THE SUDORBEDABONIRAC EFT

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Why?

Chiral “EFT” potentials based on Weinberg’s power counting widely used in nuclear physics because of their supposed link to QCD

Problem: Weinberg’s power counting inconsistent with renormalization
Solution: Certain counterterms appear at lower order than expected; subleading terms should be treated in perturbation theory

Kaplan, Savage + Wise ’96, …, Nogga, Timmermans + Nogga ’05, …

VS.

The problem doesn’t exist: Renormalization not important
Epelbaum + Meissner ’06, …, Epelbaum + Gegelia ’09, …

Anyway, there is a solution for the problem that doesn’t exist:
Relativity essential in a non-relativistic problem
Epelbaum + Gegelia ’12

(Oh, yeah, this solution doesn’t completely solve the problem that doesn’t exist
---counterterms still need to be promoted--- but that is a detail
which barely needs acknowledgement…)
the talk yesterday
Outline

- Effective field theory & model spaces
- Pre-story: ChiPT
- The story
- Conclusion & Outlook
$E$

$M$

$\Lambda$

$m$

$\mathcal{L}_{EFT} = \sum_{d=0}^{\infty} \sum_{i(d,n)} c_i(M, \Lambda) O_i \left( (\partial, m)^{d} \varphi^{n} \right)$

underlying dynamics
renormalization-group invariance

$\frac{\partial Z}{\partial \Lambda} = 0$

local underlying symmetries

$Z = \int \mathcal{D}\Phi \exp \left( i \int d^{4}x \mathcal{L}_{\text{und}}(\Phi) \right) \times \int \mathcal{D}\phi \delta(\phi - f_{\Lambda}(\Phi)) \int \mathcal{D}\phi \exp \left( i \int d^{4}x \mathcal{L}_{EFT}(\varphi) \right)$

Weinberg, Wilson, ...
\[
T = T^{(\infty)}(Q) \sim N(M) \sum_{\nu=\nu_{\text{min}}}^{\infty} \sum_{i} \tilde{c}_{\nu,i}(\Lambda) \left[ \frac{Q}{M} \right]^{\nu} F_{\nu,i} \left( \frac{Q}{m}, \frac{\Lambda}{m} \right)
\]

\[
\frac{\partial T}{\partial \Lambda} = 0
\]

\[
\nu = \nu(d, n, \ldots) \quad \text{"power counting"}
\]

\[\text{e.g. \# loops } \ell\]

For \( Q \sim m \), truncate ...

\[T = T^{(\nu)} \left[ 1 + \mathcal{O} \left( \frac{Q}{M}, \frac{Q}{\Lambda} \right) \right] \quad \Rightarrow \quad \frac{\Lambda}{T^{(\nu)}} \frac{\partial T^{(\nu)}}{\partial \Lambda} = \mathcal{O} \left( \frac{Q}{\Lambda} \right) \ll 1\]

- controlled
- model independent

If so \{\text{want } \Lambda \geq M\}

realistic estimate of errors comes from variation \( \Lambda \in [M, \infty) \)
To limit the number of one-particle states, introduce IR cutoff in addition to UV cutoff momentum $\lambda$ and $\Lambda$. Cutoffs define “model spaces”

$$T = T^{(V)} \left[ 1 + \mathcal{O} \left( \frac{Q}{M}, \frac{Q}{\Lambda}, \frac{\lambda}{Q} \right) \right]$$

To minimize “model space” error (to “converge”), want

$$\begin{cases} 
\Lambda \geq M \\
\lambda \ll Q 
\end{cases}$$
Popular examples

Lattice Box
“Lattice Field Theory”

\[ L = N\alpha \]

Harmonic-Oscillator Box
“No-Core Shell Model”

\[ \frac{N^2\pi^2}{mL^2} \]
\[ \frac{\Lambda^2}{2m} \]
\[ \frac{N_{\text{max}}}{mb^2} \]
\[ \frac{\lambda^2}{2m} \]
\[ \frac{1}{mb^2} \]

\[ L \approx \sqrt{N_{\text{max}}} \ b \]
\[ b = \sqrt{\frac{2}{m\omega}} \]

- Nuclear matter: Müller et al. ’99
- Few nucleons: Lee et al. ’05 ...
- Atomic matter: Bulgac et al. ’06 ...
- Few atoms: Kaplan et al. ’10 ...

