Electro-Magnetic Reactions in Few-Body Systems
From Nuclei to Cold-Atoms

Nir Barnea

The Racah institute for Physics
The Hebrew University, Jerusalem, Israel

INT Program
Light Nuclei From First Principles
5 October 2012
Collaboration

Jerusalem, Israel
B. Bazak*, D. Gazit, E. Liverts, N. Nevo*

Trento, Italy
W. Leidemann, G. Orlandini

Moscow, Russia
V. Efros

TRIUMF, Canada
S. Bacca
What can we learn from photo reactions?

1. Understanding of the systems at hand.
2. A test of the Hamiltonian at regimes not accessible by elastic reactions.
3. Reaction rates as input for experiments or applications (e.g. astrophysics).
4. Underlying degrees of freedom.
5. The transition from single particle to collective behavior.
What can we learn from photo reactions?

1. Understanding of the systems at hand.
2. A test of the Hamiltonian at regimes not accessible by elastic reactions.
3. Reaction rates as input for experiments or applications (e.g. astrophysics).
4. Underlying degrees of freedom.
5. The transition from single particle to collective behavior.
What can we learn from photo reactions?

1. Understanding of the systems at hand.
2. A test of the Hamiltonian at regimes not accessible by elastic reactions.
3. Reaction rates as input for experiments or applications (e.g. astrophysics).
4. Underlying degrees of freedom.
5. The transition from single particle to collective behavior.
What can we learn from photo reactions?

1. Understanding of the systems at hand.
2. A test of the Hamiltonian at regimes not accessible by elastic reactions.
3. Reaction rates as input for experiments or applications (e.g. astrophysics).
4. Underlying degrees of freedom.
5. The transition from single particle to collective behavior.
What can we learn from photo reactions?

1. Understanding of the systems at hand.
2. A test of the Hamiltonian at regimes not accessible by elastic reactions.
3. Reaction rates as input for experiments or applications (e.g. astrophysics).
4. Underlying degrees of freedom.
5. The transition from single particle to collective behavior.
What can we learn from photo reactions?

1. Understanding of the systems at hand.
2. A test of the Hamiltonian at regimes not accessible by elastic reactions.
3. Reaction rates as input for experiments or applications (e.g. astrophysics).
4. Underlying degrees of freedom.
5. The transition from single particle to collective behavior.
The Interaction Hamiltonian between the photon field $A(x)$ and the atomic/nuclear system

\[ H_I = -\frac{e}{c} \int dx A(x) \cdot J(x) \]

The current is a sum of convection and spin currents

\[ J(x) = J_c(x) + \nabla \times \mu(x) \]

\[ H_I = -\frac{e}{c} \int dx \{ A(x) \cdot J_c(x) + B(x) \cdot \mu(x) \} \]
Photo Reactions

The Interaction Hamiltonian between the photon field $A(x)$ and the atomic/nuclear system

$$H_I = -\frac{e}{c} \int dA(x) \cdot J(x)$$

The current is a sum of convection and spin currents

$$J(x) = J_c(x) + \nabla \times \mu(x)$$

$$H_I = -\frac{e}{c} \int dA(x) \cdot J_c(x) + B(x) \cdot \mu(x)$$
Theoretical Considerations

Nuclear Physics

Ultra Cold Atoms

Multipole Expansion

Conclusions

Photo Reactions

The Interaction Hamiltonian between the photon field $A(x)$ and the atomic/nuclear system

$$H_I = -\frac{e}{c} \int dA(x) \cdot J(x)$$

The current is a sum of convection and spin currents

$$J(x) = J_c(x) + \nabla \times \mu(x)$$

$$H_I = -\frac{e}{c} \int dA(x) \{ A(x) \cdot J_c(x) + B(x) \cdot \mu(x) \}$$
Photo reactions - Theoretical considerations

The Wave Functions

- We solve the $A$-body non-realtivistic Schroedinger equation.
- The Hamiltonian

\[ H = T + \sum_{ij} V_{ij}^{(2)} + \sum_{ijk} V_{ijk}^{(3)} + \ldots \]

High precision two-nucleon potentials, well constraint by NN phaseshifts
Less established 3NF

- EFT provides a solid theoretical framework for construction of the potentials.
- Phenomenological potential models are not that bad either.
Photo reactions - Theoretical considerations

The Wave Functions

- We solve the $A$-body non-relativistic Schroedinger equation.
- The Hamiltonian

$$H = T + \sum_{ij} V_{ij}^{(2)} + \sum_{ijk} V_{ijk}^{(3)} + \ldots$$

High precision two-nucleon potentials, well constraint by NN phaseshifts
Less established 3NF

- EFT provides a solid theoretical framework for construction of the potentials.
- Phenomenological potential models are not that bad either.
The Electro-Magnetic Current

- The EM current is a sum of convection and spin currents

\[ J(x) = J_c(x) + J_s(x) = J_c(x) + \nabla \times \mu(x) \]

- Classically, the convection current \( J_c = \sum_i Z_i \mathbf{v}_i \) is the flow of the charged particles.
- In nuclei \( J_c(x) \) is mainly due to proton movement.
- Meson exchange between nucleons leads to 2, 3, \ldots-body currents \( J = J_1 + J_2 + \ldots \)
- Cold atoms are neutral \( J_c(x) = 0 \) and the current \( \mu(x) \) is dominated by the electronic spins.
Nuclear Physics - A tale of two potentials

- The nuclear Hamiltonian

\[ H = - \sum_i \frac{\hbar^2}{2m_N} \nabla_i^2 + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk} + \ldots \]

- The Potential is composed of EM and NUCLEAR terms.
- The NUCLEAR force cannot be derived from QCD and must be modeled.

The AV18 NN-Force

- The 2-body potential \( V_{ij} = v_{\pi ij} + v_{2\pi ij} + v_{s ij} \)
- \( v_{\pi ij} \) - A Yukawa type interaction \( e^{-\mu r}/r \), \( v_{2\pi ij} \propto e^{-2\mu r}/r \)
- \( v_{s ij} \) is expanded into a series of operators dictated by the symmetries
- NNN force must be supplemented to reproduce 3-body binding energies

The JISP16 Potential

- A formal expansion of the potential

\[ V_f = \sum_{\ell mn} |(\ell mn)\rangle \frac{\alpha_{\ell mn}}{\mu_{\ell mn}} \langle \ell mn| \]

- The HO basis is used, \( \frac{\alpha_{\ell mn}}{\mu_{\ell mn}} \) fitted to reproduce NN scattering data
Nuclear Physics - A tale of two potentials

The nuclear Hamiltonian

\[
H = -\sum_i \frac{\hbar^2}{2m_N} \nabla_i^2 + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk} + \ldots
\]

• The Potential is composed of EM and NUCLEAR terms.
• The NUCLEAR force cannot be derived from QCD and must be modeled.

The AV18 NN-Force

• The 2-body potential \( V_{ij} = v_s^{ij} + v_\pi^{ij} + v_\omega^{ij} \)
• \( v_\pi^{ij} \) - A Yukawa type interaction \( e^{-\mu r}/r \), \( v_\omega^{ij} \propto e^{-2\mu r}/r \)
• \( v_s^{ij} \) is expanded into a series of operators dictated by the symmetries
• NNN force must be supplemented to reproduce 3,4-body binding energies.

