High finesse optical cavities

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References

Laser Beams and Resonators

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Abstract—This paper is a review of the theory of laser beams and resonators. It is meant to be tutorial in nature and useful in scope. No attempt is made to be exhaustive in the treatment. Rather, emphasis is placed on formulations and derivations which lead to basic understanding and on results which bear practical significance.

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http://en.wikipedia.org/wiki/Gaussian_beam

In optics, a Gaussian beam is a beam of electromagnetic radiation which, when the laser is said to be operating on the fundamental transverse mode (characterized by a different set of parameters), which explains why it is Gaussian. The mathematical function that describes the Gaussian beam is a solution to the wave equation, which represents the complex amplitude of the electric field, which propagates through space.
Cavity parameters

- Gaussian beams
- Strawman parameters
- Items governing finesse
- Items governing length
A gaussian beam is described in the \textit{paraxial} approximation (\(\sin \theta = \theta\)) by

\[
E(\rho, z) = A \frac{w_0}{w(z)} e^{ikz} e^{-\tan^{-1}(z/z_0)} e^{ik\rho^2/2R(z)} e^{-\rho^2/w^2(z)}
\]

where \(w_0\) is the beam waist dimension (a \textit{radius}) and

\[
z_0 = \frac{\pi w_0^2}{\lambda}
\]

is the Rayleigh range. The beam is \(\sqrt{2}\) bigger at \(z = z_0\) from the waist. The beam has a “diameter” of \(2w(z)\), with

\[
w^2(z) = w_0^2 \left[ 1 + \left( \frac{\lambda z}{\pi w_0^2} \right)^2 \right] = w_0^2 \left[ 1 + \left( \frac{z}{z_0} \right)^2 \right]
\]

the beam “size,” and a curvature

\[
R(z) = z \left[ 1 + \left( \frac{\pi w_0^2}{\lambda z} \right)^2 \right] = z + \frac{z_0^2}{z}.
\]

Finally,

\[
\theta = \frac{\lambda}{\pi w_0}
\]

is the beam divergence angle.
Intensities

At the waist, $z = 0$, $w = w_0$, $R = \infty$, and

$$E = Ae^{-\rho^2/w_0^2}$$

The intensity $\propto E^2$, so

$$I = I_0 e^{-2\rho^2/w_0^2}$$

and the power enclosed by a circle of diameter $D$ is

$$P(D) = P_0 \left[ 1 - e^{-D^2/2w_0^2} \right]$$

with $P_0$ the total power of the beam.
How do we find the waist? Set up a cavity, with curved mirrors of radii $R_1$ and $R_2$ and with a distance $L$ between them. The resonant beam will have radii of curvature of $R_i$ at each mirror, and a waist between them. For us, with curve/flat, $R_1 = R$ and $R_2 = \infty$. Then,

$$g = 1 - \frac{L}{R}$$

and

$$w_o^2 = \frac{\lambda L}{\pi} \sqrt{\frac{g}{1 - g}}$$

$g = g_1 g_2$ is called the stability product. We have $g_2 = 1$. Want $0 < |g| < 1$. 
Stability

- Hemispherical: (0, 1)
- (-1, 0)
- Concentric: (-1, -1)
- Plane-parallel: (1, 1)
- Confocal: (0, 0)
- Concave-convex: (2, 1/3)
Cavities have an infinite set of modes

- Hermite Gauss \((r)\) or Laguerre Gauss \((l)\)
Finesse and free spectral range

• Finesse is like $Q$ of a metal cavity
• Effectively, it is the number of bounces a beam makes before leaking out or being absorbed.

$$\mathcal{F} = \frac{4\pi T_1}{(T_1 + V)^2}$$

• with $T_1$ the transmission of the input mirror (assumed to be larger than that of the end mirror) and $V$ the round-trip fractional power loss from power absorption in both mirrors, scattering due to mirror defects, diffraction from the finite mirror size, etc.

