<table>
<thead>
<tr>
<th>Name</th>
<th>Affiliation</th>
</tr>
</thead>
<tbody>
<tr>
<td>F. Barranco</td>
<td>Sevilla University</td>
</tr>
<tr>
<td>R.A. Broglia</td>
<td>Milano University and INFN; NBI, Copenhagen</td>
</tr>
<tr>
<td>G. Colo’, A. Idini, E. Vigezzi</td>
<td>Milano University and INFN</td>
</tr>
<tr>
<td>K. Mizuyama</td>
<td>Osaka University</td>
</tr>
<tr>
<td>G. Potel</td>
<td>CEA, Saclay</td>
</tr>
</tbody>
</table>
Outline

- A model for one- ($^{11}\text{Be}$, $^{10}\text{Li}$, $^{9}\text{He}$..) and two-neutron halo nuclei ($^{12}\text{Be}$, $^{11}\text{Li}$, $^{10}\text{He}$...) including core polarization effects

- Test of the model: two-nucleon transfer reactions

- Calculation of single-particle self-energy in coordinate space with effective forces; optical potentials

- Renormalization of the pairing field in neutron stars
A key challenge for ab-initio theory is to describe and predict properties of medium mass nuclei from the valley of stability towards the driplines, especially in relation to the wealth of new experimental data now coming from radioactive beam facilities. The nuclear many-body problem is a difficult undertaking from both the computational and theoretical points of view. Techniques such as Green’s function Monte Carlo (GFMC) and no-core shell model (NCSM) allow essentially exact calculations, but are limited to light nuclei. For mid-mass isotopes above $A=16$, the challenge posed by the numerical scaling demands innovative many-body theory techniques and computational approaches. This is especially true for the extensions to nuclei with an open-shell character. Techniques such as self-

<table>
<thead>
<tr>
<th>E (MeV)</th>
<th>CDB2k</th>
<th>INOY</th>
<th>Exp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^6$Li</td>
<td>29.07(41)</td>
<td>32.33(19)</td>
<td>[32.07]</td>
</tr>
<tr>
<td>$^7$Li</td>
<td>35.56(23)</td>
<td>39.62(40)</td>
<td>[38.89]</td>
</tr>
<tr>
<td>$^8$Li</td>
<td>35.82(22)</td>
<td>41.27(51)</td>
<td>[39.94]</td>
</tr>
<tr>
<td>$^9$Li</td>
<td>37.88(82)</td>
<td>45.86(74)</td>
<td>[42.30]</td>
</tr>
<tr>
<td>$^{11}$Li</td>
<td>37.72(45)</td>
<td>42.50(95)</td>
<td>[40.44]</td>
</tr>
</tbody>
</table>

The exponential convergence rate is not fully reached.

Still a challenge: $^{11}$Li

C. Forssen, E. Caurier, P. Navratil, PRC 79 021303 (2009)
To what extent is this picture correct?

Talk by K. Hagino DCEN 2011
A GENERALIZATION OF THE INERT CORE MODEL

Three-body model with density-dependent delta force

G.F. Bertsch and H. Esbensen,
*Ann. of Phys.* 209(’91)327

H. Esbensen, G.F. Bertsch, K. Hencken,
*Phys. Rev. C56*(’99)3054

\[ v(r_1, r_2) = v_0(1 + \alpha \rho(r)) \times \delta(r_1 - r_2) \]

\[ H = \frac{p_1^2}{2m} + \frac{p_2^2}{2m} + V_{nC}(r_1) + V_{nC}(r_2) + V_{nn} + \frac{(p_1 + p_2)^2}{2A_cm} \]
... WE INCLUDE CORE SURFACE DYNAMICS: CORE POLARIZATION AND CORE FLUCTUATIONS:

\[ H = \frac{p_1^2}{2m} + \frac{p_2^2}{2m} + V_{nc}(r_1) + V_{nc}(r_2) + V_{nn}(r_{12}) + \frac{(p_1 + p_2)^2}{2A_cm} + \]

