Ab initio description of the unbound $^7$He

INT program INT-12-3
“Light nuclei from first principles”
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Outline

- Why $^7$He?
- No-core shell model calculations for neutron rich He isotopes
- Introducing no-core shell model with continuum (NCSMC)
- $^7$He calculations: Comparison of NCSM/RGM and NCSMC
- $^7$He predictions and comparison to experiment
- Outlook
Unbound exotic $^7\text{He}$

- **Experimental situation**
  - $3/2^-$ g.s. resonance at 0.43 MeV above $n + ^6\text{He}$
    - $^6\text{He}$ Borromean halo system
  - $5/2^-$ resonance established
  - Controversy about $1/2^-$ resonance
    - Low-lying narrow
    - Broad at 3 MeV
    - Extremely broad

Experiments very challenging: three-body background
Unbound exotic $^7$He

- **Experimental situation**
  - Controversy about $1/2^-$ resonance
    - Low-lying narrow [$^8$He+$^{12}$C fragmentation]
    - Broad at 3 MeV [$d(^6$He,$p)^7$He,$^2$H($^8$Li,$^3$He)$^7$He]
    - Extremely broad [$p+^6$He: isospin analog]

<table>
<thead>
<tr>
<th>$J^\pi$</th>
<th>experiment</th>
<th>Ref.</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>$E_R$</td>
<td>$\Gamma$</td>
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<tr>
<td>$3/2^-$</td>
<td>0.430(3)</td>
<td>0.182(5)</td>
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<tr>
<td>$5/2^-$</td>
<td>3.35(10)</td>
<td>1.99(17)</td>
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<td>$1/2^-$</td>
<td>3.03(10)</td>
<td>2</td>
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<tr>
<td></td>
<td>3.53</td>
<td>10</td>
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<td></td>
<td>1.0(1)</td>
<td>0.75(8)</td>
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*Ab initio* calculations based on bound-state techniques cannot give any insight.
Chiral Effective Field Theory

- **First principles for Nuclear Physics:**
  - QCD
    - Non-perturbative at low energies
    - Lattice QCD in the future

- **For now a good place to start:**
- Inter-nucleon forces from chiral effective field theory
  - Based on the symmetries of QCD
    - Chiral symmetry of QCD ($m_u\approx m_d\approx 0$), spontaneously broken with pion as the Goldstone boson
    - Degrees of freedom: nucleons + pions
  - Systematic low-momentum expansion to a given order ($Q/\Lambda^\chi$)
    - Hierarchy
    - Consistency
  - Low energy constants (LEC)
    - Fitted to data
    - Can be calculated by lattice QCD

$\Lambda^\chi \sim 1$ GeV: Chiral symmetry breaking scale
The \textit{ab initio} no-core shell model (NCSM)

- The NCSM is a technique for the solution of the $A$-nucleon bound-state problem
- Realistic nuclear Hamiltonian
  - High-precision nucleon-nucleon potentials
  - Three-nucleon interactions
- Finite harmonic oscillator (HO) basis
  - $A$-nucleon HO basis states
  - Complete $N_{\text{max}} \hbar \Omega$ model space
- \textit{Effective interaction tailored to model-space truncation} for NN(+NNN) potentials
  - Okubo-Lee-Suzuki unitary transformation
- \textbf{Or a sequence of unitary transformations in momentum space:}
  - Similarity-Renormalization-Group (SRG) evolved NN(+NNN) potential

\[ \Psi^A = \sum_{N=0}^{N_{\text{max}}} \sum_i c_{Ni} \Phi^A_{Ni} \]

Convergence to exact solution with increasing $N_{\text{max}}$ for bound states. \textit{No coupling to continuum.}
$^4\text{He}$ from chiral EFT interactions: g.s. energy convergence

Chiral N$^3$LO NN plus N$^2$LO NNN potential

- Bare interaction (black line)
  - Strong short-range correlations
    - Large basis needed
- SRG evolved effective interaction (red line)
  - Unitary transformation
    
    $$H_\alpha = U_\alpha H U_\alpha^+ \Rightarrow \frac{dH_\alpha}{d\alpha} = [[T,H_\alpha],H_\alpha] (\alpha = \frac{1}{\lambda^4})$$
  - Two- plus three-body components, four-body omitted
  - Softens the interaction
    - Smaller basis sufficient

- $^4\text{He}$ binding energy and half-life constraint
  - $A=3$ binding energy and half life constraint $c_D=-0.2$, $c_E=-0.205$, $\Lambda=500$ MeV

**Figure Caption:**

- **4 He**
- **N$^3$LO (500 MeV) NN + NNN**

**Graph Details:**

- **Axes:**
  - **X-axis:** $N_{\text{max}}$
  - **Y-axis:** $E$ [MeV]