- Finite nuclei: Stetcu et al. ’06 ...
- Few atoms: Stetcu et al. ’07 ...
Extrapolations in a HO basis

\[ \frac{\Delta E}{E} = \Lambda \]

for much more see
Furnstahl, Hagen + Papenbrock '12
More et al. '13
Chiral EFT

\[ Q \sim m_\pi \ll M_{QCD} \sim 1 \text{ GeV} \]

- d.o.f.s: pions, nucleons, deltas \((m_\Delta - m_N \sim 2m_\pi)\)

\[
\pi = \begin{pmatrix}
\pi_1 \\
\pi_2 \\
\pi_3
\end{pmatrix} = \begin{pmatrix}
(\pi^+ + \pi^-)/\sqrt{2} \\
-i(\pi^+ - \pi^-)/\sqrt{2} \\
\pi^0
\end{pmatrix}
\]

\[
N = \begin{pmatrix}
P \\
n
\end{pmatrix}
\]

\[
\Delta = \begin{pmatrix}
\Delta^{++} \\
\Delta^+ \\
\Delta^0 \\
\Delta^-
\end{pmatrix}
\]

- symmetries: Lorentz, P, T, chiral

\[
f_\pi = 92 \text{ MeV} = O \left( \frac{M_{QCD}}{4\pi} \right)
\]

spontaneously broken: non-linear realization

+ chiral breaking
as in quark mass terms

\[
m_\pi^2 = O \left( \left( m_u + m_d \right) M_{QCD} \right)
\]

Weinberg '68

Callan, Coleman, Wess + Zumino '69
Pre-story: ChiPT

Example: pion sector (similar in one-nucleon sector)

\[
\mathcal{L}_{f=0} = 2 f_\pi^2 D_\mu \cdot D^\mu - \frac{1}{2} m_\pi^2 \pi^2 \left(1 - \frac{\pi^2}{4 f_\pi^2} + \ldots\right)
\]

\[
+ c_1 f_\pi^2 \left(D_\mu \cdot D^\mu\right)^2 + c_2 f_\pi^2 D_\mu \cdot D_\nu D^\mu \cdot D^\nu + c_3 m_\pi^2 D_\mu \cdot D^\mu \pi^2 (1+\ldots) + c_4 \frac{m_\pi^4}{f_\pi^2} \pi^4 (1+\ldots)
\]

\[
\mathcal{T}_{\pi\pi} = \ldots
\]

Weinberg '66

... current algebra

quantum corrections
\[
\begin{align*}
\left\{ \begin{array}{l}
\sum_{i=1}^{4} + \ldots = 1 \int_{\pi}^{4} \frac{d^4 l}{(2\pi)^4} \frac{(l, k, m_\pi)^2}{(l + k)^2 - m_\pi^2 - i \varepsilon}
\sum_{i=1}^{4} + \ldots = 1 \int_{\pi}^{4} \frac{d^4 l}{(2\pi)^4} \frac{(l, k, m_\pi)^2}{(l + k)^2 - m_\pi^2 - i \varepsilon}
\end{array} \right.
\end{align*}
\]

\[
\sim \frac{1}{f_\pi^2 (4\pi f_\pi)^2} \left\{ \begin{array}{l}
\Lambda^4 + \Lambda^2 \left( \# k^2 + \# m_\pi^2 \right) + \left( \# k^4 + \# m_\pi^2 k^2 + \# m_\pi^4 \right)
\end{array} \right\}
\]

absorbed in non-analytic

forbidden by chiral sym

\[
\sim \frac{1}{f_\pi^2} \left( \# k^2 + \# m_\pi^2 \right) \sim \frac{Q^2}{f_\pi^2}
\]

