Matthias Kaminski (University of Washington)
Hydrodynamics, non-AdS/non-CFT Correspondence & the Ridge


Matthias Kaminski (University of Washington)
Hydrodynamics, Gauge/Gravity Correspondence & the Ridge


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Invitation

Properties of Gauge/Gravity

**Negative**
- only toy models
- no model of QCD or SM
- no quantitative results (mass)
- QCD in this universality class?

**Positive**
- strong coupling effects
- models thermalization, etc
- exact solutions exist
- qualitative results (scaling)
- some universal results
Invitation

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The Ridge Phenomenon

- strong coupling effects?
- pre-thermalization?
- needs qualitative explanation
- some “universal” results?
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The Ridge Phenomenon
- strong coupling effects?
- pre-thermalization?
- needs qualitative explanation
- some “universal” results?

Gauge/Gravity seems like an appropriate tool.
### Invitation

**Gauge/Gravity Dictionary**

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<thead>
<tr>
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<td>Hawking T ~ horizon radius</td>
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Background geometry (metric, gauge fields, ...)

\[ g_{\mu\nu}(r) \]

\[ g_{\mu\nu}(r; r_{\text{Horizon}}) \]
Invitation

Gauge/Gravity Dictionary

Gauge Theory

“Medium” after collision

Gravity Theory

Background geometry
(metric, gauge fields, ...)

Temperature

Hawking
T ~ horizon radius

Thermalization

Horizon formation

Pre-Equilibrium (difficult)

Shock-wave collision

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\[ g_{\mu\nu}(r) \]
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\[ \delta g_{\mu\nu}(r, t, \vec{x}) \]
Outline

✓ Invitation

I. Review: Gauge/Gravity & Heavy-Ion-Collisions
   • Gauge/Gravity
   • Completed Hydrodynamics

II. Gauge/Gravity Models for the Ridge
   • Shock-Wave Metric yields Pre-Equilibrium
   • Fluctuations give Correlation Functions

III. Other Possibilities

IV. Conclusions
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   • Corrections to “(causal) viscous hydro”, new methods
   • A “first guess”
   • Toy models of full collision

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III. Other Possibilities
     Toy models of full collision
     Systematic scan for origin of ridge
     Toy models for hydrodynamic flow vs. toy models of jets

IV. Conclusions
I. Gauge/Gravity & Heavy-Ion-Collisions

What has been done to holographically model HIC?

A lot

Not much

We are going to discuss only examples here. This is not a full review. Review: [Gubser, Karch 0901.0935]
I. Gauge/Gravity & Heavy-Ion-Collisions

Chiral vortex effect

Heavy-ion-collision
I. Gauge/Gravity & Heavy-Ion-Collisions

Chiral vortex effect

Fluid/Gravity

\[ \text{Einstein equations} = \text{hydrodyn. conservation} + \text{EOMs for gravity fields} \]

Heavy-ion-collision
I. Gauge/Gravity & Heavy-Ion-Collisions

Chiral vortex effect

Heavy-ion-collision

Fluid/Gravity

\[
\text{Einstein equations} = \text{hydrodyn.} + \text{dynamical conservation} + \text{EOMs for equations} + \text{gravity fields}
\]

\[
\rightarrow \text{Complete constitutive relation for EM-tensor, values for transport coefficients. (completes Israel-Stewart)}
\]

[Baier et al. 2007]
[Bhattacharyya et al. 0712.2456]
I. Gauge/Gravity & Heavy-Ion-Collisions

**Chiral vortex effect**

**Heavy-ion-collision**

---

**Fluid/Gravity**

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Complete constitutive relation for EM-tensor, values for transport coefficients.

(Completes Israel-Stewart)

*It gives you all there is!*

---

[Baier et al. 2007]

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Fluid/Gravity derivation of chiral vortex effect.

Computed all first/second order transport coefficients in a gravity dual without B.

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[Banerjee et al. 0809.2596]
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Pure field theory derivation.

[Son, Surowka 0906.5044]

In parallel: chiral magnetic effect.

[Kharzeev et al., 2007]
[Fukushima et al., 2008]
I. Gauge/Gravity & Heavy-Ion-Collisions

Hydrodynamics

Hydrodynamics is an effective field theory, an expansion in gradients (equivalently: low frequencies and large momenta).

Constitutive equations

\[
T^{\mu\nu} = \frac{\epsilon}{3} (4u^\mu u^\nu + g^{\mu\nu}) + \tau^{\mu\nu}
\]

\[
j^\mu = \nu u^\mu - \sigma T (g^{\mu\nu} + u^\mu u^\nu) \partial_\nu \left( \frac{\mu}{T} \right) + \xi \omega^\mu
\]

\[
\omega^\mu = \frac{1}{2} \epsilon^{\mu\nu\lambda\rho} u_\nu \partial_\lambda u_\rho
\]

Example: Relativistic fluids with one conserved charge, with an anomaly (chiral)
I. Gauge/Gravity & Heavy-Ion-Collisions

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Hydrodynamics is an effective field theory, an expansion in gradients (equivalently: low frequencies and large momenta).