**Graph Elements:**

- **Data Points:**
  - Bare (36)
  - SRG (2.0/28)

**Equation:**

$$H_\alpha = U_\alpha H U_\alpha^+ \Rightarrow \frac{dH_\alpha}{d\alpha} = [[T,H_\alpha],H_\alpha] (\alpha = \frac{1}{\lambda^4})$$
6He and 8He with SRG-evolved chiral N^3LO NN + N^2LO NNN

- 3N matrix elements in coupled-J single-particle basis:
  - Introduced and implemented by Robert Roth et al.
  - Now also in my codes: Jacobi-Slater-Determinant transformation & NCSD code
  - Example: 6He, 8He NCSM calculations up to N_{max}=10 done with moderate resources

![Graphs showing energy levels](image1.png)

A=3 binding energy & half life constraint 
\(c_0=-0.2, \ c_E=-0.205, \ \Lambda=500 \text{ MeV}\)
3N interaction effects in neutron rich nuclei: He isotopes

- $^6\text{He}$ and $^8\text{He}$ with SRG-evolved chiral N$^3$LO NN + N$^2$LO 3N
  - chiral N$^3$LO NN: $^4\text{He}$ underbound, $^6\text{He}$ and $^8\text{He}$ unbound
  - chiral N$^3$LO NN + N$^2$LO 3N(500): $^4\text{He}$ OK, both $^6\text{He}$ and $^8\text{He}$ bound

$A=3$ binding energy & half life constraint $c_D=-0.2$, $c_E=-0.205$, $\Lambda=500$ MeV

NNN interaction important to bind neutron rich nuclei
3N interaction effects in neutron rich nuclei: He isotopes

- $^6$He and $^8$He with SRG-evolved chiral $N^3$LO NN + $N^2$LO 3N
  - chiral $N^3$LO NN: $^4$He underbound, $^6$He and $^8$He unbound
  - chiral $N^3$LO NN + $N^2$LO 3N(400): $^4$He fitted, $^6$He barely unbound, $^8$He unbound
    - describes quite well binding energies of $^{12}$C, $^{16}$O, $^{40}$Ca, $^{48}$Ca
  - chiral $N^3$LO NN + $N^2$LO 3N(500): $^4$He OK, both $^6$He and $^8$He bound
    - does well up to $A=10$, overbinds $^{12}$C, $^{16}$O, Ca isotopes
  - SRG-$N^3$LO NN $\Lambda=2.02$ fm$^{-1}$: $^4$He OK, both $^6$He and $^8$He bound
    - $^{16}$O, Ca strongly overbound

$^4$He binding energy & $^3$H half life constraint
$c_D=-0.2, c_E=+0.098, \Lambda=400$ MeV

$A=3$ binding energy & half life constraint
$c_D=-0.2, c_E=-0.205, \Lambda=500$ MeV

NNN interaction important to bind neutron rich nuclei

Our knowledge of the 3N interaction is incomplete
NCSM calculations of $^6$He and $^7$He g.s. energies

- $^6$He: $E_{g.s.} = -29.25(15)$ MeV (Expt. -29.269 MeV)
- $^7$He: $E_{g.s.} = -28.27(25)$ MeV (Expt. -28.84(30) MeV)
- $^7$He unbound (+0.430(3) MeV), width 0.182(5) MeV
- NCSM: no information about the width

**N_max** convergence OK

Extrapolation feasible

- $^6$He: $E_{g.s.} = -29.25(15)$ MeV (Expt. -29.269 MeV)
- $^7$He: $E_{g.s.} = -28.27(25)$ MeV (Expt. -28.84(30) MeV)
- $^7$He unbound (+0.430(3) MeV), width 0.182(5) MeV
- NCSM: no information about the width
The \textit{ab initio} NCSM/RGM in a snapshot

**Ansatz:**
\[
\Psi^{(A)} = \sum_{\nu} \int d\vec{r} \varphi_{\nu}(\vec{r}) \hat{A} \Phi^{(A-a,a)}_{\nu \vec{r}}
\]

- Many-body Schrödinger equation:
\[
H \Psi^{(A)} = E \Psi^{(A)}
\]
\[
\sum_{\nu} \int d\vec{r} \left[ H^{(A-a,a)}_{\mu \nu}(\vec{r}', \vec{r}) - E \mathcal{N}^{(A-a,a)}_{\mu \nu}(\vec{r}', \vec{r}) \right] \varphi_{\nu}(\vec{r}) = 0
\]