The JISP16 Potential

• A formal expansion of the potential

\[
V_L = \sum_{l} \langle l \mid V^{(l)} \mid j \rangle \langle j \mid n \rangle \langle n \mid l \rangle
\]

• The HO basis is used, \( V^{(l)} \) fitted to reproduce EN scattering data.
Nuclear Physics - A tale of two potentials

- The nuclear Hamiltonian

\[
H = - \sum_i \frac{\hbar^2}{2m_N} \nabla_i^2 + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk} + \ldots
\]

- The Potential is composed of EM and NUCLEAR terms.
- The NUCLEAR force cannot be derived from QCD and must be modeled.

The AV18 NN-Force

- The 2-body potential \( V_{ij} = v_s^{ij} + v_\pi^{ij} + v_\sigma^{ij} \)
- \( v_\pi^{ij} \) - A Yukawa type interaction \( e^{-\mu r}/r \), \( v_\sigma^{ij} \propto e^{-2\mu r}/r \)
- \( v_s^{ij} \) is expanded into a series of operators dictated by the symmetries.
- NNN force must be supplemented to reproduce 3,4-body binding energies.

The JISP16 Potential

- A formal expansion of the potential

\[
V_f = \sum_{l=0}^{\infty} f(l,\alpha) \phi_l^{\alpha}(\rho) \phi_0(\rho) + \text{other terms}
\]

- The HO basis is used, \( \phi_0^{\alpha}(\rho) \) fitted to reproduce NN scattering data.
Nuclear Physics - A tale of two potentials

- The nuclear Hamiltonian

\[
H = -\sum_i \frac{\hbar^2}{2m_N} \nabla_i^2 + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk} + \ldots
\]

- The Potential is composed of EM and NUCLEAR terms.
- The NUCLEAR force cannot be derived from QCD and must be modeled.

The AV18 NN-Force

- The 2-body potential \( V_{ij} = v^{s}_{ij} + v^{2\pi}_{ij} + v^{\pi}_{ij} \)
- \( v^{\pi}_{ij} \) - A Yukawa type interaction \( e^{-\mu r} / r \), \( v^{2\pi}_{ij} \propto e^{-2\mu r} / r \).
- \( v^{s}_{ij} \) is expanded into a series of operators dictated by the symmetries.
- NNN force must be supplemented to reproduce 3,4-body binding-energies.

The JISP16 Potential

- A formal expansion of the potential

\[
V_{ij} = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} V^{l,m}_{ij} (l^m) \psi_{ij}^{l,m}
\]

- The HO basis is used, \( V^{l,m}_{ij} \) fitted to reproduce EM scattering data.
Nuclear Physics - A tale of two potentials

- The nuclear Hamiltonian

\[
H = - \sum_i \frac{\hbar^2}{2m_N} \nabla_i^2 + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk} + \ldots
\]

- The Potential is composed of EM and NUCLEAR terms.
- The NUCLEAR force cannot be derived from QCD and must be modeled.

The AV18 NN-Force

- The 2-body potential \( V_{ij} = v^s_{ij} + v^{2\pi}_{ij} + v^\pi_{ij} \)
  - \( v^\pi_{ij} \) - A Yukawa type interaction \( e^{-\mu r}/r \), \( v^{2\pi}_{ij} \propto e^{-2\mu r}/r \).
  - \( v^s_{ij} \) is expanded into a series of operators dictated by the symmetries.
  - NNN force must be supplemented to reproduce 3,4-body binding-energies.

The JISP16 Potential

- A formal expansion of the potential

\[
V_{ij} = \sum_{l,m,n} (l,m,n) V^{(l,m,n)}_{ij} \langle l,m,n |\n\]

- The HO basis is used, \( V^{(l,m,n)} \) fitted to reproduce EM scattering data.
Nuclear Physics - A tale of two potentials

• The nuclear Hamiltonian

\[ H = -\sum_i \frac{\hbar^2}{2m_N} \nabla_i^2 + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk} + \ldots \]

• The Potential is composed of EM and NUCLEAR terms.
• The NUCLEAR force cannot be derived from QCD and must be modeled.

The AV18 NN-Force

• The 2-body potential \( V_{ij} = v_{ij}^s + v_{ij}^{2\pi} + v_{ij}^\pi \)
• \( v_{ij}^\pi \) - A Yukawa type interaction \( e^{-\mu r}/r \), \( v_{ij}^{2\pi} \propto e^{-2\mu r}/r \).
• \( v_{ij}^s \) is expanded into a series of operators dictated by the symmetries.
• NNN force must be supplemented to reproduce 3,4-body binding-energies.

The JISP16 Potential

• A formal expansion of the potential

\[ V_r = \sum_{l\alpha m} \langle l\alpha m | V_{ij}^{\alpha \beta} | l\alpha m \rangle \delta_ {\delta\mu} \]

• The HO basis is used, \( V_{ij}^{\alpha \beta} \) fitted to reproduce NN scattering data.
Nuclear Physics - A tale of two potentials

- The nuclear Hamiltonian

\[ H = - \sum_i \frac{\hbar^2}{2m_N} \nabla_i^2 + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk} + \ldots \]

- The Potential is composed of EM and NUCLEAR terms.
- The NUCLEAR force cannot be derived from QCD and must be modeled.

The AV18 NN-Force

- The 2-body potential \( V_{ij} = v_{ij}^s + v_{ij}^{2\pi} + v_{ij}^\pi \)
- \( v_{ij}^\pi \) - A Yukawa type interaction \( e^{-\mu r}/r \), \( v_{ij}^{2\pi} \propto e^{-2\mu r}/r \).
- \( v_{ij}^s \) is expanded into a series of operators dictated by the symmetries.
- NNN force must be supplemented to reproduce 3,4-body binding-energies.

The JISP16 Potential

- A formal expansion of the potential

\[ V_{ij} = \sum_{j=0}^{\infty} \sum_{\text{sym}} \sum_{\text{nn}} V_{ij}^{j\text{nn}}(l_j) \langle j\text{nn} | V_{ij} | l_j \rangle \]

- The HO basis is used, \( V_{ij}^{j\text{nn}}(l_j) \) fitted to reproduce NN scattering data.
Nuclear Physics - A tale of two potentials

- The nuclear Hamiltonian

\[ H = - \sum_i \frac{\hbar^2}{2m_N} \nabla_i^2 + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk} + \ldots \]

- The Potential is composed of EM and NUCLEAR terms.
- The NUCLEAR force cannot be derived from QCD and must be modeled.

The AV18 NN-Force

- The 2-body potential \( V_{ij} = v_{ij}^s + v_{ij}^{2\pi} + v_{ij}^\pi \)
- \( v_{ij}^\pi \) - A Yukawa type interaction \( e^{-\mu r}/r \), \( v_{ij}^{2\pi} \propto e^{-2\mu r}/r \).
- \( v_{ij}^s \) is expanded into a series of operators dictated by the symmetries.
- NNN force must be supplemented to reproduce 3,4-body binding-energies.