\[
\sum_{i=1}^{4} + \ldots = 1 \int_{\pi}^{4} \frac{d^4 l}{(2\pi)^4} \frac{(l, k, m_\pi)^2}{(l + k)^2 - m_\pi^2 - i \varepsilon}
\sum_{i=1}^{4} + \ldots = 1 \int_{\pi}^{4} \frac{d^4 l}{(2\pi)^4} \frac{(l, k, m_\pi)^2}{(l + k)^2 - m_\pi^2 - i \varepsilon}
\end{align*}
\]

\[
f_i (\Lambda) = - \frac{\#}{(4\pi f_\pi)^2} \ln \left( \frac{\Lambda}{m_\pi} \right) + c_i^{(R)}
\]

four parameters; if omitted:
- cutoff becomes physical
- only one parameter = model

c_i (\alpha \Lambda) = \frac{\#}{(4\pi f_\pi)^2} \ln \left( \frac{\Lambda}{m_\pi} \right) + \frac{\# \ln \alpha}{(4\pi f_\pi)^2} + c_i^{(R)}

\[
\sim \frac{Q^6}{f_\pi^2 M_{QCD}^4}
\]

error not dominant as long as

\[
\Lambda \geq M_{QCD}
\]

cf.

NDA: naïve dimensional analysis

\[
c_i^{(R)} = O \left( (4\pi f_\pi)^{-2} \right) = O \left( M_{QCD}^{-2} \right)
\]
Generalizing,

\[ \mathcal{L}_{EFT} = \sum_{\{n,p,f\}} \mathcal{C}_{\{n,p,f\}} \left( \frac{D_D, m_\Delta - m_N}{M_{QCD}} \right)^n \left( \frac{m_\pi^2}{M_{QCD}^2} \frac{\pi^2}{f_\pi^2} \right)^{p/2} \left( \frac{\psi^+ \psi}{f_\pi^2 M_{QCD}} \right)^{f/2} \]

\[ = \mathcal{O}(1) \]  
\[ = \mathcal{O} \left( \epsilon, \frac{\alpha}{4\pi} \right) \]  
\[ \text{isospin conserving} \]  
\[ \text{isospin breaking} \]  
\[ \text{chiral symmetry} \]  
\[ \text{(NDA)} \]  

\[ \Delta \equiv n + p + \frac{f}{2} - 2 \equiv d + \frac{f}{2} - 2 \geq 0 \]  
\[ \text{“chiral index”} \]

\[ T = T^{(\infty)}(Q) \sim N(M_{QCD}) \sum_{\nu=\nu_{\text{min}}}^{\nu} \sum_i \tilde{c}_{\nu,i}(\Lambda) \left[ \frac{Q}{M_{QCD}} \right]^\nu F_{\nu,i} \left( \frac{Q}{m_\pi}; \frac{\Lambda}{m_\pi} \right) \]

\[ \nu = 2 - A + 2L + \sum_i V_i \Delta_i \geq \nu_{\text{min}} = 2 - A \]

\[ \# \text{ nucleons} = 0,1 \quad \# \text{ loops} \quad \# \text{ vertices of type } i \]
The story*

The era of the scriptures

\[ V \]

\[ E = \frac{k^2}{m_N} \]

\[ V \]

\[ = i \int \frac{d^4 l}{(2\pi)^4} V \frac{1}{l^0 + k^2/m_N - l^2/m_N - i\varepsilon} \]

\[ = \int \frac{d^3 l}{(2\pi)^3} V \frac{m_N}{l^2 - k^2} V + \ldots \]

infrared enhancement: potential = sum of subdiagrams without IR enhancement: amenable to ChiPT expansion, cutoff absorbed in counterterms of NDA size

Weinberg’s recipe (“W PC”):

truncate potential, solve dynamical equation exactly

[and, as always, check assumptions...]

