Towards Exploring Fundamental Symmetries with Lattice QCD

Brian Tiburzi
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Towards Exploring Fundamental Symmetries with Lattice QCD

- **Lattice QCD:**
  compute single and few-body couplings

- **Hadronic Parity Violation:**
  isovector and isotensor

- **B Violation:** neutron-antineutron oscillations

- **T Violation:** nucleon EDMs

**Goal:** provide a sense of what challenges lattice QCD must confront
Towards Exploring Fundamental Symmetries with Lattice QCD

- **Lattice QCD:** compute single and few-body couplings
- **Hadronic Parity Violation:** isovector and isotensor
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**Goal:** provide a sense of what challenges lattice QCD must confront
Quark Interactions to Hadronic Couplings

- **Textbook**: gauge theories defined in perturbation theory
- **QCD**: short distance perturbative, long distance non-perturbative

\[ \bar{\psi} (\mathcal{D} + m_q) \psi + \frac{1}{4} G_{\mu\nu} G^{\mu\nu} \]

Non-perturbative definition of asymptotically free gauge theories

- **Quarks couple to other fundamental interactions**: e.g. weak interaction

\[ J(x) D(x, 0) J(0) = \sum_i C_i(\mu) \mathcal{O}_i(x, \mu) \]


- **Hadronic weak (& BSM) interactions require all the Wilson brand names**
Example: $K \rightarrow \pi\pi$ and $\Delta I = 1/2$ Rule

- **Old Puzzle:** $I = 0$ weak decay channel experimentally observed ~500x over $I = 2$
  
  Amplitude level: $A_0/A_2 \sim 22.5$
  
  pQCD contributes a factor of ~2
  
  Rest non-perturbative?

  **PRL 110, 152001 (2013)**

- **Almost There?**
  
  $A_0/A_2(m_\pi = 330 \text{ MeV}) = 12.0(1.7)$

\[
\mathcal{A} = \sum_i C_i(\mu) \langle \pi\pi | \mathcal{O}_i(\mu) | K \rangle_{\text{Lattice}}
\]

Emerging understanding of the $\Delta I = 1/2$ Rule from Lattice QCD

P.A. Boyle,¹ N.H. Christ,² N. Garron,³ E.J. Goode,⁴ T. Janowski,⁴ C. Lehner,⁵ Q. Liu,² A.T. Lytle,⁴ C.T. Sachrajda,⁴ A. Soni,⁶ and D. Zhang²

(The RBC and UKQCD Collaborations)

- **Theoretical Challenges $\Delta S = 1$ Processes**
  
  *Usual Suspects:* pion mass, lattice spacing, lattice volume  
  - underway
  
  *Additional Challenges:* Physical Kinematics  
  - underway
  
  Multi-Hadron States and Normalization  
  - ✓
  
  Operator Renormalization & Scale Invariance  
  - ✓
  
  Statistically Noisy Operator Self-Contractions  
  - ✓

- **Can such success carry over to weak nuclear processes?**
Example: $N \rightarrow (N\pi)_S$ and $\Delta I = 1$ Parity Violation

- **Old Problem**: hadronic neutral weak interaction is the least constrained SM current

- **New experiments**: parity violation in few-body systems, map out NN weak interaction?

\[ A = \sum_i C_i(\mu) \langle (\pi N)_S | O_i(\mu) | N \rangle_{\text{Lattice}} \]

Signal Found $h_{\pi NN}^1 = 1.1(5) \times 10^{-7}$

- **Theoretical Challenges $\Delta I = 1$ Processes**

  *Usual Suspects*: pion mass, lattice spacing, lattice volume

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  Multi-Hadron States and Normalization

  Operator Renormalization & Scale Invariance

  Statistically Noisy Operator Self-Contractions

- **How many lattice advances carry over to weak nuclear processes?**
Particle Physics \((B=0)\) vs. Nuclear Physics \((B>0)\)

### Pion Correlation Function

**Signal**

\[
\sum_{\{A_\mu\}} \langle q\bar{q}(t)q\bar{q}(0) \rangle \sim e^{-m_\pi t}
\]

**Noise^2**

\[
\sum_{\{A_\mu\}} \langle q\bar{q}(t)q\bar{q}(t)q\bar{q}(0)q\bar{q}(0) \rangle \sim e^{-2m_\pi t}
\]

