Nuclear Forces and Few-Nucleons
Dynamics in Break-up Reactions

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TRIUMF Theory Group

INT program on “Nuclei and Fundamental Symmetries: Theory Needs of the Next-Decade Experiments”
August 6th, 2013
Motivations

- For few-nucleons one can perform exact calculations both for bound and scattering observables to test the nuclear theory on light nuclei and extend it to heavier mass number.

- Electroweak probes (coupling constant <<1)
  
  "With the electro-magnetic probe, we can immediately relate the cross section to the transition matrix element of the current operator, thus to the structure of the target itself."

  \[ \sigma \propto |\langle \Psi_f | J^\mu | \Psi_0 \rangle|^2 \]

  [De Forest-Walecka, Ann. Phys. 1966]

- Provide important informations in other fields of physics, where nuclear physics plays a crucial role:
  - Astrophysics: $\gamma$ interactions with nucleonic matter, radiative capture reactions, $\nu$ interactions with nucleonic matter (vector current as em)
  - Atomic physics (nuclear corrections to atomic levels, etc.)
  - Particle physics (neutrino experiments, ...)

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Photo-nuclear Reactions

Reactions resulting from the interaction of a photon with the nucleus

For photon energy 15-25 MeV stable nuclei across the periodic table show wide and large peak
Electromagnetic Reactions

Photo-nuclear Reactions

Reactions resulting from the interaction of a photon with the nucleus

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\[ E_\gamma \text{ [MeV]} \]

\[ \sigma_\gamma \text{ [mb]} \]

Ahrens et al.

Giant Dipole Resonance

Coulomb excitations

Inelastic scattering between two charged particles. Can use unstable nuclei as projectiles.

Neutron-rich nuclei show fragmented low-lying strength

\[ \sigma_{\gamma,\alpha} \text{ (mb)} \]

Leistenschneider et al.

Can we give a microscopic explanation of these observations?

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Electromagnetic Reactions

Monopole Resonance in $^4$He

Electron scattering
interaction of a virtual photon with the nucleus

$E_x = 20.10(5)$ MeV
$\Gamma = 0.27(5)$ MeV

$\alpha$ - scattering
interaction of a virtual photon with the nucleus + strong interaction

$E_x = 20.29(2)$ MeV
$\Gamma = 0.89(4)$ MeV

Can we understand this difference? Can microscopic theories help?

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**Ab-initio Theory Tools**

**High precision two-nucleon potentials:**
well constraint on NN phase shifts

Three nucleon forces:
less known, constraint on A>2 observables

\[ H \psi_i = E_i \psi_i \]
\[ H = T + V_{NN} + V_{3N} + \ldots \]

Traditional Nuclear Physics
AV18+UIX, ..., J\(_2\)

Effective Field Theory
N\(^2\)LO, N\(^3\)LO ...

Exact Initial state & Final state in the continuum at different energies and for different A

\[ \sigma \propto \left| \langle \Psi_f | J^\mu | \Psi_0 \rangle \right|^2 \]

two-body currents (or MEC) subnuclear d.o.f.

\[ J^\mu = J^\mu_N + J^\mu_{NN} + \ldots \]

\[ \nabla \cdot J = -i[V, \rho] \]

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Final State Interaction

Exact evaluation of the final state in the continuum is limited in energy and $A$

Solution: The Lorentz Integral Transform Method


Response in the continuum

$$R(\omega) = \sum_f \left| \left\langle \psi_f \, | \, J^\mu \, | \, \psi_0 \right\rangle \right|^2 \delta(E_f - E_0 - \omega)$$

$$L(\sigma, \Gamma) = \int d\omega \frac{R(\omega)}{(\omega - \sigma)^2 + \Gamma^2} = \langle \tilde{\psi} | \tilde{\psi} \rangle$$

$$(H - E_0 - \sigma + i\Gamma) | \tilde{\psi} \rangle = J^\mu | \psi_0 \rangle$$

- Due to imaginary part $\Gamma$ the solution $| \tilde{\psi} \rangle$ is unique
- Since the r.h.s. is finite, then $| \tilde{\psi} \rangle$ has bound state asymptotic behavior