- Hamiltonian kernel:
\[
\langle \Phi_{\mu \vec{r}'}^{(A-a,a)} | \hat{A} | \Phi_{\nu \vec{r}}^{(A-a,a)} \rangle
\]

- Norm kernel:
\[
\langle \Phi_{\mu \vec{r}'}^{(A-a,a)} | \hat{A}^2 | \Phi_{\nu \vec{r}}^{(A-a,a)} \rangle
\]

- Eigenstates of \( H_{(A-a)} \) and \( H_{(a)} \) in the \textit{ab initio} NCSM basis:
\[
\Psi^{(A-a)}_{1 \nu} \Psi^{(a)}_{2 \nu} \delta(\vec{r} - \vec{r}_{A-a,a})
\]

- Realistic nuclear Hamiltonian:
\[
T_{\text{rel}}(r) + V_{\text{rel}} + V_{\text{Coul}}(r) + H_{(A-a)} + H_{(a)}
\]
Consider the $T = \frac{1}{2}$ case: $^5\text{He (}^5\text{Li)}$

- Five-nucleon cluster unbound; $^4\text{He}$ tightly bound, not easy to deform

Satisfactory description of $n - ^4\text{He (}p - ^4\text{He)}$ scattering at low excitation energies within single-channel approximation.

However, both $n(p) + ^4\text{He}$ and $d + ^3\text{H}(^3\text{He})$ channels needed to describe $^3\text{H}(d,n)^4\text{He}$ [$^3\text{He}(d,p)^4\text{He}]$ fusion!
Unbound $A=5$ nuclei: $^{5}\text{He} \rightarrow n + ^{4}\text{He}$, $^{5}\text{Li} \rightarrow p + ^{4}\text{He}$

- **NCSM/RGM calculations**
  - SRG-$N^3\text{LO}$ NN potential with $\Lambda=2.02$ fm$^{-1}$

- **Differential cross section and analyzing power @17 MeV neutron energy**
  - Polarized neutron experiment at Karlsruhe

NNN missing: Good agreement only for energies beyond low-lying $3/2^-$ resonance
How about $^7$He as $n^+^6$He?

- All $^6$He excited states above $2^+$ broad resonances or states in continuum
- Convergence of the NCSM/RGM $n^+^6$He calculation slow with number of $^6$He states
  - Negative parity states also relevant
  - Technically not feasible to include more than ~ 5 states
New approach: NCSM with continuum

\[ |\Psi_A^{J^\pi T}\rangle = \sum_{Ni} c_{Ni} |ANiJ^\pi T\rangle \]
The idea behind the NCSM is
\[ |\Psi_A^{J\pi T}\rangle = \sum_{Ni} c_{Ni} |ANiJ\pi T\rangle \]

New developments: NCSM with continuum

\[ |\Psi_A^{J\pi T}\rangle = \sum_{\nu} \int d\vec{r} \chi_{\nu}(\vec{r}) \hat{A} \Phi_{\nu\vec{r}}^{J\pi T(A-a,a)} \]

\[ \mathcal{H}\chi = E\mathcal{N}\chi \]

\[ \bar{\chi} = \mathcal{N}^{+\frac{1}{2}} \chi \]

\[ \mathcal{N}^{-\frac{1}{2}} \mathcal{H} \mathcal{N}^{-\frac{1}{2}} \bar{\chi} = E\bar{\chi} \]
New developments: NCSM with continuum

\[ |\Psi_A^{J^T}\rangle = \sum_{Ni} c_{Ni} |ANiJ^T\rangle \]

**NCSM/RGM**

\[ |\Psi_A^{J^T}\rangle = \sum_{\nu} \int d\vec{r} \chi_{\nu}(\vec{r}) \hat{A} \Phi_{\nu r}^{J^T(A-a,a)} \]
\[ \mathcal{H} \chi = EN \chi \]
\[ \bar{\chi} = \mathcal{N}^{+\frac{1}{2}} \chi \]

\[ (\mathcal{N}^{\frac{1}{2}} \mathcal{H} \mathcal{N}^{\frac{1}{2}}) \bar{\chi} = E \bar{\chi} \]