\[ \delta V_{nc}(r_1, \theta_1, \phi_1, \{\alpha_{\lambda\mu}\}) + \delta V_{nc}(r_2, \theta_2, \phi_2, \{\alpha_{\lambda\mu}\}) \]

where \( \delta V_{nc} \) is the change in \( V_{nc} \) due to (core) surface-like deformation \( \{\alpha_{\lambda\mu}\} \):

\[ \delta V_{nc}(r, \theta, \phi, \{\alpha_{\lambda\mu}\}) = - \sum_{\lambda\mu} r \frac{dV_{nc}}{dr} \gamma_{\lambda\mu}(\theta, \phi) \cdot \alpha_{\lambda\mu} \]

where, for example, \( \alpha_{2\mu} \) is the dynamical quadrupole deformation of the core, described (harmonic oscillator formalism) in terms of creation and annihilation of surface oscillation quanta

\[ \alpha_{\lambda\mu} = \beta_{\lambda}(2\lambda + 1)^{1/2}(\Gamma_{\lambda-\mu}^+ + \Gamma_{\lambda\mu}) \; ; \; H_{coll} = \sum_{\lambda\mu} (\Gamma_{\lambda\mu}^+ \Gamma_{\lambda\mu} + \frac{1}{2}) \hbar \omega_{\lambda} \]

\( \beta_{\lambda} \) is determined from experiment (inelastic scattering or \( B(E\lambda) \)), analyzed via a RPA calculation with a multipole-multipole force
Parity inversion in N=7 isotones

Experimental systematics

Typical mean-field results

If one ignores core-polarizability/deformability

\[ H = \frac{p^2}{2m} + V_{nc}(r_1) + \frac{(p_1)^2}{2A_c m} + \delta V_{nc}(r_1, \theta_1, \phi_1, \{\alpha_{\lambda\mu}\}) \]

A different \( V_{nc}(r) \) is needed for each parity
Let us now consider the effects of $\delta V_{nc}(r_1, \theta_1, \varphi_1, \{\alpha_{\lambda\mu}\})$ on the self-energy!

\begin{align*}
\begin{array}{c}
s_{1/2} \\
d_{5/2} \\
s_{1/2}
\end{array}
\end{align*}

$\rightarrow$

\begin{align*}
\begin{array}{c}
p_{1/2} \\
p_{3/2} \\
p_{1/2}
\end{array}
\end{align*}

1/2- Eshift = - 2.5 MeV

\begin{align*}
\begin{array}{c}
2^+
\end{array}
\end{align*}

5 MeV

\begin{align*}
\begin{array}{c}
\frac{1}{2}^+
\end{array}
\end{align*}

$\rightarrow$

\begin{align*}
\begin{array}{c}
0\text{ MeV}
\end{array}
\end{align*}

\begin{align*}
\begin{array}{c}
\frac{1}{2}^-
\end{array}
\end{align*}

\begin{align*}
\begin{array}{c}
\frac{1}{2}^+
\end{array}
\end{align*}

+ 2.5 MeV

Pauli blocking of core ground state correlations

H. Sagawa et al., PLB 309 (1993)
Forbidden if both particles have the same quantum numbers

ELIMINATE!
Relax some of the assumptions of the inert core model:

**Inert core**
- Different potentials for s- and p-waves
- Zero range interaction, with ad hoc density dependence

**Low-lying collective modes of the core taken into account**
- Standard mean field potential
- Bare N-N interaction (Argonne)


F. Barranco et al. EPJ A11 (2001) 385

G. Gori et al. PRC 69 (2004) 041302(R)
Main ingredients of our calculation

**Fermionic degrees of freedom:**
- $s1/2$, $p1/2$, $d5/2$ Wood-Saxon levels up to 150 MeV (discretized continuum) from a standard (Bohr-Mottelson) Woods-Saxon potential

