Renormalization and power counting of chiral nuclear forces

龙炳蔚
(Bingwei Long)

in collaboration with Chieh-Jen “Jerry” Yang (U. Arizona)
Correcting Weinberg's scheme about NN contact interactions using renormalization group invariance, (cutoff independence) as the guideline

However, naïve dimensional analysis sets the lower bound
Outline

- Brief intro. to chiral effective field theory
- Dr. W's prescription for chiral nuclear forces
- What went wrong
- What need to change
- Summary
EFT recipe

- Degrees of freedom relevant at low energies
- Symmetries
- Power counting
- Renormalization

Observables independent of renormalization group (RG) invariance

Model independence

Energy

High-energy theory

Low-energy EFT
What does chiral effective field theory look like

- Chiral symmetry of QCD
  - $SU(2)_L \times SU(2)_R$
  - Spontaneously broken
  - Approximate $m_q > 0$

\[
L_{\text{int}} = N^\dagger \left( -\frac{1}{4f_\pi^2} \epsilon_{abc} \tau_a \pi_b \pi_c - \frac{g_A}{2f_\pi} \tau_a \vec{\sigma} \cdot \vec{\nabla} \pi_a \right) N - \frac{1}{2} C_S (N^\dagger N)^2 - \frac{1}{2} C_T (N^\dagger \vec{\sigma} N)^2
\]

- Perturbative QCD
  - Few GeV
  - $\sim 1$ GeV
  - $\sim 100$ MeV

- Lattice QCD
  - $M_{hi}$

- Chiral EFT
  - Q: Generic external momentum
  - D.o.f.s
    - Nucleons
    - Pions

- mass of non-Goldstone mesons $\sigma, \rho, \ldots$
Pros and cons

Pros

- Most general Lagrangian w/ chiral symmetry
  - A unified framework to study strong interactions and electroweak probes
- Can estimate theoretical error, but power counting must be consistent

\[ M = \sum_n \left( \frac{Q}{M_{hi}} \right)^n \mathcal{F}_n \left( \frac{Q}{M_{lo}} \right) \]

Non-analytical functions from loops

Cons

- Break down below \( Q \sim 500 \text{ MeV} \)
Basics of chpt

OPE

Leading irreducible TPE

\[ V_{1\pi} = \frac{g_A^2}{4f_\pi^2} \frac{\vec{q} \cdot \vec{\sigma}_1 \vec{q} \cdot \vec{\sigma}_2}{m_\pi^2 + q^2} \]

\[ V_{2\pi} = -\frac{3g_A^4}{4f_\pi^2 (4\pi f_\pi)^2} \frac{w}{q} \ln \frac{w + q}{2m_\pi} \frac{\vec{q} \cdot \vec{\sigma}_1 \vec{q} \cdot \vec{\sigma}_2}{q} + \cdots \]

\[ + Aq^2 + Bk^2 \quad \text{primordial c.t.} \]

Long-range non-polynomials follow naïve dimensional analysis:

\[ \frac{V_{2\pi}}{V_{1\pi}} \sim \frac{Q^2}{(4\pi f_\pi)^2} F \left( \frac{Q}{m_\pi} \right) \]

Weinberg's prescription

→ assuming resummed OPE does not change anything

→ c.t. follow naïve dimensional analysis, too
But, is there a real problem?

Large subleading corrections in $3P_0$
Entem et al (2001)
- Dashed: N3LO Idaho
- Band: several models
- Dotted: modified Idaho
- Dot-dashed: NLO by Epelbaum

Why does N3LO work worse at lower energies?
Mass scale of OPE's strength

\[ \frac{m_N}{4\pi f_\pi} \frac{Q}{a(l)f_\pi} \]

For lower p.w. where \( a(l) \sim 1 \):
\[ Q \sim a(l)f_\pi \sim 100 \text{ MeV} \rightarrow \text{nonperturbative OPE} \]

This is a good thing
→ no need to put in by hand low-energy mass scale in order to generate bound states