**NCSMC**

\[ |\Psi_A^{J^T}\rangle = \sum_{\lambda} c_{\lambda} |A\lambda J^T\rangle + \sum_{\nu} \int d\vec{r} \left( \sum_{\nu'} \int d\vec{r}' \mathcal{N}_{\nu\nu'}^{\frac{1}{2}}(\vec{r}, \vec{r}') \bar{\chi}_{\nu'}(\vec{r}') \right) \hat{A} \Phi_{\nu r}^{J^T(A-a,a)} \]

\[
\begin{pmatrix}
H_{NCSM} & \bar{h} \\
\bar{h} & \mathcal{N}^{\frac{1}{2}} \mathcal{H} \mathcal{N}^{\frac{1}{2}}
\end{pmatrix}
\begin{pmatrix}
c \\
\bar{\chi}
\end{pmatrix}
= E
\begin{pmatrix}
1 & \bar{g} \\
\bar{g} & 1
\end{pmatrix}
\begin{pmatrix}
c \\
\bar{\chi}
\end{pmatrix}
\]
When computing the above kernel, the “exchange” term is obtained by expanding the radial dependence arising from the permutations in the cluster states and its size is consistent with the model-space NCSM/RGM norm in the NCSM/RGM sector. This HO basis has the same frequency used in the identity is obtained by expanding the radial dependence of HO excitation for the harmonic oscillator and by the coupling form factor for the orthogonalized NCSMC Hamiltonian.

\[
\left( \begin{array}{cc} H_{\text{NCSM}} & \bar{h} \\ \bar{h} & \bar{H} \end{array} \right) \left( \begin{array}{c} c \\ \chi \end{array} \right) = E \left( \begin{array}{c} 1 \\ \bar{g} \end{array} \right) \left( \begin{array}{c} c \\ \chi \end{array} \right)
\]

NCSM sector:

\[
(H_{\text{NCSM}})_{\lambda\lambda'} = \langle A\lambda J^\pi T | \hat{H} | A\lambda' J^\pi T \rangle = \epsilon^\pi T \delta_{\lambda\lambda'}
\]

NCSM/RGM sector:

\[
\bar{H}_{\nu\nu'}(r, r') = \sum_{\mu\mu'} \int \int dy dy' y^2 y'^2 N^{-\frac{1}{2}}_{\nu\mu}(r, y) \mathcal{H}_{\mu\mu'}(y, y') N^{-\frac{1}{2}}_{\nu'\nu'}(y'|r')
\]
How to calculate the NCSM/RGM kernels?

\[ |\psi_{J^T}\rangle = \sum_v \int \frac{g_v^{J^T}(r)}{r} \hat{A}_v \left[ \left( |A-a \alpha_1 I_1^{\pi_1} T_1\rangle |a \alpha_2 I_2^{\pi_2} T_2\rangle \right)^{(sT)} Y_\ell (\hat{r}_{A-a,a}) \right]^{(J^T)} \delta(r-r_{A-a,a}) r^2 dr \]

- Since we are using NCSM wave functions, it is convenient to introduce Jacobi channel states in the HO space

\[ |\Phi_{J^T}\rangle \]

(Jacobi) channel basis

\[ |\Phi_{vn}\rangle = \left[ \left( |A-a \alpha_1 I_1^{\pi_1} T_1\rangle |a \alpha_2 I_2^{\pi_2} T_2\rangle \right)^{(sT)} Y_\ell (\hat{r}_{A-a,a}) \right]^{(J^T)} R_{n\ell} (r_{A-a,a}) \]

- The coordinate space channel states are given by

\[ |\Phi_{vr}\rangle = \sum_n R_{n\ell} (r) |\Phi_{vn}\rangle \]

- We used the closure properties of HO radial wave functions

\[ \delta(r-r_{A-a,a}) = \sum_n R_{n\ell} (r) R_{n\ell} (r_{A-a,a}) \]

- Target and projectile wave functions are both translational invariant NCSM eigenstates calculated in the Jacobi coordinate basis
Norm kernel (Pauli principle)

Single-nucleon projectile

\[
\begin{align*}
\langle \Phi^J_{\nu r'} | \hat{A}_\nu \hat{A}_v | \Phi^J_{\nu r} \rangle &= \langle (A-1) | (a'=1) | 1 - \sum_{i=1}^{A-1} \hat{P}_{iA} | (a=1) \rangle \\
N^J_{\nu v} (r', r) &= \delta_{\nu \nu'} \frac{\delta(r' - r)}{r' r} - (A-1) \sum_{n'n} R_{n'\ell'}(r') R_{n\ell}(r) \langle \Phi^J_{\nu' n'} | \hat{P}_{A-1,A} | \Phi^J_{\nu n} \rangle \\
\delta(r - r_{A-a,a}) &= \sum_n R_{n\ell}(r) R_{n\ell}(r_{A-a,a})
\end{align*}
\]

Direct term: Treated exactly!
(in the full space)

Exchange term:
Obtained in the model space!
(Many-body correction due to the exchange part of the inter-cluster antisymmetrizer)
Hamiltonian kernel (projectile-target potentials)

