Large-\(x\) structure functions and OAM

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Outline

- Why large-\(x\) quarks are important
  - valence quarks, relation with high-\(t\) form factors

- \(x \rightarrow 1\) behavior from perturbative QCD
  - \(L_z = 0\) analysis; suppression of helicity-flip

- Role of OAM
  - log enhancement of helicity-flip amplitudes

- Phenomenological implications
  - CJ (CTEQ-JLab) large-\(x\) global analysis
  - challenges for empirical \(x \rightarrow 1\) analysis
Why large $x$?
Most direct connection between quark distributions and models of nucleon structure (e.g. leading Fock state of wfn) is via \textit{valence} quarks

\[ \rightarrow \text{most cleanly revealed at } x > 0.4 \]
Large-$x$ PDFs

- Ideal testing ground for nonperturbative & perturbative models of the nucleon

$\rightarrow$ e.g. ratio of $d$ to $u$ PDFs sensitive to spin-flavor dynamics

**SU(6) proton wave function**

\[
p^\uparrow = -\frac{1}{3} d^\uparrow (uu)_1 - \frac{\sqrt{2}}{3} d^\downarrow (uu)_1
\]

\[
+ \frac{\sqrt{2}}{6} u^\uparrow (ud)_1 - \frac{1}{3} u^\downarrow (ud)_1 + \frac{1}{\sqrt{2}} u^\uparrow (ud)_0
\]

- Interacting quark
- Diquark spin
- Spectator “diquark”
Large-$x$ PDFs

- Ideal testing ground for nonperturbative & perturbative models of the nucleon

→ e.g. ratio of $d$ to $u$ PDFs sensitive to spin-flavor dynamics

- $d/u \to 1/2$ \quad SU(6) symmetry
- $d/u \to 0$ \quad $S = 0$ \quad $qq$ dominance
- $d/u \to 1/5$ \quad $S_z = 0$ \quad $qq$ dominance
- $d/u \to \frac{4 \mu_n^2 / \mu_p^2 - 1}{4 - \mu_n^2 / \mu_p^2}$ \quad local quark-hadron duality* \quad ($\mu_{p,n}$ magnetic moments)

*structure function at $x \to 1$ given by elastic form factor at $Q^2 \to \infty$

see e.g. WM, Ent, Keppel Phys. Rep. 406, 127 (2005)
Large-$x$ PDFs

- Ideal testing ground for nonperturbative & perturbative models of the nucleon

  $\rightarrow$ *e.g.* ratio $\Delta q/q$ even more sensitive

  - $\Delta u/u \rightarrow 2/3$
    $\Delta d/d \rightarrow -1/3$
    
  - $\Delta u/u \rightarrow 1$
    $\Delta d/d \rightarrow -1/3$
    
  - $\Delta u/u \rightarrow 1$
    $\Delta d/d \rightarrow 1$
    
    *SU(6) symmetry*

    *S = 0  qq dominance*

    *$S_z = 0$  qq dominance*

    *or local duality*
Inclusive-exclusive connection

- **Drell-Yan-West relation**

\[ G_M(Q^2) \sim \left( \frac{1}{Q^2} \right)^n \iff F_2(x) \sim (1 - x)^{2n-1} \]

- **Drell & Yan:** field-theoretical model of strongly interacting $N, \bar{N} \& \pi$ “partons” in infinite momentum frame

- **West:** covariant model with single *scalar* quark, assuming amplitude for proton $\rightarrow$ quark + spectator behaves as

\[ f(p_i^2, p_{\text{spec}}^2) \sim \left( \frac{1}{p_i^2} \right)^n g(p_{\text{spec}}^2), \quad p_i^2 \to \infty \]

- for several flavors, in general \[ \sum_i e_i^2 \neq \left( \sum_i e_i \right)^2 \]

- how does duality arise?

*Close, Isgur, PLB 509, 81 (2001)*
Perturbative QCD
In QCD, “exceptional” $x \rightarrow 1$ configurations of proton wave function generated from “typical” wave function (for which $x_i \sim 1/3$) by exchange of $\geq 2$ hard gluons, with mass $k^2 \sim -\langle k^2_\perp \rangle/(1 - x)$.