The JISP16 Potential

- A formal expansion of the potential

\[ V_{ij} = \sum_{lsjm'n'} \langle(ls)jm'|V_{ij}^{(ls)jm'}(ls)jm| \]

- The HO basis is used, \( V_{ij}^{(ls)jm'} \) fitted to reproduce NN scattering data.
Nuclear Physics - A tale of two potentials

- The nuclear Hamiltonian

\[
H = - \sum_i \frac{\hbar^2}{2m_N} \nabla_i^2 + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk} + \ldots
\]

- The Potential is composed of EM and NUCLEAR terms.
- The NUCLEAR force cannot be derived from QCD and must be modeled.

The AV18 NN-Force

- The 2-body potential \( V_{ij} = v_{ij}^s + v_{ij}^{2\pi} + v_{ij}^\pi \)
- \( v_{ij}^\pi \) - A Yukawa type interaction \( e^{-\mu r}/r \), \( v_{ij}^{2\pi} \propto e^{-2\mu r}/r \).
- \( v_{ij}^s \) is expanded into a series of operators dictated by the symmetries.
- NNN force must be supplemented to reproduce 3,4-body binding-energies.

The JISP16 Potential

- A formal expansion of the potential

\[
V_{ij} = \sum_{lsjnn'} |(ls)jn'\rangle V_{nn'}^{(ls)j} \langle (ls)jn|
\]

- The HO basis is used, \( V_{nn'}^{(ls)j} \) fitted to reproduce NN scattering data.
Nuclear Physics - A tale of two potentials

- The nuclear Hamiltonian

\[ H = - \sum_i \frac{\hbar^2}{2m_N} \nabla_i^2 + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk} + \ldots \]

- The Potential is composed of EM and NUCLEAR terms.
- The NUCLEAR force cannot be derived from QCD and must be modeled.

The AV18 NN-Force

- The 2-body potential \( V_{ij} = v_{ij}^s + v_{ij}^{2\pi} + v_{ij}^\pi \)
- \( v_{ij}^\pi \) - A Yukawa type interaction \( e^{-\mu r}/r \), \( v_{ij}^{2\pi} \propto e^{-2\mu r}/r \).
- \( v_{ij}^s \) is expanded into a series of operators dictated by the symmetries.
- NNN force must be supplemented to reproduce 3,4-body binding-energies.

The JISP16 Potential

- A formal expansion of the potential

\[ V_{ij} = \sum_{lsjn'} |(ls)jn'\rangle V_{nn'}^{(ls)j} \langle (ls)jn| \]

- The HO basis is used, \( V_{nn'}^{(ls)j} \) fitted to reproduce NN scattering data.
Nuclear Physics - A tale of two potentials

- The nuclear Hamiltonian

\[ H = - \sum_i \frac{\hbar^2}{2m_N} \nabla_i^2 + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk} + \ldots \]

- The Potential is composed of EM and NUCLEAR terms.
- The NUCLEAR force cannot be derived from QCD and must be modeled.

The AV18 NN-Force

- The 2-body potential \( V_{ij} = v_{ij}^s + v_{ij}^{2\pi} + v_{ij}^\pi \)
- \( v_{ij}^\pi \) - A Yukawa type interaction \( e^{-\mu r}/r \), \( v_{ij}^{2\pi} \propto e^{-2\mu r}/r \).
- \( v_{ij}^s \) is expanded into a series of operators dictated by the symmetries.
- NNN force must be supplemented to reproduce 3,4-body binding-energies.

The JISP16 Potential

- A formal expansion of the potential

\[ V_{ij} = \sum_{lsjn'j'n} |(ls)jn'\rangle V_{nn'}^{(ls)} j j'n \langle (ls)jn| \]

- The HO basis is used, \( V_{nn'}^{(ls)} j j'n \) fitted to reproduce NN scattering data.
A tale of two potentials

- **AV18+UBIX** Argonne V18 NN force
  + Urbana IX NNN force
- **JISP16** J-matrix Inverse Scattering Potential, Shirokov *et al.*

### Binding Energies

<table>
<thead>
<tr>
<th></th>
<th>AV18+UBIX</th>
<th>JISP16</th>
<th>Nature</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>2.24</td>
<td>2.24</td>
<td>2.24</td>
</tr>
<tr>
<td>$^3$H</td>
<td>8.48</td>
<td>8.35</td>
<td>8.48</td>
</tr>
<tr>
<td>$^3$He</td>
<td>7.74</td>
<td>7.65</td>
<td>7.72</td>
</tr>
<tr>
<td>$^4$He</td>
<td>28.5</td>
<td>28.3</td>
<td>28.3</td>
</tr>
</tbody>
</table>
A tale of two potentials

- AV18+UBIX Argonne V18 NN force + Urbana IX NNN force
- JISP16 J-matrix Inverse Scattering Potential, Shirokov et al.

**Binding Energies**

<table>
<thead>
<tr>
<th></th>
<th>AV18+UBIX</th>
<th>JISP16</th>
<th>Nature</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>2.24</td>
<td>2.24</td>
<td>2.24</td>
</tr>
<tr>
<td>(^3)H</td>
<td>8.48</td>
<td>8.35</td>
<td>8.48</td>
</tr>
<tr>
<td>(^3)He</td>
<td>7.74</td>
<td>7.65</td>
<td>7.72</td>
</tr>
<tr>
<td>(^4)He</td>
<td>28.5</td>
<td>28.3</td>
<td>28.3</td>
</tr>
</tbody>
</table>

Photodisintegration cross-section for \(A=2,3,4\)

JISP vs AV18+UBIX

![Graph showing photodisintegration cross-section for different nuclei.](image)
The Experimental Verdict!

S. Quaglioni, and P. Navratil PLB 652, 370 (2007)
R. Raut et al., PRL 108, 042502 (2012)
W. Tornow et al., PRC 85, 061001 (2012)
The Experimental Verdict?

S. Quaglioni, and P. Navratil PLB 652, 370 (2007)
R. Raut et al., PRL 108, 042502 (2012)
W. Tornow et al., PRC85, 061001 (2012)
Effective Field Theory

- Expansion in small momentum $Q$.
- Contains all terms compatible with QCD up to a given order.
- NNN and NNNN forces come in naturally at orders N2LO and N3LO.

\[
V = -\left(\frac{g_A}{2f_\pi}\right)^2 \frac{(\sigma_1 \cdot q)(\sigma_2 \cdot q)}{q^2 + m_\pi^2} \tau_1 \cdot \tau_2 + C_S + C_T \sigma_1 \cdot \sigma_2 + V_{NLO} + V_{N2LO} + \ldots
\]

Effective Field Theory potentials

**Effective Field Theory**

- Expansion in small momentum $Q$.
- Contains all terms compatible with QCD up to a given order.
- NNN and NNNN forces come in naturally at orders N2LO and N3LO.

\[
V = - \left( \frac{g_A}{2f_\pi} \right)^2 \frac{(\sigma_1 \cdot q)(\sigma_2 \cdot q)}{q^2 + m_\pi^2} \tau_1 \cdot \tau_2 \\
+ C_S + C_T \sigma_1 \cdot \sigma_2 \\
+ V_{NLO} + V_{N2LO} + \ldots
\]