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\[
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\]

\[
j^\mu = nu^\mu - \sigma T(g^{\mu\nu} + u^\mu u^\nu)\partial_\nu \left( \frac{\mu}{T} \right) + \xi \omega^\mu \\
= : \Delta^{\mu\nu}
\]

from writing down all possible terms (respecting symmetries) with one derivative, built from \( \{ u, \epsilon, T, n, \mu, \epsilon^{\mu\nu\rho} \ldots \} \).

Examples

\[
\{ \nabla^\nu \mu, \nabla^\nu T, nu^\nu, \\
u^\nu u^\kappa \nabla_\kappa n, u^\nu n \nabla_\kappa u^\kappa, \ldots \}
\]

Example: Relativistic fluids with one conserved charge, with an anomaly (chiral)

\[
\omega^\mu = \frac{1}{2} \epsilon^{\mu\nu\lambda\rho} u_\nu \partial_\lambda u_\rho
\]

NEW!

Vorticity
I. Gauge/Gravity & Heavy-Ion-Collisions

Hydrodynamics: first order traditional procedure

1. Write down all first order (pseudo)vectors and (pseudo)tensors

2. Restricted by conservation equations

\[ \nabla_\mu T^{\mu\nu} = F^{\nu\lambda} j_\lambda \quad \nabla_\mu j^\mu = CE^\mu B_\mu \]

**Example:** no external fields

\[ 0 = \nabla_\mu nu^\mu = n \nabla_\mu u^\mu + u^\mu \nabla_\mu n \]

Possibly restricted by conformal symmetry

**Example:** invariant under Weyl rescaling

3. Further restricted by positivity of entropy production

\[ \nabla_\mu J^\mu_s \geq 0 \]

[Landau, Lifshitz]
I. Gauge/Gravity & Heavy-Ion-Collisions

*(non-conformal) hydrodynamics in 3+1*

[Son,Surowka 0906.5044]

Complete constitutive equations in 3+1 (with external gauge field)

\[
T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu + Pg^{\mu\nu} - \eta \Delta^{\mu\alpha} \Delta^{\nu\beta}(\partial_\alpha u_\beta + \partial_\beta u_\alpha) - (\zeta - \frac{2}{3}\eta)\Delta^{\mu\nu}\nabla_\gamma u^\gamma
\]

\[
j^{\mu} = nu^\mu + \sigma V^\mu + \xi \omega^\mu + \xi_B B^\mu
\]

\[
V^\mu = E^\mu - T \Delta^{\mu\nu}\nabla_\nu \left( \frac{\mu}{T} \right)
\]

\[
E^\mu = F^{\mu\nu}u_\nu
\]

\[
B^\mu = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta}u_\nu F_{\alpha\beta}
\]

\[
\omega^\mu = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma}u_\nu \nabla_\rho u_\sigma
\]
Complete constitutive equations in 3+1 (with external gauge field)

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\[ j^\mu = nu^{\mu} + \sigma V^\mu + \xi \omega^\mu + \xi_B B^\mu \]

New transport coefficients restricted

\[ \xi = C \left( \mu^2 - \frac{2}{3} \frac{n\mu^3}{\epsilon + P} \right), \quad \xi_B = C \left( \mu - \frac{1}{2} \frac{n\mu^2}{\epsilon + P} \right) \]

Chiral vortex effect

Chiral magnetic effect

Observable in heavy-ion collisions

Predicted values:

[Kharzeev, Son 1010.0038]
I. Gauge/Gravity & Heavy-Ion-Collisions

Un-biased predictive power

What we did not know:

Chiral magnetic effect predicted: [Kharzeev 2004]
Chiral vortical effect proposed: [Kharzeev, Zhitnitsky 2007]
Needs corrections: [Landau, Lifshitz]

Ignorance is bliss:
Complete first order constitutive equations in 3+1dim discovered in gravity without prejudice.

Gauge/Gravity method gives you everything there is inside a model.