**Bosonic degrees of freedom:**
- $2^+$ and $3^-$ QRPA solutions with energy up to 50 MeV; residual interaction: multipole-multipole separable with the coupling constant tuned to reproduce $E(2^+)=3.36$ MeV and $0.6<\beta_2<0.7$
Calculated ground state

\[ |{1/2}^+\rangle = \sqrt{0.87} |s_{1/2}\rangle + \sqrt{0.13} |d_{5/2} \otimes 2^+\rangle \]


\[ |{1/2}^+\rangle = \sqrt{0.84} |s_{1/2}\rangle + \sqrt{0.16} |d_{5/2} \otimes 2^+\rangle \]

\(^{11}\text{Be}(p,d)^{10}\text{Be}\) in inverse kinematic detecting both the ground state and the 2+ excited state of \(^{10}\text{Be}\).
New result for $S[1/2+]$: $0.28^{+0.03}_{-0.07}$

Spectroscopic factors from $(12\text{Be},11\text{Be}+\gamma)$ reaction to $\frac{1}{2}^+$ and $\frac{1}{2}^-$ final states:

$S[1/2-]= 0.37\pm0.10 \quad S[1/2+]= 0.42\pm0.10$

Good agreement also between theory and experiment concerning energies and “spectroscopic” factors in $12\text{Be}$

Kanungo et al. PLB 682 (2010) 39

A. Navin et al., PRL 85(2000)266
Good agreement between theory and experiment concerning energies and spectroscopic factors in $^{11}\text{Be}$

<table>
<thead>
<tr>
<th></th>
<th>Expt.</th>
<th>Particle vibration</th>
<th>Mean field</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{3/2}$</td>
<td>$-0.504\text{ MeV}$</td>
<td>$-0.48\text{ MeV}$</td>
<td>$\sim0.14\text{ MeV}$</td>
</tr>
<tr>
<td>$E_{p1/2}$</td>
<td>$-0.18\text{ MeV}$</td>
<td>$-0.27\text{ MeV}$</td>
<td>$-3.12\text{ MeV}$</td>
</tr>
<tr>
<td>$E_{d3/2}$</td>
<td>$1.28\text{ MeV}$</td>
<td>$\sim0\text{ MeV}$</td>
<td>$\sim2.4\text{ MeV}$</td>
</tr>
<tr>
<td>$S[1/2^+]$</td>
<td>$0.65-0.80\text{ [19]}$</td>
<td>0.87</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>$0.73\pm0.06\text{ [20]}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.77 [21]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S[1/2^-]$</td>
<td>$0.63\pm0.15\text{ [20]}$</td>
<td>0.96</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>$0.96\text{ [21]}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S[5/2^+]$</td>
<td>0.72</td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>
$^{10}\text{Li}$ results

\[\begin{array}{|c|c|c|}
\hline
\text{State} & \text{Exp.} & \text{Theory} \\
\hline
^{10}\text{Li}_7 & s & 0.1-0.2 \text{ MeV} \\
\text{(not bound)} & p & 0.5-0.6 \text{ MeV} \\
\hline
\end{array}\]
A dynamical description of two-neutron halos

$^{11}\text{Li}$
F. Barranco et al. EPJ A11 (2001) 385

$^{12}\text{Be}$
G. Gori et al. PRC 69 (2004) 041302(R)

Diagonalization of $H_{\text{eff}}(E)$

Bare interaction

Induced interaction
B(E1) calculated with separable force; coupling constant tuned to reproduce experimental strength; part of the strength comes from admixture of GDR

<table>
<thead>
<tr>
<th>Quadr.</th>
<th>Core transitions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soft dipole</td>
<td>Valence transitions</td>
</tr>
</tbody>
</table>
Theoretical calculation for $^{11}$Li

Low-lying dipole strength

s-p strong mixing

also

Strong Pauli correction is needed:

About 50% in each vertex

The recoil term $p_1^*p_2/AM$ is incorporated as a dipole-dipole term.
The excitation of the $^9$Li core is also important to reproduce the total breakup strength, because about 15% of the strength escapes to the higher energy region as the component of the core excitation in the present coupled-channel approach. This...
$^{10}\text{Li}$ and $^{11}\text{Li}$ results