A tale of two potentials II
AV18+UIX ⇔ EFT

Binding Energies

<table>
<thead>
<tr>
<th></th>
<th>AV18+UBIX</th>
<th>EFT</th>
<th>Nature</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>2.24</td>
<td>2.24</td>
<td>2.24</td>
</tr>
<tr>
<td>$^3$H</td>
<td>8.48</td>
<td>8.47</td>
<td>8.48</td>
</tr>
<tr>
<td>$^3$He</td>
<td>7.74</td>
<td>7.73</td>
<td>7.72</td>
</tr>
<tr>
<td>$^4$He</td>
<td>28.5</td>
<td>28.5</td>
<td>28.30</td>
</tr>
<tr>
<td>$^4$He*</td>
<td>7.3(1)</td>
<td>7.1(2)</td>
<td>8.21</td>
</tr>
</tbody>
</table>

Electron scattering on $^4$He, the $0^+_2$ resonance

A tale of two potentials II

AV18+UIX ⇔ EFT

Electron scattering on $^4$He, the $0^+_2$ resonance

The transition form factor $0^+_1 \rightarrow 0^+_2$

<table>
<thead>
<tr>
<th></th>
<th>AV18+UIX</th>
<th>N3LO+N2LO</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>2.24</td>
<td>2.24</td>
</tr>
<tr>
<td>$^3$H</td>
<td>8.48</td>
<td>8.47</td>
</tr>
<tr>
<td>$^3$He</td>
<td>7.74</td>
<td>7.73</td>
</tr>
<tr>
<td>$^4$He</td>
<td>28.5</td>
<td>28.5</td>
</tr>
<tr>
<td>$^4$He*</td>
<td>7.3(1)</td>
<td>7.1(2)</td>
</tr>
</tbody>
</table>
A tale of two potentials II

AV18+UIX ⇔ EFT

Binding Energies

<table>
<thead>
<tr>
<th></th>
<th>AV18+UIX</th>
<th>EFT</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>2.24</td>
<td>2.24</td>
</tr>
<tr>
<td>$^3$H</td>
<td>8.48</td>
<td>8.48</td>
</tr>
<tr>
<td>$^3$He</td>
<td>7.74</td>
<td>7.72</td>
</tr>
<tr>
<td>$^4$He</td>
<td>28.5</td>
<td>28.30</td>
</tr>
<tr>
<td>$^4$He*</td>
<td>7.3(1)</td>
<td>7.1(2)</td>
</tr>
</tbody>
</table>

The transition form factor $0^+_1 \rightarrow 0^+_2$

Electron scattering on $^4$He, the $0^+_2$ resonance
The $^4\text{He} \, 0^+_2$ state - A short summary
Ultra Cold atoms

Bose systems, short range force, energy scale $10^{-9}$ eV

- A 3-body bound state $E_3 < 0$ exists even if the 2-body systems is unbound $E_2 > 0$.

- When $E_2 = 0$, $a_s \to \infty$.

- In 1970 V. Efimov found out that if $E_2 = 0$ the 3-body system will have an infinite number of bound states.

- The 3-body spectrum is $E_n = E_0 e^{-2\pi n/s_0}$ with $s_0 = 1.00623$.

- In atomic traps $a_s$ can be manipulated through the Fesbach resonance.

- Particle losses in traps are closely related to Efimov’s physics through the 3-body recombination process

$$A + A + A \rightarrow A_2 + A$$

Physics Today, March 2010
Ultra Cold atoms

Bose systems, short range force, energy scale $10^{-9}$ eV

- A 3-body bound state $E_3 < 0$ exists even if the 2-body systems is unbound $E_2 > 0$.

- **When** $E_2 = 0$, $a_s \rightarrow \infty$.

- In 1970 V. Efimov found out that if $E_2 = 0$ the 3-body system will have an infinite number of bound states.

- The 3-body spectrum is $E_n = E_0 e^{-2\pi n/s_0}$ with $s_0 = 1.00623$.

- In atomic traps $a_s$ can be manipulated through the Fesbach resonance.

- Particle losses in traps are closely related to Efimov’s physics through the 3-body recombination process

  \[ A + A + A \rightarrow A_2 + A \]
Ultra Cold atoms

Bose systems, short range force, energy scale $10^{-9}$ eV

- A 3-body bound state $E_3 < 0$ exists even if the 2-body systems is unbound $E_2 > 0$.
- When $E_2 = 0$, $a_s \rightarrow \infty$.
- In 1970 V. Efimov found out that if $E_2 = 0$ the 3-body system will have an infinite number of bound states.
  - The 3-body spectrum is $E_n = E_0 e^{-2\pi n/s_0}$ with $s_0 = 1.00623$.
  - In atomic traps $a_s$ can be manipulated through the Fesbach resonance.
  - Particle losses in traps are closely related to Efimov’s physics through the 3-body recombination process $A + A + A \rightarrow A_2 + A$.
Ultra Cold atoms

Bose systems, short range force, energy scale $10^{-9}$ eV

- A 3-body bound state $E_3 < 0$ exists even if the 2-body systems is unbound $E_2 > 0$.
- When $E_2 = 0$, $a_s \rightarrow \infty$.
- In 1970 V. Efimov found out that if $E_2 = 0$ the 3-body system will have an infinite number of bound states.
- The 3-body spectrum is $E_n = E_0 e^{-2\pi n/s_0}$ with $s_0 = 1.00623$.
- In atomic traps $a_s$ can be manipulated through the Fesbach resonance.
- Particle losses in traps are closely related to Efimov’s physics through the 3-body recombination process $A + A + A \rightarrow A_2 + A$.
Ultra Cold atoms
Bose systems, short range force, energy scale $10^{-9}$ eV

- A 3-body bound state $E_3 < 0$ exists even if the 2-body systems is unbound $E_2 > 0$.
- When $E_2 = 0$, $a_s \to \infty$.
- In 1970 V. Efimov found out that if $E_2 = 0$ the 3-body system will have an infinite number of bound states.
- The 3-body spectrum is $E_n = E_0 e^{-2\pi n/s_0}$ with $s_0 = 1.00623$.
- In atomic traps $a_s$ can be manipulated through the Fesbach resonance.
- Particle losses in traps are closely related to Efimov’s physics through the 3-body recombination process $A + A + A \to A_2 + A$.

Physics Today, March 2010

Universal insights from few-body land
Chris H. Greene
The ability to tune atomic interactions has inspired theorists and experimentalists to investigate those properties of few-particle systems that hold universally, regardless of the specific nature of the interparticle force.
Ultra Cold atoms

Bose systems, short range force, energy scale $10^{-9}$ eV

- A 3-body bound state $E_3 < 0$ exists even if the 2-body systems is unbound $E_2 > 0$.
- When $E_2 = 0$, $a_s \to \infty$.
- In 1970 V. Efimov found out that if $E_2 = 0$ the 3-body system will have an infinite number of bound states.
- The 3-body spectrum is $E_n = E_0 e^{-2\pi n/s_0}$ with $s_0 = 1.00623$.
- In atomic traps $a_s$ can be manipulated through the Fesbach resonance.
- Particle losses in traps are closely related to Efimov’s physics through the 3-body recombination process

$$A + A + A \to A_2 + A$$
Few-Body Universality in a Bosonic $^7\text{Li}$ system
Photoassociation of Atomic Molecules

The quest for the Efimov Effect

RF-induce atom loss resonances for different values of bias magnetic fields.