Word of caution:
Gauge/Gravity is not entirely universal. Values of e.g. transport coefficients and features are generally model-dependent. But within the model you get “everything”.
I. Gauge/Gravity & Heavy-Ion-Collisions

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Take a model, check for ridge, change model

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Matthias Kaminski  Hydrodynamics, Gauge/Gravity Correspondence & the Ridge
I. Gauge/Gravity & Heavy-Ion-Collisions

**Hydrodynamic two-point-functions**

*Simplified example in 2+1 dim:*

\[ J^\mu = \rho_0 u^\mu + \sigma E^\mu \]

External sources \( A_t, A_x \propto e^{-i\omega t + ikx} \)

Allow response \( \rho_0 = \delta \rho \) \((\text{fix T and } u)\)
I. Gauge/Gravity & Heavy-Ion-Collisions

Hydrodynamic two-point-functions

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Allow response \( \rho_0 = \delta \rho \) (fix T and u)

One-point-functions from solving \( \nabla_\mu J^\mu = 0 \)

\[
\langle J^t \rangle = \delta \rho = -\frac{i\sigma k}{\omega + ik^2 \frac{\sigma}{\chi}} (\omega A_x + k A_t)
\]

\[
\langle J^x \rangle = \delta \rho = -\frac{i\sigma \omega}{\omega + ik^2 \frac{\sigma}{\chi}} (\omega A_x + k A_t)
\]

\[
\langle J^y \rangle = 0
\]
I. Gauge/Gravity & Heavy-Ion-Collisions

Hydrodynamic two-point-functions

Simplified example in 2+1 dim:

\[ J^\mu = \rho_0 u^\mu + \sigma E^\mu \]

External sources \[ A_t, A_x \propto e^{-i\omega t + ikx} \]

\[ u^\mu = (1, 0, 0) \]

Allow response \[ \rho_0 = \delta \rho \quad (fix \ T \ and \ u) \]

One-point-functions from solving \[ \nabla_\mu J^\mu = 0 \]

\[ \langle J^t \rangle = \delta \rho = -\frac{i\sigma k}{\omega + ik^2 \sigma \chi} (\omega A_x + kA_t) \quad \text{Einstein relation for diffusion:} \quad D = \frac{\sigma}{\chi} \]

\[ \langle J^x \rangle = \delta \rho = -\frac{i\sigma \omega}{\omega + ik^2 \sigma \chi} (\omega A_x + kA_t) \]

\[ \langle J^y \rangle = 0 \]

⇒ Two-point-functions \[ \langle J^t J^x \rangle = \frac{\delta \langle J^t \rangle}{\delta A_x} = -\frac{i\sigma \omega k}{\omega + iDk^2} \]

⇒ Kubo formulae for transport coefficients
I. Gauge/Gravity & Heavy-Ion-Collisions

Hydrodynamic two-point-functions

Simplified example in 2+1 dim:

\[ J^\mu = \rho_0 u^\mu + \sigma E^\mu \]

External sources \( A_t, A_x \propto e^{-i\omega t + ikx} \)

possible: more sources

Allow response \( \rho_0 = \delta \rho \) (fix \( T \) and \( u \))

generally: \( T \) and \( u \) respond as well

One-point-functions from solving \( \nabla_\mu J^\mu = 0 \)

\[
\langle J^t \rangle = \delta \rho = -\frac{i\sigma k}{\omega + ik^2/\chi} (\omega A_x + kA_t) \\
\langle J^x \rangle = \delta \rho = -\frac{i\sigma \omega}{\omega + ik^2/\chi} (\omega A_x + kA_t) \\
\langle J^y \rangle = 0
\]

Einstein relation for diffusion: \( D = \frac{\sigma}{\chi} \)

Two-point-functions

\[
\langle J^t J^x \rangle = \frac{\delta \langle J^t \rangle}{\delta A_x} = -\frac{i\sigma \omega k}{\omega + iDk^2}
\]

Kubo formulae for transport coefficients
I. Gauge/Gravity & Heavy-Ion-Collisions

Hydrodynamic Frames

Decomposition

\( T_{\mu\nu} = \mathcal{E} u_\mu u_\nu + \mathcal{P} \Delta_{\mu\nu} + (q_\mu u_\nu + q_\nu u_\mu) + t_{\mu\nu} \)

\( J_\mu = \mathcal{N} u_\mu + j_\mu \)

\( u_\mu q^\mu = 0, \quad u_\mu t^{\mu\nu} = 0, \quad u_\mu j^\mu = 0 \)

Example: Temperature gradient

\( j_\mu = \cdots + \chi T \Delta_\mu \nabla_\nu T + \cdots \)

Field redefinition ambiguity out-of-equilibrium

\( u_\nu(x) \to \hat{u}_\nu(x) \)

\( T(x) \to \hat{T}(x) \)

\( \mu(x) \to \hat{\mu}(x) \)

Fix by choice of a particular hydrodynamic frame

Example: Landau frame

\( q_\mu = 0 \quad \mathcal{E} = \epsilon_0 \quad \mathcal{N} = \rho_0 \)
I. Gauge/Gravity & Heavy-Ion-Collisions

Hydro without entropy current

Two-point functions together with “equilibrium correlators“ replace the entropy argument.