<table>
<thead>
<tr>
<th></th>
<th>Exp.</th>
<th>Theory</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>particle-vibration + Argonne</td>
</tr>
<tr>
<td>$^{10}\text{Li}_7$</td>
<td>$s$</td>
<td>$0.1-0.2$ MeV</td>
</tr>
<tr>
<td></td>
<td>$p$</td>
<td>$0.5-0.6$ MeV</td>
</tr>
<tr>
<td>$^{11}\text{Li}_8$</td>
<td>$S_{2n}$</td>
<td>$0.369$ MeV</td>
</tr>
<tr>
<td></td>
<td>$s^2p^2$</td>
<td>$50%, 50%$</td>
</tr>
<tr>
<td></td>
<td>$\langle r^2 \rangle^{1/2}$</td>
<td>$3.55\pm0.1$ fm</td>
</tr>
<tr>
<td></td>
<td>$\Delta p_{\perp}$</td>
<td>$48\pm10$ MeV/c</td>
</tr>
</tbody>
</table>

$^{11}\text{Li}$ correlated wave function

$$|\tilde{0}\rangle = |0\rangle + 0.7|(ps)_{1^-} \otimes 1^-; 0\rangle + 0.1|(sd)_{2^+} \otimes 2^+; 0\rangle$$

$$|0\rangle = 0.45|s_{1/2}^2(0)\rangle + 0.55|p_{1/2}^2(0)\rangle + 0.04|d_{5/2}^2(0)\rangle$$
Correlated halo wavefunction

Uncorrelated
Role of coupling to continuum

\[ |\tilde{0}\rangle = |0\rangle + 0.7|(ps)_{1^{-}} \otimes 1^{-}; 0\rangle + 0.1|(sd)_{2^{+}} \otimes 2^{+}; 0\rangle \]

\[ |0\rangle = 0.45|s_{1/2}^{2}(0)\rangle + 0.55|p_{1/2}^{2}(0)\rangle + 0.04|d_{5/2}^{2}(0)\rangle \]

Mixing \(n/n'\) (\([\phi_{n_{lj}} \times \phi_{n'_{lj}}]0^+\)) in the continuum creates bound waves
Comparison with the model by Bertsch and Esbensen

**OUR MODEL**

Parity independent potential (Bohr-Mottelson)

Depth adjusted to experimental $p_{1/2}$ single particle energy

**Single-particle potential**

Bare Argonne interaction + particle-vibration coupling with phenomenological parameters (low-lying vibrations)

Strength fitted to $S_{2n}$ in $^{12}\text{Be}$

\[ v_{\text{eff}}(r_1, r_2) = \delta(r_1 - r_2) \left( v_0 + v_p \left( \frac{\rho_c ((r_1 + r_2)/2)}{\rho_0} \right)^p \right). \]

**2-body interaction**

**Results**

Good reproduction of binding energies in $^{12}\text{Be}$ and $^{11}\text{Li}$

50% $(s_{1/2})^2$

Good reproduction of binding energy

Low $(s_{1/2})^2$ admixture unless two different s.p. potentials are used
Comparison with the model by Ikeda, Myo et al.


$p_{1/2}$ orbit is pushed up by pairing correlations and tensor force. Only 3/2-configurations are included: coupling to core vibrations (1/2-) is not considered. Binding energy is given as input. 50%($s^2$)-50%($p^2$) wavefunction is obtained.
How to probe the particle-phonon coupling?
Test the microscopic correlated wavefunction with phonon admixture

\[ |\tilde{0}\rangle = |0\rangle + 0.7|(ps)_{1^{-}} \otimes 1^{-}; 0\rangle + 0.1|(sd)_{2^{+}} \otimes 2^{+}; 0\rangle \]

\[ |0\rangle = 0.45|s_{1/2}^{2}(0)\rangle + 0.55|p_{1/2}^{2}(0)\rangle + 0.04|d_{5/2}^{2}(0)\rangle \]