Since $|k^2|$ is large, coupling at $q$-$g$ vertex is small

$\rightarrow$ use lowest-order perturbation theory!

Assume wave function vanishes sufficiently fast as $|k^2| \rightarrow \infty$ and unperturbed wave function dominated by 3-quark Fock component with $SU(2) \times SU(3)$ symmetry
Perturbative QCD

- If spectator “diquark” spins are anti-aligned (helicity of struck quark = helicity of proton)
  - can exchange transverse or longitudinal gluon

- If spectator “diquark” spins are aligned (helicity of struck quark ≠ helicity of proton)
  - can exchange only longitudinal gluon

- Coupling of (large-$k^2$) longitudinal gluon to (small-$p^2$) quark is suppressed by $(p^2/k^2)^{1/2} \sim (1 - x)^{1/2}$ w.r.t. transverse
  - $q^\perp \sim (1 - x)^2$ $q^\uparrow \sim (1 - x)^5$
Perturbative QCD

 Phenomenological consequences of $S_z = 0$ $qq$ dominance* 

→ assuming unperturbed SU(6) wave function, 
\[ \frac{F_2^n}{F_2^p} \to \frac{3}{7} \]

→ dominance of helicity-1/2 photoproduction cross section 
\[ \sigma_{1/2} \gg \sigma_{3/2} \]

→ for all quark flavors $q$, 
\[ \Delta q/q \to 1 \]
and therefore all polarization asymmetries $A_1 \to 1$

→ for pion, expect 
\[ F_2^\pi \sim (1 - x)^2 \]

* valid in Abelian & non-Abelian theories
Role of orbital angular momentum

- Above results assume quarks in lowest Fock state are in relative $s$-wave

  → higher Fock states and nonzero quark OAM will in general introduce additional suppression in $(1-x)$

- **BUT** nonzero OAM can provide logarithmic enhancement of helicity-flip amplitudes!

  → quark OAM modifies asymptotic behavior of nucleon’s Pauli form factor

  \[
  F_2(Q^2) \sim \log^2(Q^2/\Lambda^2) \frac{1}{Q^6}
  \]

  [Belitsky, Ji, Yuan, PRL 91, 092003 (2003)]

  → consistent with surprising $Q^2$ dependence of proton’s $G_E/G_M$ form factor ratio
Role of orbital angular momentum

For $L_z = 1$ Fock state, expand hard scattering amplitude in powers of $k_\perp$ ("collinear expansion")

- logarithmic singularities arise when integrating over longitudinal momentum fractions $x_i$ of soft quarks

- leads to additional $\log^2(1-x)$ enhancement of $q^\downarrow$

$$ q^\downarrow \sim (1-x)^5 \log^2(1-x) $$

Avakian, Brodsky, Deur, Yuan, PRL 99, 082001 (2007)

(similar contributions to positive helicity $q^\uparrow$ are power-suppressed)
Role of orbital angular momentum

- $k_\perp$-odd transverse momentum dependent (TMD) distributions (vanish after $k_\perp$ integration)
  → arise from interference between $L_z = 0$ and $L_z = 1$ states

- $T$-even TMDs
  → $g_{1T}$ (longitudinally polarized $q$ in a transversely polarized $N$)
  $h_{1L}$ (transversely polarized $q$ in a longitudinally polarized $N$)

- $T$-odd TMDs
  → $f_{1T}^\perp$ (unpolarized $q$ in a transversely polarized $N$ – “Sivers”)
  $h_{1}^\perp$ (transversely polarized $q$ in an unpolarized $N$ – “Boer-Mulders”)

- Each behaves in $x \rightarrow 1$ limit as
  $\text{TMD} \sim (1 - x)^4$

Brodsky, Yuan
PRD 74, 094018 (2006)
Phenomenological implications
Phenomenological implications

Power counting rule constraints used in exploratory fit to limited set of inclusive DIS spin structure function data

\[ q^\uparrow = x^\alpha [A(1 - x)^3 + B(1 - x)^4] \]
\[ q^\downarrow = x^\alpha [C(1 - x)^5 + D(1 - x)^6] \]