Proven for 2+1 dimensions:
[Jensen, MK, Kovtun, Meyer, Ritz, Yarom 1112.4498]

Proven for “equality type” conditions in d dimensions:
[Jensen, MK, Kovtun, Meyer, Ritz, Yarom 1203.3556]
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Proven for “equality type” conditions in d dimensions:
[Jensen, MK, Kovtun, Meyer, Ritz, Yarom 1203.3556]

Inequality type: \( \sigma \geq 0 \quad \eta \geq 0 \) (from two-point functions)

\[
W_m = \int d^d x \mathcal{L}[\text{sources}(x)].
\]

Example: \textit{Equality type} \( \chi_T = 0 \)

generally: \( m \)-point functions, simplifies higher order hydro (zero frequency)

Example: Ideal superfluid
\[
W_0 = \int d^d x \sqrt{-g} P(T, \mu, \xi^2)
\]
I. Gauge/Gravity & Heavy-Ion-Collisions

Summary of part I

- Relativistic hydrodynamics was completed at first and second order (Careful with “Causal Viscous Hydro”).
  [Baier et al, Minwalla et al 2007]
  [Erdmenger, Haack, MK, Yarom 0809.2488]
  [Banerjee et al. 0809.2596]

- Chiral transport effects measured in heavy-ion-collisions?
  [Kharzeev, Son]

- New methods for hydrodynamic correlation functions
- New method restricting transport coefficients

- Gauge/Gravity provides playground without prejudice

- Various models of particle collisions exist
II. Gauge/Gravity Models for the Ridge
   • Shock-Wave Metric yields Pre-Equilibrium
   • Fluctuations give Correlation Functions

III. Other Possibilities

IV. Conclusions
II. Gauge/Gravity Models for the Ridge

Pre-Equilibrium Model I

Single gravitational shock-wave metric

\[ ds^2 = \frac{L^2}{z^2} \left\{ -2\, dx^+ \, dx^- + t_1(x^-) \, z^4 \, dx^{-2} + dx_+^2 + dz^2 \right\} \]

\[ t_1(x^-) \equiv \frac{2\, \pi^2}{N_c^2} \langle T_{--}(x^-) \rangle \]

\( z \) is the radial AdS-direction

\( L \) is the AdS-radius

Energy-momentum tensor component

Solves Einstein’s equations in AdS5

\[ R_{\mu\nu} + \frac{4}{L^2} \, g_{\mu\nu} = 0 \]
II. Gauge/Gravity Models for the Ridge

Pre-Equilibrium **Model I**

Single gravitational shock-wave metric

\[ ds^2 = \frac{L^2}{z^2} \left\{ -2 \, dx^+ \, dx^- + t_1(x^-) \, z^4 \, dx^{-2} + dx_\perp^2 + dz^2 \right\} \]

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Energy-momentum tensor component

Solves Einstein’s equations in AdS5

\[ R_{\mu\nu} + \frac{4}{L^2} \, g_{\mu\nu} = 0 \]

Collide two shock waves with

\[ t_1(x^-) = \mu_1 \, \delta(x^-), \quad t_2(x^+) = \mu_2 \, \delta(x^+) \]

This gives

\[ ds^2 = \frac{L^2}{z^2} \left\{ - \left[ 2 + G(x^+, x^-, z) \right] \, dx^+ \, dx^- + \left[ t_1(x^-) \, z^4 + F(x^+, x^-, z) \right] \, dx^{-2} \right. \]

\[ + \left[ t_2(x^+) \, z^4 + F(x^+, x^-, z) \right] \, dx_\perp^2 + \left[ 1 + H(x^+, x^-, z) \right] \, dz^2 \right\}. \]

which is analytically known (perturbatively)
Evolution of two colliding initial states with finite energy density, finite thickness, Gaussian profile, in N=4 Super-Yang-Mills theory at strong coupling.

Full planar shock-wave, non-singular, time-dependent, numerical solution to Einstein’s equations.

Contains strong coupling and “medium” effects.

\[
\text{Ansatz: } \quad ds^2 = -A \, dv^2 + \Sigma^2 \left[ e^B \, dx_\perp^2 + e^{-2B} \, dz^2 \right] + 2dv \, (dr + F \, dz)
\]
II. Gauge/Gravity Models for the Ridge

Pre-Equilibrium Model II

“Holography and colliding gravitational shock waves in asymptotically AdS\(_5\) spacetime” [Chesler, Yaffe 1011.3562]

Initial data:

\[ ds^2 = r^2[-dx_+ dx_- + dx_\perp^2] + \frac{1}{r^2} [dr^2 + h(x_\pm) \, dx_\perp^2] \]

Pick Gaussian (arbitrary)

\[ h(x_\pm) \equiv \mu^3 \left(2\pi w^2\right)^{-1/2} e^{-\frac{1}{2} x_\pm^2 / w^2} \]
II. Gauge/Gravity Models for the Ridge