We will try to draw information about the halo structure of \(^{11}\text{Li}\) from the reactions \(^{1}\text{H}(^{11}\text{Li},^{9}\text{Li})^{3}\text{H}\) and \(^{1}\text{H}(^{11}\text{Li},^{9}\text{Li}^{*}(2.69 \text{ MeV}))^{3}\text{H}\) (I. Tanihata et al., Phys. Rev. Lett. 100, 192502 (2008))
Probing $^{11}$Li halo-neutrons correlations via (p,t) reaction
The cross section for transitions to the first excited state (Ex = 2.69 MeV) is shown also in Fig. 3. If this state were populated by a direct transfer, it would indicate that a 1\(^+\) or 2\(^+\) halo component is present in the ground state of \(^{11}\text{Li}(\frac{3}{2}^-)\), because the spin-parity of the \(^9\text{Li}\) first excited state is \(\frac{1}{2}^-\). This is new information that has not yet been observed in any of previous investigations. A compound

---

**TABLE I.** Optical potential parameters used for the present calculations.

<table>
<thead>
<tr>
<th>System</th>
<th>V MeV</th>
<th>(r_v) fm</th>
<th>(a_v) fm</th>
<th>W MeV</th>
<th>(W_D) MeV</th>
<th>(r_w) fm</th>
<th>(a_w) fm</th>
<th>(V_\infty) MeV</th>
<th>(r_{so}) fm</th>
<th>(a_{so}) fm</th>
</tr>
</thead>
<tbody>
<tr>
<td>(p + ^{11}\text{Li}) [10]</td>
<td>54.06</td>
<td>1.17</td>
<td>0.75</td>
<td>2.37</td>
<td>16.87</td>
<td>1.32</td>
<td>0.82</td>
<td>6.2</td>
<td>1.01</td>
<td>0.75</td>
</tr>
<tr>
<td>(d + ^{10}\text{Li}) [11]</td>
<td>85.8</td>
<td>1.17</td>
<td>0.76</td>
<td>1.117</td>
<td>11.863</td>
<td>1.325</td>
<td>0.731</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(t + ^9\text{Li}) [12]</td>
<td>1.42</td>
<td>1.16</td>
<td>0.78</td>
<td>28.2</td>
<td>0</td>
<td>1.88</td>
<td>0.61</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Calculation of absolute two-nucleon transfer cross section by finite-range DWBA calculation

the initial and final channel wave functions are

$$|\alpha\rangle = \phi_a(\xi_b, r_1, r_2) \phi_A(\xi_A) \chi_{aA}(r_{aA})$$

$$|\beta\rangle = \phi_b(\xi_b) \phi_B(\xi_A, r_1, r_2) \chi_{bB}(r_{bB})$$

very schematically, the first order (simultaneous) contribution is

$$T^{(1)} = \langle \beta | V | \alpha \rangle,$$

while the second order contribution can be separated in a successive and a non-orthogonality term

$$T^{(2)} = T^{(2)}_{\text{succ}} + T^{(2)}_{\text{NO}}$$

$$= \sum_{\gamma} \langle \beta | V | \gamma \rangle G(\gamma | V | \alpha) - \sum_{\gamma} \langle \beta | \gamma \rangle \langle \gamma | V | \alpha \rangle.$$
\[
\sum_{n_1,n_2} a_{n_1,n_2} [\psi_{n_1}(r_1)\psi_{n_2}(r_2)]_{00}
\]
\begin{table}
\begin{tabular}{|c|c|c|c|}
\hline
\textbf{i} & \textbf{$\Delta L$} & \textbf{Theory} & \textbf{Experiment} \\
\hline
\text{gs (3/2$^-$)} & 0 & 6.1 & 5.7 $\pm$ 0.9 \\
\text{2.69 MeV (1/2$^-$)} & 2 & 0.5 & 1.0 $\pm$ 0.36 \\
\hline
\end{tabular}
\end{table}
Channels $c$ leading to the first $1/2^-$ excited state of $^9\text{Li}$

$c = 1$: Transfer of the two halo neutrons
$c = 2$: Transfer of a $p_{1/2}$ halo neutron and a $p_{3/2}$ core neutron
$c = 3$: Transfer to the ground state + inelastic excitation

\[
\sigma_c = \frac{\pi}{k^2} \sum_l (2l + 1)|S_l^{(c)}|^2, \quad P^{(c)} = \sum_l |S_l^{(c)}|^2 \quad (c = 1, 2, 3).
\]

Small probabilities $\Rightarrow$ use of second order perturbation theory.