Brodsky, Burkardt, Schmidt
NPB 441, 197 (1995)
Phenomenological implications

- Power counting rule constraints used in exploratory fit to limited set of inclusive DIS spin structure function data

\[
q^\uparrow = x^\alpha [A(1-x)^3 + B(1-x)^4]
\]

\[
q^\downarrow = x^\alpha [C(1-x)^5 + D(1-x)^6 + C'(1-x)^5 \log^2(1-x)]
\]

Avakian, Brodsky, Deur, Yuan

LSS’98

ABDY’07

improved fit for $\Delta d/d$

additional $L_{\bar{z}}=1$ term
Phenomenological implications

Determining $x \rightarrow 1$ behavior experimentally is problematic

- simple $x^\alpha (1 - x)^\beta$ parametrizations inadequate for describing high-precision data, and global fits typically require more complicated $x$ dependence, e.g.

$$q \sim x^\alpha (1 - x)^\beta (1 + \gamma \sqrt{x} + \eta x)$$

- recent global fits of spin-dependent PDFs find (at $Q^2 \sim 5 \text{ GeV}^2$)

$$\beta \approx 3.3 \ (\Delta u_V), \ 3.9 \ (\Delta d_V)$$

but with $\gamma, \eta \sim O(10-100)$

Challenge to perform constrained global fit to all DIS, SIDIS & $\vec{p} \vec{p}$ scattering data
Phenomenological implications

- **Determining** \( x \to 1 \) behavior experimentally is problematic

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\[
q \sim x^\alpha (1 - x)^\beta (1 + \gamma \sqrt{x} + \eta x)
\]

- recent global fits of spin-dependent PDFs find (at \( Q^2 \sim 5 \text{ GeV}^2 \))

\[
\beta \approx 3.3 \ (\Delta u_V), \ 4.1 \ (\Delta d_V)
\]

  but with \( \gamma, \eta \sim \mathcal{O}(10-100) \)

- **Challenge to perform constrained global fit to all DIS, SIDIS & \( \vec{p} \vec{p} \) scattering data**

  Leader, Sidorov, Stamenov
  
  PRD 82, 114018 (2010)
Phenomenological implications

Determining $x \to 1$ behavior experimentally is problematic.

- Simple $x^\alpha (1 - x)^\beta$ parametrizations inadequate for describing high-precision data, and global fits typically require more complicated $x$ dependence, e.g.

$$q \sim x^\alpha (1 - x)^\beta (1 + \gamma \sqrt{x} + \eta x)$$

- Recent global fits of spin-dependent PDFs find (at $Q^2 \sim 5 \text{ GeV}^2$)

$$\beta \approx 3.0 (\Delta u_V), \ 4.1 (\Delta d_V)$$

but with $\gamma, \eta \sim \mathcal{O}(10-100)$

- Challenge to perform constrained global fit to all DIS, SIDIS & $\vec{p} \vec{p}$ scattering data

Bluemlein, Boettcher
NPB 841, 205 (2010)
Phenomenological implications

**Challenges for large-\(x\) PDF analysis**

- at fixed \(Q^2\), increasing \(x\) corresponds to decreasing \(W\)
  - eventually run into nucleon *resonance* region as \(x \to 1\)
  - impose cuts (usual solution) or utilize quark-hadron duality (theoretical bias)

- subleading \(1/Q^2\) corrections (target mass, higher twists)

- nuclear corrections in extraction of *neutron* information from nuclear (deuterium, \(^3\)He) data

- dependence on choice of PDF parametrization

**New CTEQ-JLab (“CJ”) global PDF analysis* (unpolarized) dedicated to describing large-\(x\) region**

CJ global analysis

- **Cut 0**: $Q^2 > 4\text{ GeV}^2$, $W^2 > 12.25\text{ GeV}^2$
- **Cut 1**: $Q^2 > 3\text{ GeV}^2$, $W^2 > 8\text{ GeV}^2$
- **Cut 2**: $Q^2 > 2\text{ GeV}^2$, $W^2 > 4\text{ GeV}^2$
- **Cut 3**: $Q^2 > m_c^2$, $W^2 > 3\text{ GeV}^2$

- Factor 2 increase in DIS data from cut 0 to cut 3
CJ global analysis

- Systematically reduce $Q^2$ & $W$ cuts
- Fit includes TMCs, HT term, nuclear corrections