“Long-Range Rapidity Correlations in Heavy Ion Collisions at Strong Coupling from AdS/CFT”

[Grigoryan, Kovchegov 1012.5431]

Basic idea:

**Gauge**

Collision of nuclei

Correlations at early times

**Gravity**

Metric of Model I

Fluctuations around this

Correlations at late times

Fluctuations around dual to ideal Bjorken
II. Gauge/Gravity Models for the Ridge

Recipe: Two-point correlator from fluctuations \([Son, Starinets 2002]\)

Action for gravity scalar field fluctuation (dual to glueball)

\[
S^\phi = -\frac{N_c^2}{16\pi^2 L^3} \int d^4x \, dz \sqrt{-g} \, g^{MN} \partial_M \phi(x, z) \partial_N \phi(x, z)
\]

Solve equation of motion for that scalar

\[
\frac{1}{\sqrt{-g}} \partial_M \left[\sqrt{-g} \, g^{MN} \partial_N \phi(x, z)\right] = 0
\]

On-shell action

\[
S^\phi_{cl} = \frac{N_c^2}{16\pi^2 L^3} \int d^4x \left[\sqrt{-g} \, g^{zz} \phi(x, z) \partial_z \phi(x, z)\right] \bigg|_{z=0} = \frac{N_c^2}{16\pi^2} \int d^4x \, \phi_B(x) \left[\frac{1}{z^3} \partial_z \phi(x, z)\right] \bigg|_{z=0}
\]

Real-time retarded Green’s function

\[
G_R(x_1, x_2) = \frac{\delta^2[S^\phi_{cl} - S_0]}{\delta \phi_B(x_1) \delta \phi_B(x_2)}
\]
II. Gauge/Gravity Models for the Ridge

Implications

Large-rapidity glueball correlations in simplest background look very different from ridge data. But there are large-rapidity correlations at early times. [cf. talk by K. Dusling]

\[ C'(k_1, k_2) \big|_{|\Delta y| \gg 1} \sim \cosh(4 \Delta y) \]

Computation in background dual to ideal Bjorken hydrodynamics gives no large-rapidity correlations at late times.
Outline

✓ Invitation

✓ Review: Gauge/Gravity & Heavy-Ion-Collisions
  • Gauge/Gravity
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✓ Gauge/Gravity Models for the Ridge
  • Shock-Wave Metric yields Pre-Equilibrium
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III. Other Possibilities

Systematic scan for origin of ridge
Toy models for hydrodynamic flow vs. toy models of jets

IV. Conclusions
III. Other Possibilities

Correlations after collision of two nulei in a medium

PROPOSAL

Compute fluctuations around the full numerical background metric of model II at different times to scan the full time evolution of correlations.

Step in this direction:  

[Chesler, Teaney 2011]

Compute fluctuations around simplified version of model II (dual to two-point correlation functions). Check fluctuation dissipation theorem and equilibration.
III. Other Possibilities

Model of a jet

Take a string falling/being torn apart (backreacted)

Compute fluctuations around this background (dual to two-point correlation functions)

Initial conditions?

see also [Hofman, Maldacena 2008]
III. Other Possibilities

Model of a jet

Take a string falling/being torn apart (backreacted)

![Diagram of AdS boundary and radial AdS coordinate]

Compute fluctuations around this background (dual to two-point correlation functions)

Initial conditions?  Toy model for jets?

see also [Hofman, Maldacena 2008]
IV. Conclusions

✓ complete first and second order hydro

✓ new method for restricting transport coeffs

✓ new method for zero-frequency m-point correlators

✓ candidate model for collision (ridge)

➡ fluctuations at different times, unique features?

➡ use “more of hydro”: fluctuations, 2nd O(), methods...

➡ measure chiral transport effects
Inviting Discussions/Feedback

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➡ Safe travels!

➡ Thank you!
APPENDIX

-Entropy production

Structure of divergence

\[ \nabla_{\alpha} J_{s}^{\alpha} = \]

\[ + \text{(products of first order data)}, \]

\[ \implies \tilde{\nu}_2 = \nu_0 = \nu_1 = \nu_2 = \nu_3 = 0 \]
APPENDIX

- Entropy production

Structure of divergence

\[ \nabla_\alpha J^\alpha_s = + \left( \nu_2 - \frac{\nu_3}{T} \right) \nabla_\mu E^\mu + \nu_3 \Delta^{\mu\nu} \nabla_\mu \partial_\nu \frac{\mu}{T} \]

\[ + (\nu_0 + \nu_1) u^\alpha \nabla_\alpha \nabla_\mu u^\mu - \nu_1 u^\alpha u^\mu R_{\alpha\mu} \]

\[ - \tilde{\nu}_2 u^\alpha \nabla_\alpha B + \text{(products of first order data)} \]