$P^{(1)} = 1.3 \times 10^{-3}$
$P^{(2)} = 4.6 \times 10^{-5}$
$P^{(3)} = 2.6 \times 10^{-6}$
Convergence of the calculation

With box radius (30, 40 fm)

With number of intermediate states
Success of second order DWBA in the calculation of absolute two-neutron transfer cross sections

G. Potel et al., arXiV 1304.2569
Continuum particle-vibration coupling method


Self-consistent Skyrme Hartree-Fock

- Description of the single particle motion in a nucleus.

Self-consistent Skyrme continuum RPA

- Description of the vibration of the nucleus.

PVC Hamiltonian

\[ \hat{H}_{PVC} = \int dr \delta \hat{\rho}(r) \kappa(r) \sum_\sigma \hat{\psi}_\sigma^+(r \sigma) \hat{\psi}(r \sigma) \]

Self-energy function

\[ \Sigma(r \sigma, r' \sigma'; \omega) = \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \kappa(r) G(r \sigma, r' \sigma'; \omega - \omega') \kappa(r') iR(r, r'; \omega') \]

Continuum HF Green’s function

\[ G_{0,ij}(rr'; E) = \frac{1}{W(u, v)} u_{lj}(r<; E) v_{lj}(r>; E) \]

RPA
Self-energy function

\[
\Sigma_{l; j}(rr'; \omega) = \sum_{l'j',L} \left| \langle l | Y_L | l' j' \rangle \right|^2 \frac{1}{2j+1} \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \frac{\kappa(r)}{r^2} G_{0; l'j'}(rr'; \omega - \omega') \frac{\kappa(r')}{r'^2} i R_L(rr'; \omega')
\]

- Numerical contour integration on the complex energy plane.

\[\text{Im}[\omega' - \epsilon_F]\]

\[\text{Re}[\omega' - \epsilon_F]\]

\[\Sigma(r r') = R(r r') \cdot \frac{K(r)}{K(r')} \cdot G_{0}(rr')\]
Level density and Experimental Spectroscopic factor

PHYSICAL REVIEW C 86, 034318 (2012)

HF+PVC level density

$$\bar{\rho}_{lj}(\omega) = \frac{\pm 1}{\pi} \int dr \text{Im} \left( G_{lj}(rr, \omega) - G_{Free,lj}(rr, \omega) \right)$$

Cross section
→ DWBA analysis
(With phenomenological optical pot.)
→ Experimental Spectroscopic factor

Nuclear Data Sheets 107, 225 (2006)
TABLE VI. The same as Table III for $^{208}$Pb.