**Signal/Noise**

\[
\sim \text{const}
\]

### Nucleon Correlation Function

**Signal**

\[
\sum_{\{A_\mu\}} \langle qqq(t)\bar{q}qq(0) \rangle \sim e^{-Mt}
\]

**Noise^2**

\[
\sum_{\{A_\mu\}} \langle qqq(t)\bar{q}qq(t)qqq(0)\bar{q}qq(0) \rangle \sim e^{-3m_\pi t}
\]

**Signal/Noise**

\[
\sim e^{-(M-\frac{3}{2}m_\pi)t}
\]

Baryons are statistically noisy.... scales exponentially with \(A\).
(Un)Physical Kinematics in $N\to(N\pi)_s$

- Lattice states are created on-shell

\[ G(\tau) = \sum_{\vec{x}} e^{i\vec{p} \cdot \vec{x}} \langle N(\vec{x}, \tau) N^\dagger(0, 0) \rangle = Ze^{-\sqrt{\vec{p}^2 + M_N^2} \tau} + \cdots \quad \text{ground-state saturation} \]

- Hadronic transition matrix elements have energy insertion

\[ E_N = M_N \]
\[ E_{(\pi N)_s} = M_N + m_\pi \]

\[ \langle (\pi N)_s | O_i(\mu) | N \rangle_{\text{Lattice}} = h_{\pi NN}^1 (\Delta E) \]

- Partial solution implemented (due to Beane, Bedaque, Parreno, Savage, NUPHA:747, 55 (2005))

\[ p \to n\pi^+ \]
\[ n\pi^+ \to p \]

T-invariance

\[ h_{\pi NN}^1 (m_\pi) \]
\[ h_{\pi NN}^1 (-m_\pi) \]

\[ h_{\pi NN}^1 = \frac{1}{2} \left[ h_{\pi NN}^1 (m_\pi) + h_{\pi NN}^1 (-m_\pi) \right] + \mathcal{O}(m_\pi^2) \]

Consequence: remove via chiral extrapolation but then only can determine chiral limit coupling

Likely small \~10% at 400 MeV pion mass.

Precision demands in nuclear physics not as great as particle physics

- Full solution: determine form factors, extrapolate to zero, e.g. partially twisted BCs
Example: $N \rightarrow (N\pi)_s$ and $\Delta I = 1$ Parity Violation

- **Old Problem**: hadronic neutral weak interaction is the least constrained SM current

- **New experiments**: parity violation in few-body systems, map out NN weak interaction?

  \[ A = \sum_i C_i(\mu) \langle (\pi N)_s | O_i(\mu) | N \rangle_{\text{Lattice}} \]

  **Lattice QCD Calculation of Nuclear Parity Violation**
  
  Joseph Wasem  
  PRC 85, 022501 (2012)

  **Signal Found**  
  \[ h_{\pi NN}^1 = 1.1(5) \times 10^{-7} \]

- **Theoretical Challenges $\Delta I = 1$ Processes**

  - *Usual Suspects*: pion mass, lattice spacing, lattice volume
  - *Additional Challenges*: Physical Kinematics, Multi-Hadron States and Normalization, Operator Renormalization & Scale Invariance, Statistically Noisy Operator Self-Contractions

- **How many lattice advances carry over to weak nuclear processes?**
Multi-Hadron States and Normalization

- Multi-Hadron operator not used... Matrix element evaluated by a trick

\[ G^* (\tau) = \langle 0 | N^* (\tau) N^+ (0) | 0 \rangle = Z e^{-E(N\pi)_s \tau} + \ldots \]

three-quark operator

\[ M_{N^*} > M_N + m_\pi \]

for odd-parity N

\[ (N\pi)_s \]

four-quarks + antiquark

Method requires this condition to hold for lattice parameters

\[ \text{Unfortunately likely } Z \ll Z' \]

\[ = Z e^{-E(N\pi)_s \tau} + Z' e^{-E^* \tau} + \ldots \]

\[ \text{Lellouch-Lüscher factor requires two-particle energy} \]

\[ \text{Not Computed} \]

\[ \text{Computed} \]