* Not a history, not even Whiggish
\[ V(\Lambda) \sim N(M) \sum_{\nu=\nu_{\text{min}}}^{\infty} \sum_{i} \tilde{c}_{\nu,i}(\Lambda) \left[ \frac{Q}{M} \right]^\nu f_{\nu,i} \left( \frac{Q}{m}; \frac{\Lambda}{m} \right) \]

\[ \nu = 2 - A + 2L + \sum_{i} V_i \Delta_i \geq \nu_{\text{min}} = 2 - A \]

not an observable: in general depends on cutoff, form of dynamical equation, choice of nucleon fields, etc.
LO \[ \mathcal{O}\left( \frac{1}{f^2} \right) \]

NLO \[ \mathcal{O}\left( \frac{1}{f^2} \frac{Q}{M_{QCD}} \right) \]

NNLO \[ \mathcal{O}\left( \frac{1}{f^2} \frac{Q^2}{M_{QCD}^2} \right) \]

NNNLO \[ \mathcal{O}\left( \frac{1}{f^2} \frac{Q^3}{M_{QCD}^3} \right) \]

NNNNLO \[ \mathcal{O}\left( \frac{1}{f^2} \frac{Q^4}{M_{QCD}^4} \right) \]

2-body  3-body  4-body  ...

(parity violating)

etc.
\[ V(\Lambda) \sim N(M) \sum_{\nu=\nu_{\text{min}}}^{\infty} \sum_i \tilde{c}_{\nu,i}(\Lambda) \left[ \frac{Q}{M} \right]^{\nu} f_{\nu,i} \left( \frac{Q}{m}; \frac{\Lambda}{m} \right) \]

\[ \nu = 2 - A + 2L + \sum_i V_i \Delta_i \geq \nu_{\text{min}} = 2 - A \]

Not an observable: in general depends on cutoff, form of dynamical equation, choice of nucleon fields, etc.

- Potential to \( O(Q^3) \) with and to \( O(Q^4) \) without delta isobar derived

- Fit of NN phase shifts to \( O(Q^3) \) with delta encouraging; similar accuracy (or lack thereof) for three cutoffs from 500 to 1000 MeV

- TPE potential to \( O(Q^3) \) without delta improves Nijmegen PWA

- Pions perturbative in F waves and higher
Also, many processes with external probes:

- pion elastic scattering
- electroweak currents
- pion photoproduction
- pion production
- Compton scattering
- ...

Weinberg '92  
Rho '93  
Park, Min + Rho '94  
Beane, Lee + v.K. '95  
...
Amplitude in 1S0 solved in semi-analytic form for $W$ LO:

\[
T_Y^{(0)}(\vec{p}', \vec{p}; k) = T_Y(\vec{p}', \vec{p}; k) + \frac{\chi(\vec{p}'; k)\chi(\vec{p}; k)}{1 - I(k)}
\]

\[
\frac{4\pi}{m_N} I(k) = \# \Lambda + \# \frac{m_N}{4\pi f_\pi} \frac{m_\pi^2}{f_\pi} \ln \left( \frac{\Lambda}{m_\pi} \right) + O\left( \frac{k^2}{\Lambda} \right)
\]

\[
c(m_\pi^2) = C_0 + D_2 m_\pi^2 + \ldots
\]

$W$ PC: LO \(\not\equiv\) NNLO

NDA fails for chiral symmetry-breaking operators: $W$ PC not entirely correct.
Detailed study of renormalization, validity of NDA, perturbativity of subLOs, power counting, etc. in simpler pionless EFT for $Q < m_\pi$

Some lessons:
1) fine-tuning necessary for large scattering lengths can be incorporated into PC for amplitude
2) non-perturbative renormalization intrinsically different from renormalization of corresponding perturbative series
3) one gains no understanding of the renormalization of the $A$-body system by just monkeying around with higher-order terms in the $A-1$-body system
4) NDA has very limited usefulness; e.g., three-body force of very high order by NDA, but renormalization requires it at LO
5) subleading interactions must be treated in perturbation theory
6) fully consistent theory works well for very low-energy processes involving (at least) light nuclei and cold atoms, incorporating universal properties such as the Efimov effect, Phillips and Tjon lines, Wigner SU(4), ...; yet, mostly ignored by nuclear physics community