O. Machtey, Z. Shotan, N. Gross and L. Khaykovich

PRL 108, 210406 (2012)
The Static Response - Inelastic Reactions

- The response of an A-particle system is closely related to the static moments of the charge density
  \[ \rho(\mathbf{x}) = \sum_{i}^{A} Z_i \delta(\mathbf{x} - \mathbf{r}_i) \]

- The Fourier Transform
  \[ \rho(q) = \int d\mathbf{x} \rho(\mathbf{x}) e^{i\mathbf{q} \cdot \mathbf{x}} = \sum_{i}^{A} Z_i e^{i\mathbf{q} \cdot \mathbf{r}_i} \]

- In the long wavelength limit \( q \rightarrow 0 \)

- For a system of identical particles

  - Conclusion A: In general the Dipole is the leading term.
  - Conclusion B: For identical particles the leading terms are \( \hat{R}^2 \) and \( \hat{Q} \).
The Static Response - Inelastic Reactions

• The response of an A-particle system is closely related to the static moments of the charge density

\[ \rho(\mathbf{x}) = \sum_{i}^{A} Z_i \delta(\mathbf{x} - \mathbf{r}_i) \]

• The Fourier Transform

\[ \rho(\mathbf{q}) = \int d\mathbf{x} \rho(\mathbf{x}) e^{i\mathbf{q} \cdot \mathbf{x}} = \sum_{i}^{A} Z_i e^{i\mathbf{q} \cdot \mathbf{r}_i} \]

• In the long wavelength limit \( q \rightarrow 0 \)

• For a system of identical particles

• Conclusion A: In general the Dipole is the leading term.
• Conclusion B: For identical particles the leading terms are \( \hat{R}^2 \) and \( \hat{Q} \).
The Static Response - Inelastic Reactions

- The response of an A-particle system is closely related to the static moments of the charge density
  \[ \rho(\mathbf{x}) = \sum_{i}^{A} Z_i \delta(\mathbf{x} - \mathbf{r}_i) \]

- The Fourier Transform
  \[ \rho(q) = \int d\mathbf{x} \rho(\mathbf{x}) e^{i\mathbf{q} \cdot \mathbf{x}} = \sum_{i}^{A} Z_i e^{i\mathbf{q} \cdot \mathbf{r}_i} \]

- In the long wavelength limit \( q \to 0 \)
  \[ \rho(q) \approx \sum_{i}^{A} Z_i + i \sum_{i}^{A} Z_i \mathbf{q} \cdot \mathbf{r}_i - \frac{1}{2} \sum_{i}^{A} Z_i (\mathbf{q} \cdot \mathbf{r}_i)^2 \]

- For a system of identical particles

- **Conclusion A:** In general the Dipole is the leading term.
- **Conclusion B:** For identical particles the leading terms are \( \hat{R}^2 \) and \( \hat{Q} \).
The Static Response - Inelastic Reactions

- The response of an A-particle system is closely related to the static moments of the charge density
  \[ \rho(\mathbf{x}) = \sum_{i}^{A} Z_i \delta(\mathbf{x} - \mathbf{r}_i) \]

- The Fourier Transform
  \[ \rho(q) = \int d\mathbf{x} \rho(\mathbf{x}) e^{iq \cdot \mathbf{x}} = \sum_{i}^{A} Z_i e^{iq \cdot \mathbf{r}_i} \]

- In the long wavelength limit \( q \rightarrow 0 \)
  \[ \rho(q) \approx \sum_{i}^{A} Z_i + i \sum_{i}^{A} Z_i q \cdot \mathbf{r}_i - \frac{1}{2} \sum_{i}^{A} Z_i (q \cdot \mathbf{r}_i)^2 \]

- For a system of identical particles
  \[ \rho(q) \approx AZ_1 + iZ_1 R_{cm} - \frac{1}{2} Z_1 \sum_{i}^{A} \left( \frac{q^2 r_i^2}{6} + 4\pi \frac{q^2 r_i^2}{15} \sum_{m} Y_{2-m}(\hat{q}) Y_{2m}(\hat{r}_i) \right) \]

- Conclusion A: In general the Dipole is the leading term.
- Conclusion B: For identical particles the leading terms are \( \hat{R}^2 \) and \( \hat{Q} \).
The Static Response - Inelastic Reactions

• The response of an A-particle system is closely related to the static moments of the charge density

\[ \rho(\mathbf{x}) = \sum_{i}^{A} Z_i \delta(\mathbf{x} - \mathbf{r}_i) \]

• The Fourier Transform

\[ \rho(q) = \int d\mathbf{x} \rho(\mathbf{x}) e^{i\mathbf{q} \cdot \mathbf{x}} = \sum_{i}^{A} Z_i e^{i\mathbf{q} \cdot \mathbf{r}_i} \]

• In the long wavelength limit \( q \to 0 \)

\[ \rho(q) \approx \sum_{i}^{A} Z_i + i \sum_{i}^{A} Z_i \mathbf{q} \cdot \mathbf{r}_i - \frac{1}{2} \sum_{i}^{A} Z_i (\mathbf{q} \cdot \mathbf{r}_i)^2 \]

• For a system of identical particles

\[ \rho(q) \approx AZ_1 + iZ_1 \mathbf{R}_{cm} - \frac{1}{2} Z_1 \sum_{i}^{A} \left( \frac{q^2 r_i^2}{6} + 4\pi \frac{q^2 r_i^2}{15} \sum_{m} \mathbf{Y}_2-m(\hat{\mathbf{q}}) \mathbf{Y}_2m(\hat{\mathbf{r}}_i) \right) \]

• Conclusion A: In general the Dipole is the leading term.

• Conclusion B: For identical particles the leading terms are \( \hat{R}^2 \) and \( \hat{Q} \).
The Static Response - Inelastic Reactions

• The response of an A-particle system is closely related to the static moments of the charge density

\[ \rho(\boldsymbol{x}) = \sum_{i}^{A} Z_i \delta(\boldsymbol{x} - \boldsymbol{r}_i) \]

• The Fourier Transform

\[ \rho(q) = \int d\boldsymbol{x} \rho(\boldsymbol{x}) e^{i\boldsymbol{q} \cdot \boldsymbol{x}} = \sum_{i}^{A} Z_i e^{i\boldsymbol{q} \cdot \boldsymbol{r}_i} \]

• In the long wavelength limit \( q \rightarrow 0 \)

\[ \rho(q) \approx \sum_{i}^{A} Z_i + i \sum_{i}^{A} Z_i \boldsymbol{q} \cdot \boldsymbol{r}_i - \frac{1}{2} \sum_{i}^{A} Z_i (\boldsymbol{q} \cdot \boldsymbol{r}_i)^2 \]

• For a system of identical particles

\[ \rho(q) \approx AZ_1 + iZ_1 \mathbf{R}_{cm} - \frac{1}{2} Z_1 \sum_{i}^{A} \left( \frac{q^2 r_i^2}{6} + 4\pi \frac{q^2 r_i^2}{15} \sum_{m} Y_{2-m}(\hat{q}) Y_{2m}(\hat{r}_i) \right) \]

• Conclusion A: In general the Dipole is the leading term.