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APPENDIX

-Entropy production

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Products of first order data

\[ \partial_\alpha J_s^\alpha = + \partial_\alpha J_s^{\alpha \text{ canon}} \]

\[ - \Omega (\partial \cdot u) \]

\[ - B (\partial \cdot u) \]

\[ + U_2 \cdot \tilde{U}_3 \]

\[ + U_1 \cdot \tilde{U}_3 \]

\[ + U_1 \cdot \tilde{U}_2 \]
APPENDIX

-Entropy production

Structure of divergence

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\nabla_\alpha J^\alpha_s = + \left( \nu_2 - \frac{\nu_3}{T} \right) \nabla_\mu E^\mu + \nu_3 \Delta^{\mu\nu} \nabla_\mu \partial_\nu \frac{\mu}{T} \\
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\]

\[\implies \tilde{\nu}_2 = \nu_0 = \nu_1 = \nu_2 = \nu_3 = 0\]

Products of first order data

\[
\nabla_\alpha J^\alpha_s = + \partial_\alpha J^\alpha_{s,\text{canon}} \\
- \Omega (\partial \cdot u) \left[ T \left( \frac{\partial P_0}{\partial \epsilon_0} \right) \rho_0 (\partial_T \tilde{\nu}_5 + \tilde{\nu}_1) + \frac{1}{T} \left( \frac{\partial P_0}{\partial \rho_0} \right) _{\epsilon_0} (\partial_\mu \tilde{\nu}_5 + \tilde{\nu}_3) \right] \\
- B (\partial \cdot u) \left[ T \left( \frac{\partial P_0}{\partial \epsilon_0} \right) \rho_0 \partial_T \tilde{\nu}_4 + \frac{1}{T} \left( \frac{\partial P_0}{\partial \rho_0} \right) _{\epsilon_0} \partial_\mu \tilde{\nu}_4 \right] \\
+ U_2 \cdot \tilde{U}_3 \left[ R_0 T (\partial_T \tilde{\nu}_3 - \partial_\mu \tilde{\nu}_1) - \partial_\mu \tilde{\nu}_4 + R_0 T^2 \partial_T \tilde{\nu}_4 \right] \\
+ U_1 \cdot \tilde{U}_3 \left[ - R_0 T^2 (\partial_T \tilde{\nu}_5 + \tilde{\nu}_1) + (\partial_\mu \tilde{\nu}_5 + \tilde{\nu}_3) + T (\partial_\mu \tilde{\nu}_1 - \partial_T \tilde{\nu}_3) \right] \\
+ U_1 \cdot \tilde{U}_2 \left[ \frac{\partial_\mu \tilde{\nu}_5 + \tilde{\nu}_3}{T} + \partial_\mu \tilde{\nu}_1 - \partial_T \tilde{\nu}_3 - T \partial_T \tilde{\nu}_4 \right] ,
\]
**APPENDIX**

- **Entropy production**

Canonical part
\[
\partial_\alpha J^\alpha_{\text{canon}} = - \left( \frac{1}{2} \Delta_{\mu\nu} \tau^{\mu\nu} - \left( \frac{\partial P_0}{\partial \rho_0} \right) \frac{u_\mu u_\nu \tau^{\mu\nu}}{\rho_0} + \left( \frac{\partial P_0}{\partial \rho_0} \right) \frac{u_\mu \gamma_\mu}{\epsilon_0} \right) \frac{\partial \cdot u}{T}
\]
\[
- \left( R_0 u_\mu \tau^{\mu\nu} + \gamma_\nu \right) \Delta_{\nu\alpha} U_3^\alpha
\]
\[
- \frac{\tau^{\mu\nu} \sigma_{\mu\nu}}{2T}.
\]

Transform back to Landau frame

**Thermodynamic response parameters**

\[
\tilde{\chi}_B = \frac{\partial P_0}{\partial \epsilon_0} \left( \frac{T}{\partial T} + \mu \frac{\partial M_B}{\partial \mu} - M_B \right) + \frac{\partial P_0}{\partial \rho_0} \frac{\partial M_B}{\partial \mu},
\]
\[
\tilde{\chi}_\Omega = \frac{\partial P_0}{\partial \epsilon_0} \left( \frac{T}{\partial T} + \mu \frac{\partial M_\Omega}{\partial \mu} + f_\Omega(T) - 2M_\Omega \right) + \frac{\partial P_0}{\partial \rho_0} \left( \frac{\partial M_\Omega}{\partial \mu} - M_B \right),
\]
\[
\tilde{\chi}_E = \frac{\partial M_B}{\partial \mu} - R_0 \left( \frac{\partial M_\Omega}{\partial \mu} - M_B \right),
\]
\[
T \tilde{\chi}_T = \left( \frac{T}{\partial T} + \mu \frac{\partial M_B}{\partial \mu} - M_B \right) - R_0 \left( \frac{T}{\partial T} + \mu \frac{\partial M_\Omega}{\partial \mu} + f_\Omega(T) - 2M_\Omega \right),
\]