$L(\sigma, \Gamma)$ inversion $R(\omega)$ with the exact final state interaction

You can use any good bound state method! e.g. Hyperspherical Harmonics, No Core Shell Model, Coupled Cluster Theory

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Hyperspherical Harmonics

Starts from relative coordinates

\[ |\psi(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_A)\rangle = |\varphi(\vec{R}_{CM})\psi(\vec{\eta}_1, \vec{\eta}_2, \ldots, \vec{\eta}_{A-1})\rangle \]

Recursive definition of hyper-spherical coordinates

\[ \rho, \Omega \quad \rho^2 = \sum_{i=1}^{A} r_i^2 = \sum_{i=1}^{A-1} \eta_i^2 \]

Kinetic Energy

\[ H_0(\rho, \Omega) = T_\rho - \frac{K^2(\Omega)}{\rho^2} \]

HH eigenstates of \( K^2 \)

- Use HH as a basis to expand the wf

\[ \Psi = \sum_{[K], \nu} c_{[K]}^{[K]} e^{-\rho/2b} \rho^{n/2} L_n \left( \frac{\rho}{b} \right) \left[ \mathcal{Y}_K(\Omega) \chi_{ST}^{\mu} \right]_T \]

- Model space truncation \( K \leq K_{\text{max}} \)

- Anstisymmetrization algorithm

The LIT with Hyperspherical Harmonics

Numerical example: Dipole Response Function of $^4\text{He}$  

$$J^\mu \rightarrow \hat{D}_z = \sum_i (z_i - Z_{\text{cm}})$$  

with NN(N$^3$LO)

Inversion of the LIT

Ansatz

$$R(\omega) = \sum_{i} c_i \chi_i(\omega, \alpha)$$

$$L(\sigma, \Gamma) = \sum_{i} c_i \mathcal{L}[\chi_i(\omega, \alpha)]$$

Least square fit of the coefficients $c_i$ to reconstruct the response function
Applications
\[ \gamma + ^4\text{He} \rightarrow X \]

Traditional Hamiltonian
\[ PRL\ 96\ 112301\ (2006) \]

Hamiltonian from \( \chi\text{EFT} \)
\[ S.\text{Quaglioni\ and\ P.\text{Navratil}}\ \ PLB\ 652\ (2007) \]

Realistic NN + phenomenological central 3NF
\[ W.\text{Horiuchi\ et\ al.}\ PRC\ 85\ 054002\ (2012) \]

Moderate sensitivity to the Hamiltonian used; theory variation about 10% in peak
Theoretical precision is better than experimental error

More recent experimental activity seems to confirm higher data with peak around 27 MeV

Moderate sensitivity to the Hamiltonian used; theory variation about 10% in peak

Useful in Muonic Atoms arXiv:1307.6577

See talk by C.Ji

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\[ R_L(\omega, q) = \sum_f |\langle \Psi_f | \rho(q) | \Psi_0 \rangle|^2 \delta \left( E_f - E_0 - \omega + \frac{q^2}{2M} \right) \]

Comparison with experiment improves with 3NF and at low q the reduction of the peak is up to 50%
Comparison with experiment improves with 3NF and at low q the reduction of the peak is up to 50%.

It is not a simple binding effect!