• Conclusion B: For identical particles the leading terms are \( \hat{R}^2 \) and \( \hat{Q} \).
Photo Reactions with Cold-Atoms

- For RF photons in the MHz region the wave length is **meters** so $qR \ll 1$.
- The Atoms reside in a strong magnetic field, thus spins are “frozen”

  \[ |\Psi_0\rangle = \Phi_0(r_i)|m_1^F m_2^F \ldots m_A^F\rangle \]

- In the final state the photon can either change one of the spins or leave them untouched.
- Spin-flip reaction

  \[
  |m_1^F m_2^F \ldots m_A^F\rangle \rightarrow |m_1^F m_2^F \pm 1 \ldots m_A^F\rangle
  \]

- Frozen-Spin reaction

  \[
  |m_1^F m_2^F \ldots m_A^F\rangle \rightarrow |m_1^F m_2^F \ldots m_A^F\rangle
  \]
Photo Reactions with Cold-Atoms

- For RF photons in the MHz region the wave length is \textit{meters} so \( qR \ll 1 \).
- The Atoms reside in a strong magnetic field, thus spins are “frozen”
  \[
  |\Psi_0\rangle = \Phi_0(r_i)|m_1^Fm_2^F\ldots m_A^F\rangle
  \]
- In the final state the photon can either change one of the spins or leave them untouched.
- \textit{Spin-flip} reaction
  \[
  |m_1^Fm_2^F\ldots m_A^F\rangle \rightarrow |m_1^Fm_2^F \pm 1\ldots m_A^F\rangle
  \]
- \textit{Frozen-Spin} reaction
  \[
  |m_1^Fm_2^F\ldots m_A^F\rangle \rightarrow |m_1^Fm_2^F \ldots m_A^F\rangle
  \]
Photo Reactions with Cold-Atoms

- For RF photons in the MHz region the wave length is meters so $qR \ll 1$.
- The Atoms reside in a strong magnetic field, thus spins are “frozen”
  \[ |\Psi_0\rangle = \Phi_0(r_i)|m_1^F m_2^F \ldots m_A^F\rangle \]
- In the final state the photon can either change one of the spins or leave them untouched.
- Spin-flip reaction
  \[ |m_1^F m_2^F \ldots m_A^F\rangle \rightarrow |m_1^F m_2^F \pm 1 \ldots m_A^F\rangle \]
- Frozen-Spin reaction
  \[ |m_1^F m_2^F \ldots m_A^F\rangle \rightarrow |m_1^F m_2^F \ldots m_A^F\rangle \]
Photo Reactions with Cold-Atoms

- For RF photons in the MHz region the wave length is meters so $qR \ll 1$.

- The Atoms reside in a strong magnetic field, thus spins are “frozen”

$$|\Psi_0\rangle = \Phi_0(r_i) |m_1^F m_2^F \ldots m_A^F\rangle$$

- In the final state the photon can either change one of the spins or leave them untouched.

- **Spin-flip** reaction

$$|m_1^F m_2^F \ldots m_A^F\rangle \rightarrow |m_1^F m_2^F \pm 1 \ldots m_A^F\rangle$$

- **Frozen-Spin** reaction

$$|m_1^F m_2^F \ldots m_A^F\rangle \rightarrow |m_1^F m_2^F \ldots m_A^F\rangle$$
Photo Reactions with Cold-Atoms

- For RF photons in the MHz region the wave length is meters so $qR \ll 1$.
- The Atoms reside in a strong magnetic field, thus spins are “frozen”
  \[
  |\Psi_0\rangle = \Phi_0(r_i)|m_1^F m_2^F \ldots m_A^F\rangle
  \]
- In the final state the photon can either change one of the spins or leave them untouched.
- **Spin-flip** reaction
  \[
  |m_1^F m_2^F \ldots m_A^F\rangle \longrightarrow |m_1^F m_2^F \pm 1 \ldots m_A^F\rangle
  \]
- **Frozen-Spin** reaction
  \[
  |m_1^F m_2^F \ldots m_A^F\rangle \longrightarrow |m_1^F m_2^F \ldots m_A^F\rangle
  \]
Photo Reactions with Cold-Atoms

- For Spin-flip reactions we get the ”Fermi” operator

\[ R(\omega) = Ck^5 \sum_{f,\lambda} |\langle \Phi_f | \Phi_0 \rangle|^2 \delta(E_f - E_0 - \omega) \]

- For Frozen-Spin reactions we get a sum of the monopole operator \( \hat{M} = R^2 = \sum r_i^2 \) and the Quadrupole operator \( \hat{Q} = \sum r_i^2 Y_2(\hat{r}_i) \)

\[ O = \alpha \hat{M} + \beta \hat{Q} \]

- The response is given by

\[ R(\omega) = k^5 \sum_{f,\lambda} |\langle \Phi_f | O | \Phi_0 \rangle|^2 \delta(E_f - E_0 - \omega) \]
Photo Reactions with Cold-Atoms

- For **Spin-flip** reactions we get the ”Fermi” operator

\[
R(\omega) = Ck \sum_{f,\lambda} \left| \langle \Phi_f | \Phi_0 \rangle \right|^2 \delta(E_f - E_0 - \omega)
\]

- For **Frozen-Spin** reactions we get a sum of the monopole operator \( \hat{M} = R^2 = \sum r_i^2 \) and the Quadrupole operator \( \hat{Q} = \sum r_i^2 Y_2(\hat{r}_i) \)

\[
O = \alpha \hat{M} + \beta \hat{Q}
\]

- The response is given by

\[
R(\omega) = k^5 \sum_{f,\lambda} \left| \langle \Phi_f | O | \Phi_0 \rangle \right|^2 \delta(E_f - E_0 - \omega)
\]
Photo Reactions with Cold-Atoms

• For Spin-flip reactions we get the "Fermi" operator

\[ R(\omega) = Ck \sum_{f,\lambda} |\langle \Phi_f | \Phi_0 \rangle|^2 \delta(E_f - E_0 - \omega) \]

• For Frozen-Spin reactions we get a sum of the monopole operator \( \hat{M} = R^2 = \sum r_i^2 \) and the Quadrupole operator \( \hat{Q} = \sum r_i^2 Y_2(\hat{r}_i) \)

\[ O = \alpha \hat{M} + \beta \hat{Q} \]

• The response is given by

\[ R(\omega) = k^5 \sum_{f,\lambda} |\langle \Phi_f | O | \Phi_0 \rangle|^2 \delta(E_f - E_0 - \omega) \]
Photoassociation of The Atomic Dimer

- For the dimer case the response function can be written as

\[
R(\omega) = C\omega^5 \left[ \frac{1}{6^2} |\langle \varphi_0(q) | \hat{M} | \psi_0 \rangle|^2 + \frac{1}{5 \cdot 15^2} |\langle \varphi_2(q) | \hat{Q} | \psi_0 \rangle|^2 \right]
\]

- Where the G.S. wave function is given by

\[
\psi_0 = Y_0 \sqrt{2\kappa e^{-\kappa r}} / r ; \; \kappa \approx 1/a_s
\]