Matching to two-point functions later gives: \( M_B = \frac{\partial P}{\partial B} \), \( M_\Omega = \frac{\partial P}{\partial \Omega} \).
APPENDIX

-Two-point-functions-

Most general parity-violating case is more complicated

\[
\begin{pmatrix}
  k^2 \sigma - i \omega \frac{\partial \rho_0}{\partial \mu} & -k^2 \left( \frac{\mu}{T} \tau + \chi T \right) - i \omega \frac{\partial \rho_0}{\partial T} & ik\rho_0 & 0 \\
  -i \omega \frac{\partial \rho_0}{\partial \mu} & -i \omega \frac{\partial \rho_0}{\partial T} & ik(\epsilon_0 + P_0) & 0 \\
  ik\rho_0 & ik\sigma_0 & k^2(\epsilon + \zeta) - i \omega(\epsilon_0 + P_0) & k^2(\chi \Omega + \tilde{\eta}) \\
  0 & 0 & -k^2 \tilde{\eta} & k^2 \eta - i \omega(\epsilon_0 + P_0)
\end{pmatrix}
\begin{pmatrix}
  \delta \mu \\
  \delta T \\
  \delta u^x \\
  \delta u^y
\end{pmatrix}
= \text{vector containing external sources } h_{\mu \nu}, A_\mu
\]

For example, we get a Kubo formula for

\[
\lim_{k \to 0} \frac{1}{ik} \langle C^0 T^{02} \rangle_R(0, k) = \tilde{\chi} \Omega
\]
Most general parity-violating case is more complicated

\[
\begin{pmatrix}
    k^2 \sigma - i\omega \frac{\partial \rho_0}{\partial \mu} - k^2 \left( \frac{\mu}{T} \sigma + \chi T \right) - i\omega \frac{\partial \rho_0}{\partial T} \\
    -i\omega \frac{\partial \rho_0}{\partial \mu} \\
    ik \rho_0 \\
    0
\end{pmatrix}
\begin{pmatrix}
    ik \rho_0 \\
    ik (\epsilon_0 + P_0) \\
    ik s_0 \\
    0
\end{pmatrix}
\begin{pmatrix}
    0 \\
    0 \\
    k^3 (\eta + \zeta) - i\omega (\epsilon_0 + P_0) \\
    -k^2 \eta
\end{pmatrix}
\begin{pmatrix}
    k^2 (\chi \Omega + \tilde{\eta}) \\
    k^2 \eta - i\omega (\epsilon_0 + P_0)
\end{pmatrix}
= \text{vector containing external sources } h_{\mu\nu}, A_{\mu}
\]

For example, we get a Kubo formula for

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\lim_{k \to 0} \frac{1}{ik} \left\langle C^0 T^{02} \right\rangle_R(0, k) = \tilde{\chi} \Omega
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APPENDIX

-Two-point-functions

Restrictions from Onsager relations

\[ G^{i,j}_{R}(\omega, k; b_a) = n_i n_j G^{j,i}_{R}(\omega, -k; -b_a) \]

where under time-reversal \( \Theta \mathcal{O}_i \Theta^{-1} = n_i \mathcal{O}_i \)
APPENDIX

-Two-point-functions

Restrictions from Onsager relations

\[ G_{R}^{ij}(\omega, k; b_{a}) = n_{i}n_{j}G_{R}^{ii}(\omega, -k; -b_{a}) \]

where under time-reversal \( \Theta \Theta^{-1} = n_{i} \Theta_{i} \)

From time-reversal covariance plus translation invariance

\[ G_{R}^{ij}(x) \equiv i\theta(t) \text{Tr} \left( \varphi[\mathcal{O}_{i}(t, x), \mathcal{O}_{j}(0)] \right) = i\theta(t) n_{i}n_{j} \text{Tr} \left( \varphi'[\mathcal{O}_{j}(t, -x), \mathcal{O}_{i}(0)] \right) \]

Parameters \( b_{a} \) break time-reversal invariance, 

i.e. time-reversal and \( b_{a} \rightarrow -b_{a} \)

together are a symmetry
-Two-point-functions

Restrictions from susceptibility constraints

**Examples**  \[ \lim_{k \to 0} \langle J^0 J^0 \rangle(\omega = 0, k) = \left( \frac{\partial \rho_0}{\partial \mu} \right)_T \]

Partition function in grand canonical ensemble

\[ Z[T, \mu] = \text{Tr} \left[ \exp \left( -\frac{H}{T} + \frac{\mu Q}{T} \right) \right] \]