$\frac{d}{d_{\text{ref}}}$ $Q^2=10$ GeV$^2$

$\rightarrow$ stable with respect to cut reduction

$\rightarrow$ $d$ quark suppressed by $\sim 50\%$ for $x > 0.5$
(driven by nuclear corrections)

Accardi et al., PRD 81, 034016 (2010)
larger database with weaker cuts leads to significantly reduced errors, esp. at large $x$

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CJ global analysis

Accardi et al.
PRD 81, 034016 (2010)
large nuclear correction uncertainties at $x > 0.5$

$x \rightarrow 1$ limiting value depends on deuteron model
dramatic increase in $d$ PDF in $x \to 1$ limit with more flexible parametrization $d \rightarrow d + a x^b u$

(allows for finite, nonzero $d/u$ in $x = 1$ limit)
Outlook

- Nuclear correction uncertainties expected to be resolved with new experiments at JLab–12 GeV uniquely sensitive to $d$ quarks (up to $x \sim 0.85$)

  - “spectator” protons tagged in SIDIS from deuterium
    \[ e \ d \rightarrow e \ p_{\text{Spec}} \ X \quad (\text{“BoNuS”}) \]

  - DIS from $^3\text{He}$-tritium mirror nuclei
    \[ e \ ^3\text{He}(^3\text{H}) \rightarrow e \ X \quad (\text{“MARATHON”}) \]

  - PVDIS from protons
    \[ \vec{e}_L(\vec{e}_R) \ p \rightarrow e \ X \quad (\text{“SOLID”}) \]

- Constraints from $W$ production in $pp$ collisions at high (lepton & $W$ boson) rapidities

  - CDF & D0 at Fermilab, LHCb at CERN
$W$ boson asymmetries

- Large-$x$ PDF uncertainties affect observables at large rapidity $y$, with

$$y = \frac{1}{2} \ln \left( \frac{E + p_z}{E - p_z} \right) \Rightarrow x_{1,2} = \frac{M}{\sqrt{s}} e^{\pm y}$$

- \textit{e.g.} $W^\pm$ asymmetry

\[\begin{array}{c}
\text{Large-}$x$ \text{ PDF uncertainties affect observables at large rapidity } y, \text{ with } \\
y = \frac{1}{2} \ln \left( \frac{E + p_z}{E - p_z} \right) \Rightarrow x_{1,2} = \frac{M}{\sqrt{s}} e^{\pm y} \\
\text{\textit{e.g.} } W^\pm \text{ asymmetry}
\end{array}\]

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\text{\textit{e.g.} } W^\pm \text{ asymmetry}
\end{array}\]
Outlook

- New JLab–12 GeV precisions measurements of $A_1^n$ & $A_1^p$
  hope to constrain $\Delta d/d$ up to $x \sim 0.8$
  → new (non-inclusive DIS) experiments to reduce nuclear dependence

- Parametrization dependence of $x \to 1$ limit may be eliminated through e.g. “neural network” PDFs
  → thus far applied mainly to unpolarized PDFs

- New global analysis of spin-dependent PDFs dedicated to large-$x$, moderate-$Q^2$ region
  → JLab Angular Momentum (“JAM”) collaboration*
  → initial focus on helicity PDFs; later expand scope to TMDs

* JAM collaboration: P. Jimenez-Delgado, A. Accardi, WM (theory) + JLab Halls A, B, C (expt.)
Outlook

- Large-$x$ PDFs from lattice?

  → need many moments to reconstruct $x$ dependence

  ![Graph showing fit (i) and fit (vii) with their functional forms.]

  **fit (i):** $\alpha, \beta, \gamma, \eta$ unconstrained
  
  **fit (vii):** $\gamma, \eta$ constrained

  Assume functional form
  
  $$q(x) = N x^\alpha (1 - x)^\beta (1 + \gamma \sqrt{x} + \eta x)$$

  *Detmold, WM, Thomas
  MPLA 18, 2681 (2003)*

- Need new ideas

  → e.g. compute Compton scattering tensor directly by coupling to fictitious heavy quark (remove all-to-all propagators, and operator mixing)

  *Detmold, Lin
  PRD 73, 014501 (2006)*
The End