- The continuum state is given by \( \varphi_\ell(q) = Y_\ell(\hat{r}) \chi_\ell(r) / r \)

\[
\chi_\ell(r) = 2qr \left[ \cos \delta_\ell j_\ell(qr) - \sin \delta_\ell n_\ell(qr) \right]
\]

- The \( \ell = 0 \) matrix element

\[
|\langle \varphi_0(q) | \hat{M} | \psi_0 \rangle|^2 = \frac{1}{4\pi} \left( \frac{4q \sqrt{2\kappa}}{(q^2 + \kappa^2)^3} \right)^2 \left[ \cos \delta_0 (3\kappa^2 - q^2) - \sin \delta_0 \frac{\kappa}{q} (3q^2 - \kappa^2) \right]^2
\]

- The \( \ell = 2 \) matrix element, assuming \( \delta_2 = 0 \)

\[
|\langle \varphi_2(q) | \hat{Q} | \psi_0 \rangle|^2 = \frac{5}{4\pi} \left[ \frac{16q^3 \sqrt{2\kappa}}{(q^2 + \kappa^2)^3} \right]^2
\]
Photoassociation of The Atomic Dimer

• For the dimer case the response function can be written as

\[
R(\omega) = C\omega^5 \left[ \frac{1}{6^2} |\langle \varphi_0(q) | \hat{M} | \psi_0 \rangle|^2 + \frac{1}{5 \cdot 15^2} |\langle \varphi_2(q) | \hat{Q} | \psi_0 \rangle|^2 \right]
\]

• Where the G.S. wave function is given by

\[
\psi_0 = Y_0 \sqrt{2\kappa} e^{-\kappa r} / r \quad ; \quad \kappa \approx 1 / a_s
\]

• The continuum state is given by \( \varphi_\ell(q) = Y_\ell(\hat{r}) \chi_\ell(r) / r \)

\[
\chi_\ell(r) = 2qr [\cos \delta_\ell j_\ell(qr) - \sin \delta_\ell n_\ell(qr)]
\]

• The \( \ell = 0 \) matrix element

\[
|\langle \varphi_0(q) | \hat{M} | \psi_0 \rangle|^2 = \frac{1}{4\pi} \left( \frac{4q\sqrt{2\kappa}}{(q^2 + \kappa^2)^3} \right)^2 \left[ \cos \delta_0 (3\kappa^2 - q^2) - \sin \delta_0 \frac{\kappa}{q} (3q^2 - \kappa^2) \right]^2
\]

• The \( \ell = 2 \) matrix element, assuming \( \delta_2 = 0 \)

\[
|\langle \varphi_2(q) | \hat{Q} | \psi_0 \rangle|^2 = \frac{5}{4\pi} \left( \frac{16q^3\sqrt{2\kappa}}{(q^2 + \kappa^2)^3} \right)^2
\]
Photoassociation of The Atomic Dimer

- For the dimer case the response function can be written as

\[ R(\omega) = C\omega^5 \left[ \frac{1}{6^2} |\langle \varphi_0(q)\|\hat{M}\|\psi_0\rangle|^2 + \frac{1}{5 \cdot 15^2} |\langle \varphi_2(q)\|\hat{Q}\|\psi_0\rangle|^2 \right] \]

- Where the G.S. wave function is given by

\[ \psi_0 = Y_0 \sqrt{2\kappa} e^{-\kappa r} / r ; \quad \kappa \approx 1/a_s \]

- The continuum state is given by \( \varphi_\ell(q) = Y_\ell(\hat{r}) \chi_\ell(r) / r \)

\[ \chi_\ell(r) = 2qr [\cos \delta_\ell j_\ell(qr) - \sin \delta_\ell n_\ell(qr)] \]

- The \( \ell = 0 \) matrix element

\[ |\langle \varphi_0(q)\|\hat{M}\|\psi_0\rangle|^2 = \frac{1}{4\pi} \left( \frac{4q\sqrt{2\kappa}}{(q^2 + \kappa^2)^3} \right)^2 \left[ \cos \delta_0 (3\kappa^2 - q^2) - \sin \delta_0 \frac{\kappa}{q} (3q^2 - \kappa^2) \right]^2 \]

- The \( \ell = 2 \) matrix element, assuming \( \delta_2 = 0 \)

\[ |\langle \varphi_2(q)\|\hat{Q}\|\psi_0\rangle|^2 = \frac{5}{4\pi} \left[ \frac{16q^3\sqrt{2\kappa}}{(q^2 + \kappa^2)^3} \right]^2 \]
Photoassociation of The Atomic Dimer

- For the dimer case the response function can be written as

\[
R(\omega) = C\omega^5 \left[ \frac{1}{6^2} |\langle \varphi_0(q) | \hat{M} | \psi_0 \rangle|^2 + \frac{1}{5 \cdot 15^2} |\langle \varphi_2(q) | \hat{Q} | \psi_0 \rangle|^2 \right]
\]

- Where the G.S. wave function is given by

\[
\psi_0 = Y_0 \sqrt{2}\kappa e^{-\kappa r}/r \quad \kappa \approx 1/a_s
\]

- The continuum state is given by \( \varphi_\ell(q) = Y_\ell(\hat{r}) \chi_\ell(r)/r \)

\[
\chi_\ell(r) = 2qr \left[ \cos \delta_\ell j_\ell(qr) - \sin \delta_\ell n_\ell(qr) \right]
\]

- The \( \ell = 0 \) matrix element

\[
|\langle \varphi_0(q) | \hat{M} | \psi_0 \rangle|^2 = \frac{1}{4\pi} \left( \frac{4q\sqrt{2}\kappa}{(q^2 + \kappa^2)^3} \right)^2 \left[ \cos \delta_0 (3\kappa^2 - q^2) - \sin \delta_0 \frac{\kappa}{q} (3q^2 - \kappa^2) \right]^2
\]

- The \( \ell = 2 \) matrix element, assuming \( \delta_2 = 0 \)

\[
|\langle \varphi_2(q) | \hat{Q} | \psi_0 \rangle|^2 = \frac{5}{4\pi} \left[ \frac{16q^3\sqrt{2}\kappa}{(q^2 + \kappa^2)^3} \right]^2
\]
Photoassociation of The Atomic Dimer

• For the dimer case the response function can be written as

\[
R(\omega) = C\omega^5 \left[ \frac{1}{6^2} |\langle \phi_0(q) | \hat{M} | \psi_0 \rangle|^2 + \frac{1}{5 \cdot 15^2} |\langle \phi_2(q) | \hat{Q} | \psi_0 \rangle|^2 \right]
\]

• Where the G.S. wave function is given by

\[
\psi_0 = Y_0 \sqrt{2}\kappa e^{-\kappa r} / r \quad ; \quad \kappa \approx 1 / a_s
\]

• The continuum state is given by \( \phi_\ell(q) = Y_\ell(\hat{r}) \chi_\ell(r) / r \)

\[
\chi_\ell(r) = 2qr[\cos \delta_\ell j_\ell(qr) - \sin \delta_\ell n_\ell(qr)]
\]