Stimulating new experiments:
- MAMI taken data q>150
- S-DALINAC can possibly take data at lower q

\[
R_L(\omega, q) = \sum_f |\langle \Psi_f | \rho(q) | \Psi_0 \rangle|^2 \delta \left( E_f - E_0 - \omega + \frac{q^2}{2M} \right)
\]

Two-body currents are not important.
Monopole Transition Form Factor

\[ |F_M(q)|^2 = \frac{1}{Z^2} \int d\omega R_{\text{res}}^M(q, \omega) \]

First *ab-initio* calculation: Hiyama *et al.*, PRC 70 031001 (2004)
obtained good description of data with phenomenological central 3N

\[ 0_1^+ \longrightarrow 0_2^+ \]

\[ (e,e') \]

\[ 4^\text{He}(e,e')0^+ \]

AV8' + central 3NF
\[ E_0 = -28.44 \text{ MeV} \]
\[ E_0^{\text{exp}} = -28.30 \text{ MeV} \]
First *ab-initio* calculation with realistic three-nucleon forces and with the Lorentz Integral Transform method.

\[
|F_M(q)|^2 = \frac{1}{Z^2} \int d\omega R_{\text{res}}(q, \omega)
\]

F.Cappuzzello: plan to measure \(F_M\) with \(\alpha\)-scattering in Catania.
The inelastic monopole resonance acts as a prism to nuclear Hamiltonians.

- AV8’ + central 3NF: $E_0 = -28.44$ MeV
- AV18+UIX: $E_0 = -28.40$ MeV
- NN(N^3LO)+3NF(N^2LO): $E_0 = -28.36$ MeV

$E_0^{exp} = -28.30$ MeV
Analysis of this result

Realistic three-nucleon forces do not reproduce the data for $|F_M|^2$. Particularly large difference are found with chiral EFT potentials. This is unexpected! What can be the source of this behaviour?

- **Numerics?** Our calculations are well converged (few % level) in the HH basis

<table>
<thead>
<tr>
<th>$K_{\text{max}}$</th>
<th>12</th>
<th>14</th>
<th>16</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^4</td>
<td>F_M</td>
<td>^2$</td>
<td>4.59</td>
<td>4.75</td>
</tr>
</tbody>
</table>

- **Many-body charge operators?**

Conventional Nuclear Physics

Impulse approximation valid for elastic form factor below 2 fm$^{-1}$
Viviani *et al.*, PRL 99 (2007) 112002

EFT approach

Park *et al.*, Epelbaum, Koelling *et al.*, Pastore *et al.*: many-body operators appear at high order in EFT

- **Higher order 3NF ($N^3LO$)?** Unlikely...

- **Location of the resonance?**

AV8’ + central 3NF $E_R^* = 20.25$ MeV
AV18+UIX $E_R^* = 21.00(20)$ MeV
NN($N^3LO$)+3NF($N^2LO$) $E_R^* = 21.01(30)$ MeV $E_R^* = 20.21$ MeV
Extension to medium-mass nuclei

Develop new many-body methods that can extend the frontiers to heavier and neutron nuclei

Coupled Cluster Theory

CC future aims

CC theory now

- CC is optimal for closed shell nuclei ($\pm 1, \pm 2$)

Uses particle coordinates

$$|\psi_0(\vec{r}_1, \vec{r}_2, ..., \vec{r}_A)\rangle = e^T |\phi_0(\vec{r}_1, \vec{r}_2, ..., \vec{r}_A)\rangle$$

reference SD with any sp states

$$T = \sum T_{(A)} \quad \text{cluster expansion}$$

$$T_1 = \sum_{ia}^{t_a} a_i^\dagger a_i$$

$$T_2 = \frac{1}{4} \sum_{ij,ab}^{t_{ij}} a_i^\dagger a_j^\dagger a_j a_i$$

$$T_1 \quad T_2 \quad T_3$$

For the ground state energy

$$E_0 = \langle \phi_0 | e^{-T} H e^T | \phi_0 \rangle \quad \tilde{H} = e^{-T} H e^T$$

similarity transformed Hamiltonian

$$0 = \langle \phi_i^a | e^{-T} H e^T | \phi_0 \rangle$$

$$0 = \langle \phi_{ij}^{ab} | e^{-T} H e^T | \phi_0 \rangle$$

Leads to CCSD equations for the t-amplitudes
Extension to medium-mass nuclei

Develop new many-body methods that can extend the frontiers to heavier and neutron nuclei

Coupled Cluster Theory

- CC is optimal for closed shell nuclei ($\pm 1, \pm 2$)

Uses particle coordinates

\[ \psi_0(\vec{r}_1, \vec{r}_2, ..., \vec{r}_A) = e^T \phi(\vec{r}_1, \vec{r}_2, ..., \vec{r}_A) \]

reference SD with any sp states

\[ T = \sum T(A) \text{ cluster expansion} \]

CC is a very mature theory for g.s., see e.g.