This is a bad thing
→ always have to choose between two mass scales in power counting
→ NDA no longer reliable
→ WPC is the most economical choice

\[ M_{hi} = 4\pi f_\pi \sim 1 \text{GeV} \quad M_{lo} = a(l)f_\pi \sim 0.1 \text{GeV} \]

Two scales differ only by a numerical factor!
Let there be OPE

\[ V_{1\pi}(\vec{r}) = \frac{m_\pi^3}{12\pi} \left( \frac{g_A^2}{4f_\pi^2} \right) \tau_1 \cdot \tau_2 \left[ T(r)S_{12} + Y(r)\vec{\sigma}_1 \cdot \vec{\sigma}_2 \right] \]

\[ T(r) = \frac{e^{-m_\pi r}}{m_\pi r} \left[ 1 + \frac{3}{m_\pi r} + \frac{3}{(m_\pi r)^2} \right] \rightarrow \frac{1}{r^3} \text{ at } r \rightarrow 0 \]

\[ Y(r) = \frac{e^{-m_\pi r}}{m_\pi r} \rightarrow \frac{1}{r} \text{ at } r \rightarrow 0 \]

- tensor force (TF) acts on only triplet channels.
- due to \( S_{12} \), TF could be attractive or repulsive in different channels.

3S1, 3P0... 3P1...
-1/r^3 is more interesting

V_T could be attractive, e.g. in 3P0

- 1/r^3 dominates over kinetic energy ( ~ +1/r^2 ) and centrifugal barrier
  → unbounded from below, or equivalently, amplitude depends drastically on the cutoff

- NN contact interaction (counterterm) needed → 4-fermion operators

- 3P0 4-fermion operator has at least 2 derivatives, and yet has to appear in LO for renormalization purpose → not suppressed as in 1-N sector

Nogga et al (2005)

(only for illustration)

\[ \mathcal{L}_{3P0} = D_0 (N^\dagger \partial^2 N)(N^\dagger N) + \cdots, \quad D_0 \propto \frac{1}{M_{lo}} \]

\[ D_0 \propto \frac{1}{M_{hi}^2} \]
Subleading orders: triplet channels

\[ \mathcal{O}(Q^2) \sim \frac{Q^2}{M_{hi}^2} \times \]

- Insertion of TPE can be divergent → look for suitable counterterms to cancel
- Modified NDA → \( D_0, D_2(p^2) \) … are enhanced by the same amount

\[ \mathcal{L}_{3P0} = D_0(N^\dagger \partial^2 N)(N^\dagger N) \]
\[ + D_2(N^\dagger \partial^4 N)(N^\dagger N) + \cdots \]
\[ D_0 \propto \frac{1}{M_{lo}^2} , D_2 \propto \frac{1}{M_{lo}^2 M_{hi}^2} \]
Divergence of distorted-wave expansion

for LO potential $\sim -1/r^3$,

$$\psi_k^{(0)}(r) \sim \left(\frac{\lambda}{r}\right)^{\frac{1}{4}} \left[u_0(r/\lambda) + k^2 r^2 \sqrt{\frac{r}{\lambda}} u_1(r/\lambda) + \mathcal{O}(k^4)\right]$$

$$\lambda = \frac{3 g_A^2 m_N}{8\pi f_\pi^2} \quad u_{1,2}(x) \sim \mathcal{O}(1)$$

$$V_{2\pi} \sim \frac{1}{r^5} \quad r \to 0$$

$$T^{(2)} = \langle \psi^{(0)} | V_{2\pi} | \psi^{(0)} \rangle$$

$$\sim \int_{1/\Lambda} dr r^2 |\psi^{(0)}(r)|^2 \frac{1}{r^5} \sim \alpha_0(\Lambda) \Lambda^{5/2} + \beta_0(\Lambda) k^2 + \mathcal{O}(k^4 \Lambda^{-5/2})$$

Two pieces of divergences suggest two counterterms in uncoupled channels: $C & D$ terms in $^3P_0$ ...
3S1 - 3D1 phase shifts

(BwL & Yang, 2011)

