- Measure of the fragment deformation
Saddle-to-scission dynamics

- Saddle-to-scission time ($\tau^{ssc}$) depends on the dissipation
- Scission shape also depends on the dissipation
- Fragment TKE also depends on the dissipation

$$TKE = K_{pre} + V_C$$
Coordinate dependence of friction

- Reduced dissipation
  \[ \beta = \gamma / m \]
- Different dependence on elongation
  - Wall-and-Window
  - Wall
  - Two-body
Time-dependent fission rate

- Delay of fission
  - $\tau_{delay} = \tau_{tr} + \tau_{ssc}$
  - Transient time
  - Saddle-to-scission time

- Typical time
  - $\tau_{tr}, \tau_{ssc} \approx 10^{-21}$ s: two-body
  - $\tau_{tr}, \tau_{ssc} \approx 10^{-20}$ s: one-body
Fission and nuclear dissipation

- Two types of dissipation tensors
  - One-body Wall-and-Window dissipation
  - Two-body hydrodynamical dissipation

\[ \tau_{\text{delay}} = \tau_{\text{tr}} + \tau_{\text{ssc}} \]
\[ TKE = K_{\text{pre}} + V_C \]

<table>
<thead>
<tr>
<th>Energy (MeV)</th>
<th>( \sigma_{\text{fus}} )</th>
<th>( \sigma_{\text{fiss}} )</th>
<th>( \sigma_{\text{er}} )</th>
<th>( \nu_{\text{pre}} )</th>
<th>TKE</th>
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<tr>
<td>Exp.</td>
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<td>–</td>
<td>–</td>
<td>7.7±0.3</td>
<td>139</td>
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</tbody>
</table>
Fission modes

K. –H. Schmidt et al.
Shell correction energy

- **Strutinsky method**
  \[ \Delta E_{\text{shell}} = \sum_{i}^{N} \varepsilon_i - \bar{E} \]
  \[ N = \int^{\varepsilon_F} g(\varepsilon) d\varepsilon, \quad \bar{E} = \int^{\varepsilon_F} g(\varepsilon) \varepsilon d\varepsilon \]
  \[ g(\varepsilon) = \sum_{i}^{N} \frac{1}{\sqrt{2\pi}\sigma} \exp\left( -\frac{(\varepsilon - \varepsilon_i)^2}{2\sigma^2} \right) \sum_{m} H_m \left( \frac{\varepsilon - \varepsilon_i}{\sigma} \right) \]

- **Macro-microscopic approach**
  \[ \bar{E} \rightarrow E_{\text{Macro}}, \quad E = E_{\text{Macro}} + \Delta E_{\text{shell}} \]

- **Damping of shell correction**
  \[ \Delta E_{\text{shell}}(q, E_x) = \Delta E_{\text{shell}}(q, E_x = 0) \exp(-E_x/E_d) \]

- **Essential for asymmetric fission**
Extension to multi-dimension

Multi-dimensional Langevin equation

\[ \frac{dq_i}{dt} = (m^{-1})_{ij} p_j \quad i, j, k = 1, \ldots, N \]

\[ \frac{dp_i}{dt} = -\frac{\partial V}{\partial q_i} - \frac{1}{2} \frac{\partial}{\partial q_i} (m^{-1})_{jk} p_j p_k - \gamma_{ij} (m^{-1})_{jk} p_k + g_{ij} R_j(t) \]

\[ \langle R_i(t) \rangle = 0, \quad \langle R_i(t_1) R_j(t_2) \rangle = 2\delta_{ij} \delta(t_1 - t_2) \quad \sum_k g_{ik} g_{jk} = T\gamma_{ij} \]

\[ m_{ij}(q) \quad \text{Hydrodynamical inertial mass} \]

\[ \gamma_{ij}(q) \quad \text{Wall-and-Window (one-body) friction} \]

\[ V(q) \quad \text{Macro-microscopic potential} \]

\{q_i\} : collective parameters
  (elongation, fragment deformation, neck parameter, mass asymmetry)
TKE and mass distribution of fragments

Three-dimensional Langevin calculation

elongation, fragment deformation, mass-asymmetry

$^{264}$Fm, Ex=10MeV

Numbers of the peaks are different in mass and TKE
Deformation of the fragments at scission

\( \delta = 0.04 \rightarrow \delta = 0.26 \)

\( \text{Deformation Yield [%]} \)

\( \text{Mass Yield [%]} \)

\( \text{TKE Yield [%]} \)
Fission modes


δ ≤ 0.04 : Mass-symmetric & High TKE mode
0.04 < δ ≤ 0.26 : Mass-asymmetric & Medium TKE mode
δ > 0.26 : Mass-symmetric ? & Low TKE mode
Energy dependence of mass distribution

$^{264}\text{Fm}$ Ex=10MeV

Ex=25MeV

Deformation = TKE
Isotopic dependence of mass distribution

$^{244}\text{Fm}$  
$^{258}\text{Fm}$  
$^{272}\text{Fm}$  

Ex=25MeV

Deformation = TKE
**TKE and mass distribution of fragments**

Three-dimensional Langevin calculation

elongation, fragment deformation, mass-asymmetry

$^{236}\text{U}, \text{Ex}=20\text{MeV}$

![Graphs showing TKE yield and mass distribution](image)
Independent deformation of fragments