- Finite volume and infinite volume states have different normalizations

\[ |1\rangle_\infty = N_1 |1\rangle_V \]

\[ \langle n | n \rangle_\infty = N_n^2 V \langle n | n \rangle_V = N_n^2 e^{-E_n \tau} + \ldots \]

\[ |2\rangle_\infty = N_2 |2\rangle_V \]

\[ \langle 2 | \mathcal{O} | 1 \rangle_\infty = N_2 N_1 V \langle 2 | \mathcal{O} | 1 \rangle_V = N_2 N_1 (h_{1}^{NN})_V \]

\[ \text{Not needed for spectrum} \]

Example: $N \rightarrow (N\pi)_s$ and $\Delta I = 1$ Parity Violation

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  parity violation in few-body systems,  
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\[ h^1_{\pi NN} = 1.1(5) \times 10^{-7} \]

• **Theoretical Challenges** $\Delta I = 1$ Processes
  
  **Usual Suspects**: pion mass, lattice spacing, lattice volume  
  **to be done**

  **Additional Challenges**:  
  Physical Kinematics  
  partially solved  
  Multi-Hadron States and Normalization  
  to be done  
  Operator Renormalization & Scale Invariance  
  to be done  
  Statistically Noisy Operator Self-Contractions  
  to be done

• **How many lattice advances carry over to weak nuclear processes?**
Operator Renormalization and Scale Invariance

\[ A = \sum_i C_i(\mu) \langle (\pi N)_s | O_i(\mu) | N \rangle \]

\( \mu = 90 \text{ GeV} \) computable in pQCD at high scale

\( \mu = 1 - 2 \text{ GeV} \) computable on lattice at low scale

Tree Level

- \( Z^0 \)
- \( M_Z, M_W \)
- \( m_Q \)
- \( \Lambda_{\text{QCD}} \)
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Tree Level

\( M_Z, M_W \)

\( m_Q \)

\( \Lambda_{\text{QCD}} \)

One Loop

\[ \log \frac{M_Z^2}{p^2} \]

\[ \log \frac{\mu^2}{p^2} \]

\[ \log \frac{M_Z^2}{p^2} = \log \frac{\mu^2}{p^2} - \log \frac{\mu^2}{M_Z^2} \]

\[ \delta C(\mu) \sim -\alpha_s(\mu) \log \frac{\mu^2}{M_Z^2} \]

70’s Donoghue, McKellar, . . . , 90’s Dia Savage Liu Springer
Tree Level

\[ \mathcal{L}_{PV}^{I=1} = \sum_i C_i(\mu)O_i(\mu) \]

\[ \sin^2 \theta_W \quad \text{Non-Strange} \]

1 \quad \text{vs. Strange}

One Loop Results

\[
\begin{array}{c|cc}
 i & \text{LO} & \text{LO} \\
\hline
 1 & 0.403 & 0.264 \\
 2 & 0.765 & 0.981 \\
 3 & -0.463 & -0.592 \\
 4 & 0 & 0 \\
 5 & 5.61 & 5.97 \\
 6 & -1.90 & -2.30 \\
 7 & 4.74 & 5.12 \\
 8 & -2.67 & -3.29 \\
\end{array}
\]

\[ C_i(\mu = 1 \text{GeV}) / C_1^{\text{Tree}} \]

• Discrepancies

DSLS provide only ratios

\[ \alpha_s(m_c)/\alpha_s(m_b) = 1.44 \]

**Using their ratios, I get their values**

No heavy quark masses quoted in 1990 PDG
Operator Renormalization and Scale Invariance

**Tree Level**

\[
\mathcal{L}_{PV}^{I=1} = \sum_{i} C_i(\mu) O_i(\mu)
\]

\[
\sin^2 \theta_W \quad \text{Non-Strange}
\]

\[
1 \quad \text{vs. Strange}
\]

**One Loop Results**

\[
C_i(\mu = 1 \text{ GeV}) / C_1^{\text{Tree}}
\]

<table>
<thead>
<tr>
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<th>LO: 1992 PDG</th>
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<tbody>
<tr>
<td>1</td>
<td>0.403</td>
<td>0.264</td>
<td>0.54(4)</td>
</tr>
<tr>
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<td>0.981</td>
<td>0.55(6)</td>
</tr>
<tr>
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</tr>
<tr>
<td>4</td>
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<td>0</td>
<td>0</td>
</tr>
<tr>
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<td>5.97</td>
<td>5.35(7)</td>
</tr>
<tr>
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<td>-1.90</td>
<td>-2.30</td>
<td>-1.57(10)</td>
</tr>
<tr>
<td>7</td>
<td>4.74</td>
<td>5.12</td>
<td>4.45(8)</td>
</tr>
<tr>
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<td>-2.12(15)</td>
</tr>
</tbody>
</table>