Single-nucleon projectile

\[
\langle \Phi_{v'r'}^{J\pi T} | \hat{A}_v H \hat{A}_v | \Phi_{vr}^{J\pi T} \rangle = \langle (A-1) \rangle_{r'} \langle (a'=1) \rangle_{r} H \left( 1 - \sum_{i=1}^{A-1} \hat{P}_{iA} \right) \langle (a=1) \rangle_{r}
\]

\[
H_{v'v}^{J\pi T} (r', r) = \left[ T_{rel} (r) + V_{Coul} (r) + \varepsilon_{\alpha_i}^{I_i J_i T_i} \right] N_{v'v}^{J\pi T} (r', r)
\]

\[
+ (A-1) \sum_{n'n} R_{n'l'} (r') R_{n\ell} (r) \langle \Phi_{v'n'}^{J\pi T} | V_{A-1,A} \left( 1 - \hat{P}_{A-1,A} \right) | \Phi_{vn}^{J\pi T} \rangle
\]

\[
- (A-1)(A-2) \sum_{n'n} R_{n'l'} (r') R_{n\ell} (r) \langle \Phi_{v'n'}^{J\pi T} | \hat{P}_{A-1,A} V_{A-2,A-1} | \Phi_{vn}^{J\pi T} \rangle
\]

\[
+ (A-1) \times \left\{ \begin{array}{c}
\text{Direct potential: in the model space} \\
(\text{interaction is localized!})
\end{array} \right\}
\]

\[
- \begin{array}{c}
(A-1)(A-2) \times \\
\text{Exchange potential: in the model space}
\end{array}
\]
Define SD channel states in which the eigenstates of the heaviest of the two clusters (target) are described by a SD wave function:

\[
\left| \Phi_{\nu n}^{J^T} \rightangle_{SD} = \left[ \left( \left| A-a \alpha_1 I_1^{\pi_1} T_1 \rightangle_{SD} \right| a \alpha_2 I_2^{\pi_2} T_2 \right\rangle \right]^{(sT)} Y_{\ell} \left( \hat{R}^{(a)}_{c.m.} \right)^{(J^T)} R_{n\ell} \left( R^{(a)}_{c.m.} \right)
\]

\[
\left| A-a \alpha_1 I_1^{\pi_1} T_1 \right\rangle \varphi_{00} \left( \hat{R}^{(A-a)}_{c.m.} \right)
\]

Vector proportional to the c.m. coordinate of the \( A-a \) nucleons

\[
\left( \varphi_{00} \left( \hat{R}^{(A-a)}_{c.m.} \right) \varphi_{n\ell} \left( \hat{R}^{(a)}_{c.m.} \right) \right)^{\ell} = \sum_{n_r\ell_r, NL} \left\langle 00, n\ell, \ell \left| n_r\ell_r, NL, \ell \right\rangle \right\rangle_{d=A-a} \left( \varphi_{n_r\ell_r} \left( \vec{n}_{A-a} \right) \varphi_{NL} \left( \vec{\xi}_0 \right) \right)^{\ell}
\]
Translational invariant matrix elements from SD ones

• More in detail:

\[
\left| \Phi_{\nu n}^{J^\pi T} \right|_{SD} = \sum_{n_r \ell_r, NL, J_r} \hat{\ell} \hat{J}_r \left( -1 \right)^{s+\ell_r +L +J} \begin{pmatrix} s & \ell_r & J_r \\ L & J & \ell \end{pmatrix} \left\langle 0 \rangle_{d_a = a/\ell}, n \ell, \ell | \langle n_r \ell_r, NL, \ell \rangle \right|_{d = a/\ell} \left[ \Phi_{\nu n}^{J^\pi T} \right] \varphi_{NL} \left( \vec{z}_0 \right) \left( J^\pi T \right)
\]

• The spurious motion of the c.m. is mixed with the intrinsic motion

\[
\left\langle \Phi_{\nu n'}^{J^\pi T} | \hat{O} | \Phi_{\nu n}^{J^\pi T} \right\rangle_{SD} = \sum_{n_r \ell_r, n_r' \ell_r, J_r} \left\langle \Phi_{\nu n'}^{J^\pi T} | \hat{O} | \Phi_{\nu n}^{J^\pi T} \right\rangle_{SD} 
\]

× \sum_{NL} \hat{\ell} \hat{\ell}' \hat{J}_r^2 \left( -1 \right)^{s' + \ell' - s - \ell'} \begin{pmatrix} s & \ell_r & J_r \\ L & J & \ell \end{pmatrix} \begin{pmatrix} s' & \ell'_r & J_r \\ L & J & \ell' \end{pmatrix} \left\langle 0 \rangle_{d' = a'/\ell'}, n \ell', \ell' \lambda | \langle n_r \ell_r, NL, \ell' \rangle \right|_{d = a/\ell} \left[ \Phi_{\nu n}^{J^\pi T} \right] \varphi_{NL} \left( \vec{z}_0 \right) \left( J^\pi T \right)

• Translational invariance preserved (exactly!) also with SD channels

• Transformation is general: same for different \( A \)'s or different \( a \)'s
Is the SD channel basis advantageous?