What about electromagnetic reactions?

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New theoretical method aimed at extending \textit{ab-initio} calculations towards medium mass

\[
L(\sigma, \Gamma) = \int d\omega \frac{R(\omega)}{(\omega - \sigma)^2 + \Gamma^2} \langle \tilde{\Psi}_L | \tilde{\Psi}_R \rangle
\]

The LIT equation becomes EoM with \( z = E_0 + \sigma + i\Gamma \)

\[
[\hat{H}, \hat{R}(z^*)]|\Phi_0\rangle = (z^* - E_0)\hat{R}(z^*)|\Phi_0\rangle + \tilde{\Theta}|\Phi_0\rangle
\]

CCSD scheme

\[
\tilde{\Theta} = e^{-T} \Theta e^T
\]

\[
\hat{R} = \hat{R}_0 + \sum_{ia} \hat{R}_{ia} \hat{c}_a^\dagger \hat{c}_i + \frac{1}{4} \sum_{ijab} \hat{R}_{ijab} \hat{c}_a^\dagger \hat{c}_b^\dagger \hat{c}_j \hat{c}_i + \ldots
\]

Validation for \( ^4\text{He} \)

Dipole Response Functions

with NN forces from \( \chi \text{EFT (N}^3\text{LO)} \)
New theoretical method aimed at extending *ab-initio* calculations towards medium mass

**Extension to Dipole Response Function in $^{16}\text{O}$ with NN forces derived from $\chi$EFT (N$^3$LO)**

Convergence in the model space expansion

Good convergence!

Small HO dependence: use it as error bar

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New theoretical method aimed at extending *ab-initio* calculations towards medium mass

Extension to Dipole Response Function in $^{16}$O with NN forces derived from χEFT (N$^3$LO)

Comparison to the experiment

![Graph showing comparison between LIT of data and CCSD results.](image)
New theoretical method aimed at extending ab-initio calculations towards medium mass

Extension to Dipole Response Function in $^{16}$O with NN forces derived from $\chi$EFT (N$^3$LO)

Comparison to the experiment

The GDR of $^{16}$O is described from first principles for the first time!
Calcium isotopes with NN(N^3LO)

$h\Omega = 24 \text{ MeV}$

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Electric Dipole Polarizability

\[ \alpha_E = \frac{1}{2\pi^2} \int_{\omega_{th}}^{\infty} d\omega \frac{\sigma_\gamma(\omega)}{\omega^2} \]

Phys. Rev. C 85, 041302 (2012) very correlated to the neutron-skin radius

Towards an ab-initio theory for $^{48}$Ca

$^{48}$Ca $\alpha_E$ being measured at RCNP

$^{48}$Ca parity violating electron scattering CREX

Future: study correlation $\alpha_E - r_{\text{skin}}$ with ab-initio methods
Conclusions and Outlook

- Electromagnetic break up reactions are very rich observables to test our understanding on nuclear forces.
- Interesting applications to other fields of physics (muonic atoms).
- Extending these calculations to medium mass nuclei is possible and very exciting, with hopefully more applications/impact on future experiments on fundamental symmetries.

Perspectives

- Dipole response function of neutron-rich Oxygen isotope.
- Other multipole excitation (quadrupole or monopole) of medium mass nuclei (need extension of LIT/CCSD to two-body operator).
- Add triples and three-nucleon forces.
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