Tiburzi, PRD 85 054020 (2012)
Operator Renormalization and Scale Invariance

\[ A = \sum C_i(\mu) \langle (\pi N)_s | \mathcal{O}_i(\mu) | N \rangle \]

Two Loop
\[ \alpha_s(1 \text{ GeV}) \sim 0.4 \]
QCD Renormalization of Isovector ParityViolation

Results (’t Hooft-Veltman scheme)

\[ \mathcal{L}_{\text{PV}}^{I=1} = \sum_i C_i(\mu) \mathcal{O}_i(\mu) \]

Alleged: 95% probe of hadronic neutral current

| \( C_i(\mu = 1 \text{ GeV}) / C_1^{\text{Tree}} \) |
|---|---|---|---|---|
| \( i \) | LO | LO | NLO (Z) | NLO (Z + W) |
| 1 | 0.403 | 0.264 | -0.054 | -0.055 |
| 2 | 0.765 | 0.981 | 0.803 | 0.810 |
| 3 | -0.463 | -0.592 | -0.629 | -0.627 |
| 4 | 0 | 0 | 0 | 0 |
| 5 | 5.61 | 5.97 | 4.85 | 5.09 |
| 6 | -1.90 | -2.30 | -2.14 | -2.55 |
| 7 | 4.74 | 5.12 | 4.27 | 4.51 |
| 8 | -2.67 | -3.29 | -2.94 | -3.36 |

Non-singlet chirality conservation: only 5 independent operators

\[ L \otimes L - R \otimes R \]
\[ L \otimes R - R \otimes L \]

\( \sin^2 \theta_W \) Non-Strange

vs.

1 Strange

80 - 100% Dynamical Question!
Operator Renormalization and Scale Invariance

\[
A = \sum_i C_i(\mu) \langle (\pi N)_s | O_i(\mu) | N \rangle
\]

\( \mu = 90 \text{ GeV} \) computable in pQCD at high scale

\( \mu = 1 - 2 \text{ GeV} \) computable on lattice at low scale

- Scale Invariance: requires same renormalization scheme

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pQCD 't Hooft-Veltman scheme

5 independent PV operators in chiral basis

Anisotropic Lattice Regularization + Wilson Fermions

14 independent PV operators

Unphysical + unphysical chiral mixing

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- Matching calculation required...

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  Signal Found  
  $h_{\pi NN}^1 = 1.1(5) \times 10^{-7}$

- How many lattice advances carry over to weak nuclear processes?
Statistically Noisy Operator Self-Contractions

\[ G(\tau', \tau) = \langle 0| N(\tau') O_i(\tau) N^{*\dagger} (0) |0\rangle \]

Another notorious difficulty

Vector and Axial-Vector self-contractions

\[
\begin{align*}
O_1 &= (\bar{u}u - \bar{d}d) A (\bar{u}u + \bar{d}d) V, \\
O_2 &= (\bar{u}u - \bar{d}d) A [\bar{u}u + \bar{d}d] V, \\
O_3 &= (\bar{u}u - \bar{d}d) V (\bar{u}u + \bar{d}d) A, \\
O_4 &= (\bar{u}u - \bar{d}d) V [\bar{u}u + \bar{d}d] A, \\
O_5 &= (\bar{u}u - \bar{d}d) A (\bar{s}s)_V, \\
O_6 &= (\bar{u}u - \bar{d}d) A [\bar{s}s] V, \\
O_7 &= (\bar{u}u - \bar{d}d) V (\bar{s}s)_A, \\
O_8 &= (\bar{u}u - \bar{d}d) V [\bar{s}s] A
\end{align*}
\]

\[ \sin^2 \theta_W \]

\[ 1 \]

Wilson coeffs.

Flavor dependence? \( \sim m_q \)

Extend to SU(3) + chiral corrections?