- SD to Jacobi transformation is general and exact
- Can use powerful second quantization representation
  - Matrix elements of translational invariant operators can be expressed in terms of matrix elements of density operators on the target eigenstates
  - For example, for \( a = a' = 1 \)

\[
\begin{align*}
\langle \Phi^{J\pi T}_v | P_{A-1, A} | \Phi^{J\pi T}_{vn} \rangle_{SD} &= \frac{1}{A-1} \sum_{jj'kk} \hat{S} \hat{s}' \hat{j}' \hat{j} \hat{K} \hat{\tau} (-1)^{I_{1}^{+}+j+j} (-1)^{T_{1}^{1}+\frac{1}{2}+T} \\
&\times \left\{ \begin{array}{ccc}
I_1 & \frac{1}{2} & s \\
\ell & J & j
\end{array} \right\} \left\{ \begin{array}{ccc}
I'_1 & \frac{1}{2} & s' \\
\ell' & J & j'
\end{array} \right\} \left\{ \begin{array}{ccc}
I_1 & K & I'_1 \\
J & j & j'
\end{array} \right\} \left\{ \begin{array}{ccc}
T_1 & \tau & T'_1 \\
\frac{1}{2} & T & \frac{1}{2}
\end{array} \right\} \\
&\times \langle A-1 \alpha'_1 I_1^{\pi_1} T'_1 | (a^+_n \tilde{n} \tilde{a}_n^\dagger)_{\frac{1}{2}}^{(K\tau)} | A-1 \alpha_1 I_1^{\pi} T_1 \rangle_{SD}
\end{align*}
\]

One-body density matrix elements
NCSMC formalism

Start from

\[
\begin{pmatrix}
    H_{NCSM} & \tilde{h} \\
    \tilde{h} & \tilde{H}
\end{pmatrix}
\begin{pmatrix}
    c \\
    \chi
\end{pmatrix}
= E
\begin{pmatrix}
    1 & \bar{g} \\
    \bar{g} & 1
\end{pmatrix}
\begin{pmatrix}
    c \\
    \chi
\end{pmatrix}
\]

Coupling:

\[
\bar{g}_{\lambda \nu}(r) = \sum_{\nu'} \int dr' r'^2 \langle A \lambda J^{\pi} T | \hat{A}_{\nu'} \Phi^{J^{\pi} T}_{\nu' r'} \rangle \mathcal{N}^{-\frac{1}{2}}_{\nu' \nu}(r', r)
\]

\[
\bar{h}_{\lambda \nu}(r) = \sum_{\nu'} \int dr' r'^2 \langle A \lambda J^{\pi} T | \hat{H} \hat{A}_{\nu'} | \Phi^{J^{\pi} T}_{\nu' r'} \rangle \mathcal{N}^{-\frac{1}{2}}_{\nu' \nu}(r', r)
\]

Calculation of \( g \) from SD wave functions:

\[
g_{\lambda \nu n} = \langle A \lambda J^{\pi} T | \hat{A}_{\nu} \Phi^{J^{\pi} T}_{\nu n} \rangle
\]

\[
= \frac{1}{\langle n \ell 00, \ell | 00 n \ell, \ell \rangle_{(A-1)}} \sum_{j} (-1)^{I_1 + J + j} \hat{s}_j \left\{ \begin{array}{ccc}
I_1 & 1 & s \\
\ell & 2 & j
\end{array} \right\} \frac{1}{jT} \langle A \lambda J^{\pi} T || a_+^{n \ell j \frac{1}{2}} \ || \Phi^{J^{\pi} T}_{\nu n} \rangle_{SD}
\]
• Obtained as

\[ S_{\lambda \nu} = \sum_{n} g_{\lambda \nu n}^2 \]