<table>
<thead>
<tr>
<th>$J^\pi$</th>
<th>$S_{ij}(^{207}\text{Pb})$</th>
<th>$S_{ij}(^{209}\text{Pb})$</th>
<th>$J^\pi$</th>
<th>$S_{ij}(^{207}\text{Pb})$</th>
<th>$S_{ij}(^{209}\text{Pb})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_{1/2}$</td>
<td>1.07</td>
<td>0.82</td>
<td>$g_{9/2}$</td>
<td>0.76</td>
<td>0.77</td>
</tr>
<tr>
<td>$p_{3/2}$</td>
<td>1.50</td>
<td>0.84</td>
<td>$s_{1/2}$</td>
<td>0.87</td>
<td>0.47</td>
</tr>
<tr>
<td>$f_{5/2}$</td>
<td>1.07</td>
<td>0.84</td>
<td>$d_{3/2}$</td>
<td>0.93</td>
<td>0.52</td>
</tr>
<tr>
<td>$f_{7/2}$</td>
<td>1.02</td>
<td>0.84</td>
<td>$d_{5/2}$</td>
<td>0.85</td>
<td>0.75</td>
</tr>
<tr>
<td>$h_{9/2}$</td>
<td>1.06</td>
<td>0.86</td>
<td>$g_{9/2}$</td>
<td>0.90</td>
<td>0.74</td>
</tr>
<tr>
<td>$h_{11/2}$</td>
<td>0.39</td>
<td>0.39</td>
<td>$i_{11/2}$</td>
<td>0.82</td>
<td>0.82</td>
</tr>
<tr>
<td>$i_{13/2}$</td>
<td>0.90</td>
<td>0.87</td>
<td>$f_{15/2}$</td>
<td>0.54</td>
<td>0.71</td>
</tr>
</tbody>
</table>

TABLE III. Experimental spectroscopic factors $S_{ij}$ obtained from one-nucleon transfer reactions for hole and particle states in $^{39}$Ca and $^{41}$Ca, compared to the integral of the theoretical level density performed up to an excitation energy of 10 MeV (cf. Fig. 13).

<table>
<thead>
<tr>
<th>$J^\pi$</th>
<th>$S_{ij}(^{39}\text{Ca})$</th>
<th>$J^\pi$</th>
<th>$S_{ij}(^{41}\text{Ca})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_{3/2}$</td>
<td>0.88</td>
<td>0.80</td>
<td>$f_{5/2}$</td>
</tr>
<tr>
<td>$s_{1/2}$</td>
<td>0.84</td>
<td>0.80</td>
<td>$p_{1/2}$</td>
</tr>
<tr>
<td>$p_{3/2}$</td>
<td>$2.9 \times 10^{-3}$</td>
<td>0.05</td>
<td>$p_{3/2}$</td>
</tr>
<tr>
<td>$d_{5/2}$</td>
<td>0.73</td>
<td>0.75</td>
<td>$d_{5/2}$</td>
</tr>
<tr>
<td>$f_{5/2}$</td>
<td>0.88</td>
<td>0.77</td>
<td>$f_{5/2}$</td>
</tr>
<tr>
<td>$g_{9/2}$</td>
<td>0.28</td>
<td>0.36</td>
<td></td>
</tr>
</tbody>
</table>
**T-matrix and continuum PVC**

PHYSICAL REVIEW C 86, 041603(R) (2012)
Self-consistent microscopic description of neutron scattering by 16O based on the continuum particle-vibration coupling method
Kazuhiro Mizuyama and Kazuyuki Ogata

**Lippman-Schwinger equation**

\[
\Psi^{(+)}_{PVC}(r\sigma, k) = \phi_F(r\sigma, k) + \sum\int\int dr_1 dr_2 G^{(+)}(r\sigma r_1\sigma_1; \omega) [v(r_1\sigma_1)\delta(r_1 - r_2)\delta_{\sigma_1\sigma_2} + \Sigma(r_1\sigma_1, r_2\sigma_2; \omega)] \phi_F(r_2\sigma_2, k)
\]

\[
T^{PVC}_{lj}(E) = \lim_{r \to \infty} \frac{2i}{r h_l(kr)} \left[ \int dr_1 G_{lj}^{+}(rr_1; E) \bar{v}_{lj}(r_1) r_1 j_l(kr_1) + \int\int dr_1 dr_2 G_{lj}^{+}(rr_1; E) \Sigma_{lj}(r_1 r_2; E) r_2 j_l(kr_2) \right].
\]

\[
\sigma(E) = \sum_{lj} \sigma_{lj}(E),
\]

\[
\sigma_{lj}(E) = \frac{2\pi}{k^2} \frac{2j + 1}{2} [Im T_{lj}(E)]
\]

\[
\sigma^{el}(E) = \sum_{lj} \sigma^{el}_{lj}(E),
\]