• FSR:

$$FSR = \frac{c}{2L}$$

• Finesse is equal to FSR divided by the FWHM of the resonance.
Cavity with Finesse = 100,000

- Cavity transmission as function of frequency (left) or motion of one mirror (right)
- Cavity is 12 m in length, with modes at 12 MHz (=FSR)
Factors controlling finesse

• Loss
• Scatter
• Microroughness
• Coating nonuniformity
• Ultimately, transmission of input mirrors
Loss

PRM03 HR Absorption

O X -  O X +

R (mm)

HR Absorption (ppm)

Entries  202
Mean  0.5545
RMS  0.8577E-01

Count

HR Absorption (ppm)
Scatter

- Determined by dust particulates on surface, as well as by defects from polishing
- Scatter from 100 particles of $10\,\mu\text{m}$ diameter already dominates the loss budget
- Cleanliness!
Microroughness

- Low-angle scatter
- Rms ~ 0.5 nm
- “superpolish”
Coating nonuniformity

PR201 Transmission ($\lambda=1064$ nm, $\theta=2^\circ$, $\sigma_{peak}=1$ mm, Step=1 mm)

- Entries: 1257
- Mean: 237.5
- RMS: 12.66

Entries: 709
- Mean: 244.0
- RMS: 2.148

30 mm diameter aperture
Transmission

- At 1064 nm, $T \sim 3$ ppm
- Finesse $\sim 2$ million (ignoring scattering, which you cannot do)
Transmission 2

- At 1064 nm, $T \sim 5.5$ ppm
- Finesse $\sim 1$ million (ignoring scattering, which you cannot do)
Strawman cavity design

Magnets: 6+6 Tevatron dipoles
- 5 T field
- 6 m length each
- 48 mm diameter
- $B_0 L_{mag} = 180$ T-m

Cavity: curved-flat FP
- 45 m length; $\text{FSR} = c/2L_{cav} \sim 3.3$ MHz
- Mirror radii: 114 m (outer) and -4500 m (inner); $g = 0.59$
- Gaussian beam radii (field): 5.5 mm (outer); 4.3 mm (inner)
- 1 ppm diameter = 30 mm
- $\text{Finesse} = 3.1 \times 10^5$; $T = 10$ ppm; $A < 1$ ppm/mirror
- Stored power $\sim 1$ MW
- Intensity 2.2 MW/cm$^2$
Locking the cavities

- Pound-Drever-Hall locking

- Resonant regeneration experiment is complex:
  - 2 length degrees of freedom + alignment
  - Absolute position must be held to $\sim 10^{-13}$ m
Heating

- With 1 MW incident, 1 ppm loss, absorbed power would be 1 W
- Heating, deformation of mirror surface, loss of mode
Thermal Compensation

- Add heaters to perimeter of mirror
- Heat reduces thermal gradients
- Used successfully in eLIGO
DAMAGE

- An unknown unknown
- Damage thresholds said to be 1 GW/cm\(^2\)
  - vs 0.002 GW/cm\(^2\) in strawman
- Dust, sleeks, point defects seed damage
Summary

• With care, optical cavities with lengths of ~50 m and finesses of ~ 100,000 are well within the state of the art.

• Peter’s 105-110 m is OK. (1 ppm spot on curved mirror is 46.5 mm diameter. [Spot on flat is 34 mm.], g ~ 0.64)
THE END
Other issues

• Can avoid zeros of sinc function in conversion rate by alternating field directions.
• To go beyond $L \sim 90$ m would require first removing sagitta and then using larger diameter magnets. Km scales $=> 200$ mm diameters.
• For high power in production cavity, thermal management/thermal lenses become important.
• Avoid stray light.
• Must run in UHV.
• Dust elimination is critical; scatter from 100 particles of 10 $\mu$m diameter already dominates the loss budget.
• Need vibration-free mirror suspensions. Possibly suspended.
• Include quantum efficiency, photodetector dark current.
- Phase modulated light

\[ E = e^{-i\omega t} + i\gamma \cos \omega t \]

\[ = e^{-i\omega t} \left[ 1 + i\gamma \cos \omega t + \ldots \right] \]

\[ = e^{-i\omega t} + \frac{i\gamma}{2} e^{-i(\omega + \omega_L) t} + i\frac{\gamma}{2} e^{-i(\omega - \omega_L) t} \]
Reflect from cavity

\[ E_r = (r e^{i \omega} - i r \cos \Omega t) e^{-i \omega t} \]

\[ I_r = R + \Gamma^2 \cos^2 \Omega t - 2 \Gamma r \sin \Theta \cos \Omega t \]

Demodulate at \( \cos \Omega t \)

\[ \langle \cos^2 \rangle = \frac{1}{2} \]

\[ \langle I_m \rangle = -\Gamma r \sin \Theta \]