\[ ^7\text{He} J^\pi \ 6\text{He}-(l_j) \]  | NCSM | CK | VMC | GFMC | Exp. \\
--- | --- | --- | --- | --- | --- \\
3/2^- \ 0^+ - p_{3/2}^+ | 0.56 | 0.59 | 0.53 | 0.565 | 0.512(18)[36] \\
3/2^- \ 2_1^+ - p_{1/2}^+ | 0.001 | 0.06 | 0.006 | | \\
3/2^- \ 2_1^+ - p_{3/2}^+ | 1.97 | 1.15 | 2.02 | | \\
3/2^- \ 2_2^+ - p_{1/2}^+ | 0.12 | 0.09 | 0.30 | | \\
3/2^- \ 2_2^+ - p_{3/2}^+ | 0.42 | 0.30 | 0.30 | | \\
1/2^- \ 0^+ - p_{1/2}^+ | 0.94 | 0.69 | 0.91 | | \\
1/2^- \ 2_1^+ - p_{3/2}^+ | 0.34 | 0.60 | 0.26 | | \\
1/2^- \ 2_2^+ - p_{3/2}^+ | 0.93 | | | | \\
5/2^- \ 2_1^+ - p_{1/2}^+ | 0.77 | 0.85 | 0.81 | | \\
5/2^- \ 2_1^+ - p_{3/2}^+ | 0.49 | 0.52 | 0.37 | | \\
5/2^- \ 2_2^+ - p_{1/2}^+ | 0.26 | | | | \\
5/2^- \ 2_2^+ - p_{3/2}^+ | 1.30 | | | | \\
3/2^- \ 0^+ - p_{3/2}^+ | 0.06 | 0.06 | 0.05 | | \\
3/2^- \ 2_1^+ - p_{1/2}^+ | 1.10 | 1.05 | 1.07 | | \\
3/2^- \ 2_1^+ - p_{3/2}^+ | 0.08 | 0.32 | 0.03 | | \\
3/2^- \ 2_2^+ - p_{1/2}^+ | 0.03 | | | | \\
3/2^- \ 2_2^+ - p_{3/2}^+ | 0.25 | | | | \\

• Not the final result to be compared to experiment, rather input in the NCSMC calculations
NCSMC formalism

Start from

\[
\begin{pmatrix}
H_{NCSM} & \bar{h} \\
\bar{h} & \overline{\mathcal{H}}
\end{pmatrix}
\begin{pmatrix}
c \\
\chi
\end{pmatrix}
= E
\begin{pmatrix}
1 & \bar{g} \\
\bar{g} & 1
\end{pmatrix}
\begin{pmatrix}
c \\
\chi
\end{pmatrix}
\]

Orthogonalization:

\[
\overline{\mathcal{H}} = N^{-\frac{1}{2}} \begin{pmatrix}
H_{NCSM} & \bar{h} \\
\bar{h} & \overline{\mathcal{H}}
\end{pmatrix} N^{-\frac{1}{2}}
\begin{pmatrix}
\bar{c} \\
\bar{\chi}
\end{pmatrix}
= N^{\frac{1}{2}}
\begin{pmatrix}
c \\
\chi
\end{pmatrix}
\]

Solve with generalized microscopic R-matrix

\[
\left(\hat{\mathcal{H}} + \hat{L} - E\right)
\begin{pmatrix}
\bar{c} \\
\bar{\chi}
\end{pmatrix}
= \hat{L}
\begin{pmatrix}
\bar{c} \\
\bar{\chi}
\end{pmatrix}
\]

Bloch operator

\[
\hat{L}_\nu = \begin{pmatrix}
0 & 0 \\
0 & \frac{1}{2} \delta(r - a) \left(\frac{d}{dr} - \frac{B_\nu}{r}\right)
\end{pmatrix}
\]
• Separation into “internal” and “external” regions at the channel radius \( a \)

Internal region:
\[ u_c(r) = \sum_n A_{cn} f_n(r) \]

External region:
\[ u_c(r) \sim v_c^{-\frac{1}{2}} \left[ \delta_{ci} I_c(k_c r) - U_{ci} O_c(k_c r) \right] \]

– This is achieved through the Bloch operator:
\[ L_c = \frac{\hbar^2}{2\mu_c} \delta(r - a) \left( \frac{d}{dr} - \frac{B_c}{r} \right) \]

– System of Bloch-Schrödinger equations:

\[
\begin{bmatrix}
\hat{T}_{rel}(r) + L_c + \bar{V}_{Coul}(r) - (E - E_c)
\end{bmatrix} u_c(r) + \sum_{c'} \int dr' r' W_{cc'}(r,r') u_{c'}(r') = L_c u_c(r)
\]

– Internal region: expansion on square-integrable basis
\[ u_c(r) = \sum_n A_{cn} f_n(r) \]

– External region: asymptotic form for large \( r \)
\[ u_c(r) \sim C_c W(k_c r) \quad \text{or} \quad u_c(r) \sim v_c^{-\frac{1}{2}} \left[ \delta_{ci} I_c(k_c r) - U_{ci} O_c(k_c r) \right] \]