Moral: faced with W PC vs RG, choose RG
Proposal for
perturbation approach to pion exchange in chiral EFT
(“KSW PC”)  

Some Results

1) manifestly consistent PC

2) rescues NDA for chiral symmetry-breaking operators

3) converges only for $Q < 100\text{-}150$ MeV;
at that point pion tensor force no longer perturbative
\begin{align*}
\mathcal{O}\left(\frac{4\pi}{m_N Q}\right) & \quad \text{2-body} \\
\mathcal{O}\left(\frac{4\pi}{m_N M_{NN}}\right) & \quad \text{3-body} \\
\mathcal{O}\left(\frac{4\pi Q}{m_N M_{NN}^2}\right) & \quad \text{4-body} \\
\mathcal{O}\left(\frac{4\pi Q^2}{m_N M_{NN}^3}\right) & \quad \text{etc.}
\end{align*}

\[ M_{NN} \equiv \frac{4\pi f_{\pi}^2}{m_N} \]
\[ E = \frac{k^2}{m_N} \]

\[ V = \int \frac{d^3 l}{(2\pi)^3} V \frac{m_N}{l^2 - k^2} V + \ldots \sim O\left(\frac{m_N Q}{4\pi} V^2\right) \]

Weinberg's IR enhancement

\[ \sim \frac{m_N Q}{4\pi} \]

instead of \[ \sim \frac{Q^2}{(4\pi)^2} \]

4pi enhancement compared to ChPT

\[ V^{(0)} = 1 + \text{?} \sim \frac{1}{f_\pi^2} \equiv \frac{4\pi}{m_N M_{NN}} \]

\[ M_{NN} \equiv \frac{4\pi f_\pi}{m_N} f_\pi \sim f_\pi \]

Resum when \[ Q \geq M_{NN} \]

\[ T^{(0)} = V^{(0)} + V^{(0)} \]

b.s. at

\[ B \sim \frac{M_{NN}^2}{m_N} \sim \frac{f_\pi}{4\pi} \approx 10 \text{ MeV} \]

But, since Weinberg's PC inconsistent, then what?
Elevate cutoff to physical quantity constrained to \( M_{NN} < \Lambda < M_{QCD} \)

Faced with W PC vs RG, choose W's PC

Countless improvements under W PC:
1) elimination of redundant operators
2) correction of some mistakes
3) smart choice of regulator (cutoff not on transferred momentum, to decouple effects of short-range interactions on various partial waves)
4) careful treatment of relativistic corrections

... N) fits to NN data at \( O(Q^4) \) without delta of similar quality as purely phenomenological pots

(But also some steps back, e.g., no deltas until recently, different regulators for different loops)

... Goes Viral

Chiral “EFT” becomes input of choice for a new generation of \textit{ab initio} methods for light and medium-mass nuclei
Conjecture: $M_{NN} > m_\pi$

so that one can think of $T$ as an expansion around the chiral limit, only necessary resummation being that of the tensor force:

- singlet channels \(\sim KSW\)
  (solves the $W$ problem with chiral symmetry breaking)

- triplet channels \(\sim W\)
  (solves the KSW problem of convergence)

However, $W$'s PC fails also in triplets!
incorrect renormalization...

That means some counterterms deemed to be subLO because of NDA are actually LO!
Add needed counterterms at this order, e.g.,

\[ V_{l=1,j=0} = \frac{c_1}{(2\pi)^3} \, pp' \]

cf.

\[ V_{l=0,j=1} = \frac{c_t}{(2\pi)^3} \]
That means some counterterms deemed to be subNNLO because of NDA are actually NNLO or lower!
Zeoli, Machleidt + Entem ’12

\[ \Lambda = 5 \text{ GeV} \]

\[ \Lambda = 1 \text{ GeV} \]

\[ \Lambda = 0.5 \text{ GeV} \]

YOU ARE USING THE WRONG PC

incorrect renormalization...