Utilize Fierz redundancy?

\[ \bar{s}s \quad \bar{s}\gamma_\mu s \]

small nucleon strangeness

\[ \langle \bar{s}\gamma_\mu s \rangle \ll \langle \bar{q}\gamma_\mu q \rangle ? \]

0.16 from Adelaide
Isotensor Parity Violation

\[ \mathcal{O} = (\bar{q} \tau^3 q)_A (\bar{q} \tau^3 q)_V - \frac{1}{3} (\bar{q} \tau^3 q)_A \cdot (\bar{q} \tau^3 q)_V \]

- Only **one** operator & **without** self-contractions

\[ \mathcal{L}_{PV}^{\Delta I=2} = \frac{G_F}{\sqrt{2}} C(\mu) \mathcal{O}(\mu) \]

Operator Renormalization

Tiburzi, PRD86: 097501 (2012)

<table>
<thead>
<tr>
<th></th>
<th>( C(1 \text{ GeV})/C^{(0)} )</th>
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<tbody>
<tr>
<td>LO [15]</td>
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<td>0.73</td>
</tr>
<tr>
<td>RI/SMOM(( q, \gamma_{\mu} ))</td>
<td>0.81</td>
</tr>
</tbody>
</table>

Better proving ground for Lattice QCD?

\[ \mathcal{L}_{NN} = [\vec{\mathbf{n}} p^\dagger \cdot \vec{\sigma} \sigma_2 p^*] \cdot [n^T \sigma_2 n] + \ldots \]

s- to p-wave NN interaction

Operator matrix element between two hadrons (beyond current reach?)

\[ \pi N \text{ interactions} \]

\[ \mathcal{L}_{\pi \pi N} + \mathcal{L}_{\pi \gamma N} \]

External fields could “substitute” for pions

\[ \pi PV \]

Isotensor pion interactions exist

Lattice compute parameters DDH potential?

... inevitably leads to chiral parity violating potential


Wilson fermions still to do...
Fundamental Symmetries and Lattice QCD

- **Lattice QCD**: Wilsonian machinery turns high-scale interactions (both SM & Beyond) into QCD-scale hadronic couplings

- After decades of dedicated work, trustworthy results emerging e.g. $K \to \pi\pi$

  **Theory Needs for Next-Decade Lattice QCD?**

- **Hadronic Parity Violation**:  
  $\pi N$-coupling more or less challenging than $K \to \pi\pi$? 
  Methods for coupling to pions? 
  NN-interactions? 
  Isovector parity-violating lattices?
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Lellouch-Lüscher Factor

• Single Particle Energy Quantization:
  \[ E = \sqrt{p^2 + M^2} \quad \vec{p} = \frac{2\pi}{L} \vec{n} \]

• Two Particle Energy Quantization:
  \[ E_{\text{total}} = \sqrt{k^2 + M^2} + \sqrt{k^2 + m^2} \quad \vec{P} = 0 \]
  \[ n\pi - \delta_0(k) = \phi(k) \]
  (known function for a torus)

• One-to-Two Particle Amplitude:
  \[ |M_\infty|^2 = \frac{8\pi V^2 M E_{\text{total}}^2}{k^2} \left[ \delta'(k) + \phi'(k) \right] |M_V|^2 \]

Generalization for energy insertion: Lin, Martinelli, Pallante, Sachrajda, Villadoro \textbf{NuPhB}:650, 301 (2003)
Kim, Sachrajda, Sharpe \textbf{NuPhB}:727, 218 (2005)
Auxiliary Fields for Isovector Parity Violation

- Perhaps only a Gedankenexperiment until exascale computers materialize

\[ O = (\bar{q} \gamma_\mu \gamma_5 \tau^3 q) (\bar{q} \gamma_\mu q) \rightarrow -a [\bar{q} \gamma_\mu (\gamma_5 \tau^3 - b \cdot 1) q]^2 P \otimes \tau^1 \]

\[ \tau^3 \text{--chiral symmetry} \]

Introduces PC and PV four-quark operators

- Can implement all isovector PV operators in sign-problem-free ways
  Continuum limit, parameter tuning (!?!!?)

\[ \Delta \mathcal{L} = \sigma^2 + ia \sigma [\bar{q} \gamma_\mu (\gamma_5 \tau^3 - b \cdot 1) q] \]

No sign problem \( \gamma_5 \otimes \tau^1 \text{--Hermiticity} \)

- E.g. \( \mathcal{O} = (\bar{q} \gamma_\mu \gamma_5 \tau^3 q) (\bar{q} \gamma_\mu q) \)

Other PV observables:

Nucleon anapole moment: just calculate anapole form factor

PV NN interactions from PV part of NN correlators

Bodies buried in gauge field generation