**Scattering matrix**

Bound state

Scattering state
To find the Scattering matrix

- After projection on the basis $f_n(r)$:

\[
\sum_{c' \in \mathcal{C}} \left[ C_{cn,c'n'} - (E - E_c) \delta_{cn,c'n'} \right] A_{c'n'} = \frac{\hbar^2 k_c}{2 \mu_c v_c^{1/2}} \left\langle f_n | L_e | \delta_{ci} - U_{ci} O_e \right\rangle
\]

1. Solve for $A_{cn}$

2. Match internal and external solutions at channel radius, $a$

\[
\sum_{c'} R_{cc'} \frac{k_c a}{\sqrt{\mu_c v_c}} \left[ I'_c(k_c a) \delta_{ci} - U_{ci} O'_c(k_c a) \right] = \frac{1}{\sqrt{\mu_c v_c}} \left[ I_c(k_c a) \delta_{ci} - U_{ci} O_c(k_c a) \right]
\]

- In the process introduce $R$-matrix, projection of the Green’s function operator on the channel-surface functions

\[
R_{cc'} = \sum_{n,n'} \frac{\hbar}{\sqrt{2 \mu_c a}} f_n(a) [C - EI]_{cn,c'n'}^{-1} \frac{\hbar}{\sqrt{2 \mu_c a}} f_{n'}(a)
\]

Lagrange basis associated with Lagrange mesh:

\[
\{ ax_n \in [0,a] \}
\]

\[
\int_0^a g(x) dx \approx \sum_{n=1}^N \lambda_n g(x_n)
\]

\[
\int_0^a f_n(r) f_{n'}(r) dr \approx \delta_{nn'}
\]
To find the Scattering matrix

3. Solve equation with respect to the scattering matrix $U$

$$
\sum_{c'} R_{cc'} \frac{k_{c'}a}{\sqrt{\mu_c \nu_{c'}}} \left[ I_{c'}(k_{c}a) \delta_{ci} - \left( U_{ci} O'_{c'}(k_{c'}a) \right) \right] = \frac{1}{\sqrt{\mu_c \nu_{c}}} \left[ I_{c}(k_{c}a) \delta_{ci} - \left( U_{ci} O_{c}(k_{c}a) \right) \right]
$$

4. You can demonstrate that the solution is given by:

$$
U = Z^{-1}Z^*, \quad Z_{cc'} = (k_{c}a)^{-1} \left[ O_{c}(k_{c}a) \delta_{cc'} - k_{c'}a R_{cc'} O'_{c'}(k_{c'}a) \right]
$$

- Scattering phase shifts are extracted from the scattering matrix elements

$$
U = \exp(2i\delta)
$$
NCSM with continuum: $^7\text{He}$ ↔ $^6\text{He} + n$

- **NCSM/RGM** with up to three $^6\text{He}$ states
- **NCSMC** with up to three $^6\text{He}$ states and four $^7\text{He}$ eigenstates
  - More 7-nucleon correlations
  - Fewer target states needed

---

Expt.
NCSM with continuum: $^7\text{He} \leftrightarrow ^6\text{He} + n$

NCSM/RGM with up to three $^6\text{He}$ states

More 7-nucleon correlations

Fewer target states needed

Expt.
NCSM with continuum: $^7\text{He} \leftrightarrow ^6\text{He} + n$

NCSM/RGM with three $^6\text{He}$ states

NCSMC with three $^6\text{He}$ states and ten $^7\text{He}$ eigenstates
More 7-nucleon correlations
Fewer target states needed

Expt.
Experimental controversy: Existence of low-lying 1/2^- state ... not seen in these calculations

Best agreement with the neutron pick-up and proton-removal reactions experiments [11]
- Two $3/2^-$ resonances predicted at about 3.7 MeV and 6.5 MeV with widths of 2.8 MeV and 4.3 MeV, respectively
  - Experiment: State of undetermined spin and parity at 6.2(3) MeV with the width of 4(1) MeV
- Considerable mixing of $P$-waves in $3/2^-$
Conclusions and Outlook

- We developed a new unified approach to nuclear bound and unbound states
  - Merging of the NSM and the NCSM/RGM
- We demonstrated its capabilities in calculations of $^7\text{He}$ resonances
  - We find reasonable agreement with experiment for established $3/2^-$ and $5/2^-$ resonances
  - Our results do not support the existence of a low lying narrow $1/2^-$ resonance
  - We predict two broad $3/2^-$ resonances

Outlook:
- Inclusion of 3N interactions
- Extension of the formalism to composite projectiles (deuteron, $^3\text{H}$, $^3\text{He}$, $^4\text{He}$)
- Extension of the formalism to coupling of three-body clusters

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