$Q^2$: leading TPE, $Q^3$: subleading TPE. “1.5”: $\Lambda = 1.5$ GeV

Good agreement with partial-wave analysis up to $T_{lab} \sim 100$ MeV ($k_{cm} \sim 200$ MeV)
The saga of $1S_0$

\[ V^{(0)}_{1S0} = -\frac{g_A^2 m_n^2 e^{-m_\pi r}}{4 f_\pi^2 r} + C_0 \delta(r) \]

- OPE becomes regular near the origin \( \sim \frac{1}{r} \to \) no singular attraction
- Since $T_{\text{yukawa}}$ is finite, renormalization can be more easily seen

\[ \chi(p; k) = \]

\[ I_k = \]

\[ V^{(0)} = V_{Yukawa} + C_0, \quad T^{(0)}_{1S0} = T_{Yukawa} + \frac{\chi^2(k; k)}{1 - \frac{1}{C_0}} I_k, \quad I_k \sim \# \Lambda + \# m_\pi^2 \ln \Lambda \]

(Kaplan et al, 1996)
O(Q) does not vanish in 1S0

LO residual cutoff variation \( \sim \frac{k^2}{M_{lo} \Lambda} \)

For comparison, in 3S1~ \( \sim \frac{k^2 M_{lo}^{1/2}}{\Lambda^{5/2}} \)

\( \rightarrow \) LO theo. error is at least O(Q) \( \text{RG invariance enforced more strictly} \)

\( \rightarrow \) can't be provided by TPE

\( \rightarrow \) \( C_2 p^2 \) must be O(Q), rather than O(Q^2) as suggested by NDA

\( \rightarrow \) \( \frac{\tilde{r}}{2} \sim \frac{1}{M_{hi}} T^{(0)} + T^{(1)} = T_Y + \frac{4\pi}{m_N} \frac{\chi^2_k}{-\frac{1}{a(\mu)} + \frac{\tilde{r}}{2} k^2 - \frac{4\pi}{m_N} \int_k^R (\mu) \}

But PWA says \( \tilde{r} \) is rather large

\( \frac{\tilde{r}}{2} = 1.55 \text{ fm} = \frac{1}{127 \text{ MeV}} \) \( \text{Steele & Furnstahl (1999)} \)
Need to improve LO of $1S0$

- Converge a bit too slow
- Needs to promote $C_2 \delta''(r)$ to LO → fine tuning of effective range
- Not so easy as far as renormalization is concerned

Red dots are PWA

BwL & CJ Yang (2012)
To introduce energy dependence in LO counterterm, use auxiliary field (only coupled to 1S0) → s-channel exchange

Φ does not correspond to physical state

\[ V_{1S0}^{(0)} = \pi \quad \phi \]  (Kaplan, 1996, with a bit of modification by BwL)

\[ V^{(0)} = V_{Yukawa} + \frac{\sigma y^2}{E + \Delta} , \quad T_{1S0}^{(0)} = T_{Yukawa} + \frac{\chi^2(k; k)}{E + \Delta \sigma y^2} - I_k \]
Subleading orders of 1S0

Dibaryon Lagrangian doesn't need to be the most general one

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<thead>
<tr>
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<th>with dibaryon</th>
<th>w/o dibaryon</th>
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<tbody>
<tr>
<td>$O(1)$</td>
<td>$\frac{\sigma y^2}{E+\Delta}$ + Yukawa</td>
<td>$C_0 + C_2 p^2 + \text{Yukawa}$</td>
</tr>
<tr>
<td>$O(Q)$</td>
<td>$C_0$</td>
<td>$C_4 p^4$</td>
</tr>
<tr>
<td>$O(Q^2)$</td>
<td>$C_2 p^2 + \text{leading TPE}$</td>
<td>$C_6 p^6 + \text{leading TPE}$</td>
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Convergence improved, with one more para.

Fine-tuning incorporated systematically

Blue: LO
Green: $O(Q)$
Brown: $O(Q^2)$
Black: PWA
Summary

- Consistent power counting $\rightarrow$ meaningful theoretical error
- NDA may fail to capture short-range physics because of two mass scales
- RG invariance can constrain power-counting schemes
- Good fit to NN phase shifts up to $T_{\text{lab}} \sim 100$ MeV