\[
\sigma^{el}_{lj}(E) = \frac{\pi}{k^2} \frac{2j + 1}{2} |T_{lj}(E)|^2
\]

\[
G(rr') = (1 - G_0 \Sigma)^{-1} G_0(rr').
\]

\[
\Sigma_{lj}(rr'; \omega) = \sum_{l'j', L} \frac{|(lj||l'j')|^2}{2j + 1} \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \frac{\kappa(r)}{r^2} G_{0,lj'}(rr'; \omega - \omega') \frac{\kappa(r')}{r'^2} iR_L(rr'; \omega')
\]

12
Role of transfer?

No free parameter!

Compound states or Door-way states (2p-1h, or 1p-2h config.)

\[ \sigma_{el}(E) \text{ [barn]} \]

\[ ^{16}\text{O}(n,n)^{16}\text{O} \]

\[ ^{16}\text{O}(n,n)^{16}\text{O} \]

SKM*
Reaction Cross section

\[ \sigma_R(E) = \sigma_{\text{tot}}(E) - \sigma_{\text{el}}(E). \]

\[ \begin{align*}
\text{Exp.} \\
J^\pi = 2^+, 3^-, 4^+, 5^- \\
J^\pi = 2^+, 3^-, 4^+ \\
J^\pi = 2^+, 3^- \\
J^\pi = 2^+ 
\end{align*} \]

\[ \sigma_R(E) \text{ [barn]} \]

\[ E \text{ [MeV]} \]
Representative calculations of optical potentials

J.P. Jeukenne, A. Lejeune, C. Mahaux PRC 10, 80 (1977)
Energy dependent optical potential in infinite matter
+ local density approximation

Self energy calculated in RPA with effective interactions

J.M. Mueller et al., PRC 83, 064605 (2011)
Optical potentials obtained from dispersion relations fitting elastic scattering data
S.J. Waldecker, C. Barbieri, W.H. Dickhoff, PRC 84,034316 (2011)
Self-energy calculated in FRPA with G-matrix from AV18

\[ \Sigma_{n_a,n_b}^{ij}(E) = \sum_r \frac{(E - \varepsilon_r)}{(E - \varepsilon_r)^2 + [\Gamma(\varepsilon_r)]^2} m_{n_a}^r m_{n_b}^r \]
\[ + i \left[ \frac{\Gamma(\varepsilon_h)}{(E - \varepsilon_h)^2 + [\Gamma(\varepsilon_h)]^2} m_{n_a}^h m_{n_b}^h \right] \]
\[ - \frac{\Gamma(\varepsilon_p)}{(E - \varepsilon_p)^2 + [\Gamma(\varepsilon_p)]^2} m_{n_a}^p m_{n_b}^p \]

\[ \text{Im} \Sigma_{n_l}^{(p)}(r) \]
G.P.A. Nobre et al., PRC 84, 064609 (2011)
Calculation of reaction cross section with explicit inclusion of inelastic and transfer channels using transition potentials computed in QRPA
PAIRING GAP IN FINITE NUCLEI

Medium effects increase the gap

PAIRING GAP IN NEUTRON MATTER

Medium effects decrease the gap
The inner crust: coexistence of a Coulomb lattice of finite nuclei with a sea of free neutrons

J. Negele, D. Vautherin  

M. Baldo et al  
Lattice of heavy nuclei surrounded by a sea of superfluid neutrons.
Going beyond mean field within the Wigner-Seitz cell: including the effects of polarization (exchange of vibrations) and of finite nuclei at the same time.

However, the presence of the nucleus increases the gap by about 50%.

With the adopted interaction, screening suppresses the pairing gap very strongly for $k_F > 0.7$ fm$^{-1}$.
A challenge: calculation of the self-energy in the Wigner-Seitz cell. Until now, only preliminary calculations of the pairing induced interaction exist.

The gap is quenched in the interior of the nucleus, but much less than in neutron matter at the same density.

S. Baroni et al., arXiv:0805.3962