Four-dimensional energy surface

elongation, 2 fragment deformations, mass-asymmetry
Superheavy Elements

- How many elements can exist in nature?
  - Stability against fission

- Superheavy elements
  - Shell-stabilized: No macroscopic fission barrier

- Heavy-ion fusion reaction
  - Hot fusion reaction
    - Actinide target + $^{48}$Ca projectile
    - $E_{ex} = 30$-40 MeV
  - Cold fusion reaction
    - Pb, Bi target
    - $E_{ex} = 10$-15 MeV
Coulomb barrier

Entrance

Compound Nucleus

Quasi-Fission

Fusion-Fission

Evaporation Residue

Exit

A₁, Z₁ ⇋ A₂, Z₂

Evaporation
Evaporation residue cross section

\[ \sigma_{ER} = \pi \hat{\kappa}^2 \sum_{l} (2l + 1) T_l(E_{cm}) P_{l}^{\text{for}}(E_{cm}) P_{l}^{\text{sur}}(E^*) \]

- Evaporation residue cross section is extremely small in the synthesis of SHE
  - \( T_l \): sticking probability
    - Optical potential, dissipation
  - \( P_{l}^{\text{for}} \): formation probability small
    - Fluctuation-dissipation dynamics
  - \( P_{l}^{\text{sur}} \): survival probability small
    - Statistical approach

Picobarn = \(10^{-12}\) barn
Fusion reaction of two heavy nuclei

- For lighter systems, fusion occurs when two nuclei touch each other.
- For heavier systems, fusion does not always occur even when two nuclei touch.
  - Fusion hindrance
  - Extra fusion barrier inside the Coulomb barrier of the entrance channel.
- For the synthesis of SHE, the fusion hindrance plays an essential roll.
  - Fusion probability by Langevin approach
Fusion hindrance

- Extra energy is necessary for fusion
  - Two-dimensional Langevin calculation
  - $Z_1 \times Z_2 > 1600$
  - Dissipation of relative motion

![Graph showing fusion hindrance]

Tokuda, Wada (1999)
Fusion hindrance (one-dimensional model)

\[ U = U_B - \frac{1}{2} m \omega_B^2 q^2 \quad \text{Parabolic barrier} \]

- **Formation probability**

\[ P_{for} = \frac{1}{2} \text{erfc} \left( \left( \frac{aB^2}{K-B} \right)^{1/4} \sqrt{\frac{\beta + \beta'}{2 \beta}} \left( 1 - \frac{2\omega_B}{\beta + \beta'} \sqrt{\frac{K}{B}} \right) \right) \]

- **Strong friction case**

\[ P_{for} = \frac{1}{2} \text{erfc} \left( \left( \frac{aB^2}{K-B} \right)^{1/4} \left( 1 - \frac{\omega_B}{\beta} \sqrt{\frac{K}{B}} \right) \right) \quad \beta' = \sqrt{\beta^2 + 4\omega_B^2} \]

- **Definitions**

- \( B \): extra barrier height
- \( K \): extra kinetic energy (at contact)
- \( a \): level density parameter
- \( \beta \): reduced friction parameter
Overview of Dynamical Process in reaction $^{36}\text{S}+^{238}\text{U}$
Time evolution of probability distribution

$^{30}\text{Si} + ^{238}\text{U} \rightarrow ^{268}\text{Sg} \ (E^*=35.5 \text{ MeV})$

Try to clarify the origin of difference between the both cases →

$^{36}\text{S} + ^{238}\text{U} \rightarrow ^{274}\text{Hs} \ (E^*=40.5 \text{ MeV})$
$^{30}\text{Si} + ^{238}\text{U}$

- $E_{c.m.} = 129.0\ \text{MeV}, \ E^*=35.5\ \text{MeV}$
- $90\text{u}$
- $178\text{u}$
- FF and DQF
- $t > 50 \times 10^{-21}\ \text{sec}$
- $-0.2 < \delta < 0.2$ (peak 0)
- QF via mono-nucleus
- $t < 30 \times 10^{-21}\ \text{sec}$
- $0.2 < \delta < 0.5$ (peak 0.4)

$^{36}\text{S} + ^{238}\text{U}$

- $E_{c.m.} = 154.0\ \text{MeV}, \ E^*=39.5\ \text{MeV}$
- $74\text{u}$
- $137\text{u}$
- $200\text{u}$
- FF and DQF
- $t < 30 \times 10^{-21}\ \text{sec}$
- $0 < \delta < 0.4$ (peak 0.2)
- QF
- $t < 10 \times 10^{-21}\ \text{sec}$
- $0 < \delta < 0.2$ (peak 0)

(1) Origin of reaction process
(2) Building times
(3) Deformation of fragments
Summary

- The fluctuation-dissipation dynamics is a general framework to describe the dynamics of a few slow (collective) variables interacting with many fast variables that can be treated as a heat bath.
- Powerful tool to study fission and heavy-ion fusion
  - Fission
    - Time scale of fission
    - Fragment mass and TKE distributions
  - Heavy-ion fusion reaction
    - Fusion hindrance
    - Quasi-fission and fusion-fission and more