That means...

W PC at NNNLO
Root of the problem:
pion exchanges (long-ranged, contribute to waves higher than S) are singular (sensitive to short-range physics, require counterterms)

This has NOthing to do with relativity…
(For the opposite opinion, see Epelbaum + Gegelia ’12)

New, emerging PC:

- **LO:**
  OPE plus needed counterterms
  (one per wave where OPE is non-perturbative, singular, attractive)

- **subLOs:**
  NPE given by ChPT plus counterterms given by NDA
  *with respect to the lowest order they appear at*, treated in perturbation theory

(contrast with Epelbaum + Gegelia ’09, who suggest:
if you cannot take a large cutoff when treating certain subLOs non-perturbatively, don’t take a large cutoff. )
\( \mathcal{O}\left( \frac{4\pi}{m_N Q} \right) \) (LO)

\( \mathcal{O}\left( \frac{4\pi}{m_N M_{QCD}} \right) \) (NLO)

\( \mathcal{O}\left( \frac{4\pi Q}{m_N M_{QCD}^2} \right) \) (NNLO)

\( \mathcal{O}\left( \frac{4\pi Q^2}{m_N M_{QCD}^3} \right) \) (NNNLO)

\( \mathcal{O}\left( \frac{4\pi Q^3}{m_N M_{QCD}^4} \right) \) (NNNNLO)

etc.

(Details still being worked out, e.g. at ESNT Saclay workshop two weeks ago)
\[ T^{(0)} = V^{(0)} + \cdots + V^{(0)} + \cdots = V^{(0)} + \cdots \]

\[ (T + V^{(0)}) \psi^{(0)} = E^{(0)} \psi^{(0)} \]

large enhancement \( \sim 4\pi m_N / Q \)

\[ T^{(1)} = V^{(1)} + \cdots + T^{(0)} + V^{(1)} + \cdots + T^{(0)} \]

\[ E^{(1)} = \langle \psi^{(0)} | V^{(1)} | \psi^{(0)} \rangle \]

b.s.s, resonances

deeper

smaller
\[ T^{(2)} = V^{(1)} + \ldots + V^{(2)} + \ldots \]

\[ E^{(2)} = \sum_n \frac{\langle \psi^{(0)} | V^{(1)} | \psi_n^{(0)} \rangle \langle \psi_n^{(0)} | V^{(1)} | \psi^{(0)} \rangle}{E^{(0)} - E_n^{(0)}} + \langle \psi^{(0)} | V^{(2)} | \psi^{(0)} \rangle \]

\[ \text{missed} \]

\[ \text{sum even smaller} \]

\[ T = \tilde{T}^{(\tilde{V})} + O\left( f \left( \frac{\Lambda}{M} \right) \tilde{T}^{(\tilde{V})} \right) \]

\[ \frac{\Lambda}{\tilde{T}^{(\tilde{V})}} \frac{\partial \tilde{T}^{(\tilde{V})}}{\partial \Lambda} = O\left( 1 \right) \]

uncontrolled

model dependent

error estimate???
Fits to data  
Pavon Valderrama ‘10, ‘11  
Long + Yang ‘11, 12

bands (not error estimates):  
coordinate-space cutoff variation  
0.6 - 0.9 fm  
cyan:  
NNLO in Weinberg’s scheme
Conclusion & Outlook

- much has been learned about EFT in a non-perturbative context
- non-analytic parts of long-range pots derived
- a chiral EFT NN amplitude consistent with RG being constructed

compared to the NN amplitude obtained with W PC: it contains more counterterms (thus parameters) at a given order but subLOs require perturbation theory (sorry, but that is what physics asks of you)

- details still being worked out, but first results suggest possibility of better fits to data than W PC; perhaps a “realistic” amplitude emerges at NNLO?

- few-body forces and currents remain to be studied; effects could be substantial since they are tied to NN amplitude