Constant external sources \( A_0, h_{00}, h_{0i} \)
can be eliminated by shifting thermodynamic variables

\[ Z[T, \mu; A_0, h_{00}, h_{0i}] = Z \left[ T \left( 1 + \frac{h_{00}}{2} \right), \mu \left( 1 + \frac{h_{00}}{2} \right) + A_0; 0, 0, 0 \right] \]

Thus we get relations for zero-momentum limits of zero-frequency correlators.
**APPENDIX**

-**Magnetovortical frame**

Thermodynamics depending on vorticity and magnetic field

\[ P = P(T, \mu, B, \Omega) \]
\[ dP = s\,dT + \rho\,d\mu + \frac{\partial P}{\partial B}B + \frac{\partial P}{\partial \Omega}\Omega, \]
\[ \epsilon + P = sT + \mu \rho. \]

Constitutive relations

\[ T^{\mu\nu} = (\epsilon - \mathcal{M}_\Omega \Omega + f_\Omega \Omega)\, u^\mu u^\nu \]
\[ + (P - \zeta \nabla_\alpha u^\alpha - \tilde{x}_B B - \tilde{x}_\Omega \Omega)\, \Delta^{\mu\nu} - \eta \sigma^{\mu\nu} - \tilde{\eta} \tilde{\sigma}^{\mu\nu}, \]
\[ J^\mu = (\rho - \mathcal{M}_B \Omega)\, u^\mu + \sigma V^\mu + \tilde{\sigma} \tilde{V}^\mu + \tilde{\chi}_E \tilde{E}^\mu + \tilde{\chi}_T \epsilon^{\mu\nu\rho} u_\nu \nabla_\rho T, \]

where \[ \mathcal{M}_B = \frac{\partial P}{\partial B}, \quad \mathcal{M}_\Omega = \frac{\partial P}{\partial \Omega} \]

Matching

\[
\begin{align*}
\tilde{x}_B &= \frac{\partial P}{\partial B}, \\
T\tilde{x}_T &= \frac{\partial \epsilon}{\partial B} + R_0 \left( \frac{\partial P}{\partial \Omega} - \frac{\partial \epsilon}{\partial \Omega} - f_\Omega \right), \\
\tilde{x}_\Omega &= \frac{\partial P}{\partial \Omega}, \\
\tilde{\chi}_E &= \frac{\partial \rho}{\partial B} + R_0 \left( \frac{\partial P}{\partial B} - \frac{\partial \rho}{\partial \Omega} \right).
\end{align*}
\]
APPENDIX

-2+1 dimensional results

Conservation equations

\[ \nabla_\mu T^{\mu \nu} = F^{\nu \lambda} J_\lambda \]

\[ \nabla_\mu J^\mu = 0 \]

[Jensen, MK, Kovtun, Meyer, Ritz, Yarom 1112.4498]
APPENDIX

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Constitutive equations

\[
T^{\mu\nu} = \epsilon_0 u^\mu u^\nu + (P_0 - \zeta \nabla_\alpha u^\alpha - \tilde{\chi}_B B - \tilde{\chi}_\Omega \Omega) \Delta^{\mu\nu} - \eta \sigma^{\mu\nu} - \tilde{\eta} \tilde{\sigma}^{\mu\nu}
\]

\[
J^\mu = \rho_0 u^\mu + \sigma V^\mu + \tilde{\sigma} \tilde{V}^\mu + \tilde{\chi}_E \tilde{E}^\mu + \tilde{\chi}_T \epsilon^{\mu\nu\rho} u_\nu \nabla_\rho T
\]

“New” transport terms arise!
APPENDIX
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Constitutive equations

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T^{\mu\nu} = \epsilon_0 u^\mu u^\nu + (P_0 - \zeta \nabla_\alpha u^\alpha - \tilde{\chi}_B B - \tilde{\chi}_\Omega \Omega) \Delta^{\mu\nu} - \eta \sigma^{\mu\nu} - \tilde{\eta} \tilde{\sigma}^{\mu\nu}
\]
\[
J^\mu = \rho_0 u^\mu + \sigma V^\mu + \tilde{\sigma} \tilde{V}^\mu + \tilde{\chi}_E \tilde{E}^\mu + \tilde{\chi}_T \epsilon^{\mu\nu\rho} u_\nu \nabla_\rho T
\]

“New” transport terms arise!

\[ \tilde{\eta} \quad \text{Hall viscosity} \]
\[ \tilde{\chi}_E , \tilde{\sigma} \quad \text{off-diagonal conductivity} \]
\[ \text{thermodynamic interpretation of} \quad \tilde{\chi}_E , \tilde{\chi}_\Omega , \tilde{\chi}_B \]
\[ \text{“thermal Hall conductivity”} \]

[Jensen, MK, Kovtun, Meyer, Ritz, Yarom 1112.4498]