• The \( \ell = 0 \) matrix element

\[
|\langle \phi_0(q) | \hat{M} | \psi_0 \rangle|^2 = \frac{1}{4\pi} \left( \frac{4q\sqrt{2}\kappa}{(q^2 + \kappa^2)^3} \right)^2 \left[ \cos \delta_0 (3\kappa^2 - q^2) - \sin \delta_0 \frac{\kappa}{q} (3q^2 - \kappa^2) \right]^2
\]

• The \( \ell = 2 \) matrix element, assuming \( \delta_2 = 0 \)

\[
|\langle \phi_2(q) | \hat{Q} | \psi_0 \rangle|^2 = \frac{5}{4\pi} \left[ \frac{16q^3\sqrt{2}\kappa}{(q^2 + \kappa^2)^3} \right]^2
\]
Photoassociation of The Atomic Dimer

The s-wave and d-wave components in the response function:
- upper panel
  \( a/r_{\text{eff}} = 2 \)
- lower panel
  \( a/r_{\text{eff}} = 200 \)
- red - \( r^2 \) monopole
- blue - quadrupole
Photoassociation rates

Photoassociation of $^7$Li atoms

$a_s = 1000a_0$

$T = 5\mu K$ (lower panel), $T = 25\mu K$ (upper panel)

red - $r^2$ monopole, blue - quadrupole

The relative contribution to the peak
Photoassociation of $^7$Li atoms

$a_s = 1000a_0$

$T = 5\mu K$ (lower panel), $T = 25\mu K$ (upper panel)

red - $r^2$ monopole, blue - quadrupole

The relative contribution to the peak

Normalized photoassociation rate $[\text{n.d.}]$

$\frac{\hbar \omega}{E_B} [\text{n.d.}]$
Photoassociation of The Atomic Dimer

Comparison to the Khaykovich group data

- The fitted values of $a_s$ and $T$ are in reasonable agreement with the estimates of the experimental group.
- Effect of RF field on dimers not included.
- Finite time effect
- Disagreement are due to 3-body (4-body?) association.
- Effects of $\delta_2 \neq 0$ are negligible.
Summary and Conclusions

1. EM reaction provides a prism of the nuclear Hamiltonian.
2. The $0^+_2$ poses a problem to our contemporary understanding.
3. Relation to the $A_y$ problem?
4. The new RF experiments in Cold-Atoms systems carry much in common with photo-reactions and charged current reactions in nuclei.
5. For spin-flip reaction a Fermi type operator is the leading contribution to the cross-section, and $R(\omega) \propto \omega$.
6. For frozen-spin reactions the monopole $R^2$ and the Quadrupole are the leading terms, and $R(\omega) \propto \omega^5$.
7. We have studied Dimer formation and found that the reaction mechanism change from monopole to quadrupole with increasing gas temperature.
Summary and Conclusions

1. EM reaction provides a prism of the nuclear Hamiltonian.
2. The $0^+_2$ poses a problem to our contemporary understanding.
3. Relation to the $A_y$ problem?
4. The new RF experiments in Cold-Atoms systems carry much in common with photo-reactions and charged current reactions in nuclei.
5. For *spin-flip* reaction a Fermi type operator is the leading contribution to the cross-section, and $R(\omega) \propto \omega$.
6. For *frozen-spin* reactions the monopole $R^2$ and the Quadrupole are the leading terms, and $R(\omega) \propto \omega^5$.
7. We have studied Dimer formation and found that the reaction mechanism change from monopole to quadrupole with increasing gas temperature.
Summary and Conclusions

1. EM reaction provides a prism of the nuclear Hamiltonian.
2. The $0^+_2$ poses a problem to our contemporary understanding.
3. Relation to the $A_y$ problem?
4. The new RF experiments in Cold-Atoms systems carry much in common with photo-reactions and charged current reactions in nuclei.
5. For spin-flip reaction a Fermi type operator is the leading contribution to the cross-section, and $R(\omega) \propto \omega$.
6. For frozen-spin reactions the monopole $R^2$ and the Quadrupole are the leading terms, and $R(\omega) \propto \omega^5$.
7. We have studied Dimer formation and found that the reaction mechanism change from monopole to quadrupole with increasing gas temperature.
Summary and Conclusions

1. EM reaction provides a prism of the nuclear Hamiltonian.
2. The $0^+_2$ poses a problem to our contemporary understanding.
3. Relation to the $A_y$ problem?
4. The new RF experiments in Cold-Atoms systems carry much in common with photo-reactions and charged current reactions in nuclei.
5. For spin-flip reaction a Fermi type operator is the leading contribution to the cross-section, and $R(\omega) \propto \omega$.
6. For frozen-spin reactions the monopole $R^2$ and the Quadrupole are the leading terms, and $R(\omega) \propto \omega^5$.
7. We have studied Dimer formation and found that the reaction mechanism change from monopole to quadrupole with increasing gas temperature.
Summary and Conclusions

1. EM reaction provides a prism of the nuclear Hamiltonian.
2. The $0^+_2$ poses a problem to our contemporary understanding.
3. Relation to the $A_y$ problem?
4. The new RF experiments in Cold-Atoms systems carry much in common with photo-reactions and charged current reactions in nuclei.
5. For spin-flip reaction a Fermi type operator is the leading contribution to the cross-section, and $R(\omega) \propto \omega$.
6. For frozen-spin reactions the monopole $R^2$ and the Quadrupole are the leading terms, and $R(\omega) \propto \omega^5$.
7. We have studied Dimer formation and found that the reaction mechanism change from monopole to quadrupole with increasing gas temperature.
Summary and Conclusions

1. EM reaction provides a prism of the nuclear Hamiltonian.
2. The $0^+_2$ poses a problem to our contemporary understanding.
3. Relation to the $A_y$ problem?
4. The new RF experiments in Cold-Atoms systems carry much in common with photo-reactions and charged current reactions in nuclei.
5. For spin-flip reaction a Fermi type operator is the leading contribution to the cross-section, and $R(\omega) \propto \omega$.
6. For frozen-spin reactions the monopole $R^2$ and the Quadrupole are the leading terms, and $R(\omega) \propto \omega^5$.
7. We have studied Dimer formation and found that the reaction mechanism change from monopole to quadrupole with increasing gas temperature.
Summary and Conclusions

1. EM reaction provides a prism of the nuclear Hamiltonian.
2. The $0^+_2$ poses a problem to our contemporary understanding.
3. Relation to the $A_y$ problem?
4. The new RF experiments in Cold-Atoms systems carry much in common with photo-reactions and charged current reactions in nuclei.
5. For \textit{spin-flip} reaction a Fermi type operator is the leading contribution to the cross-section, and $R(\omega) \propto \omega$.
6. For \textit{frozen-spin} reactions the \textit{monopole} $R^2$ and the \textit{Quadrupole} are the leading terms, and $R(\omega) \propto \omega^5$.
7. We have studied Dimer formation and found that the reaction mechanism change from monopole to quadrupole with increasing gas temperature.
Summary and Conclusions

1. EM reaction provides a prism of the nuclear Hamiltonian.
2. The $0_2^{+}$ poses a problem to our contemporary understanding.
3. Relation to the $A_y$ problem?
4. The new RF experiments in Cold-Atoms systems carry much in common with photo-reactions and charged current reactions in nuclei.
5. For spin-flip reaction a Fermi type operator is the leading contribution to the cross-section, and $R(\omega) \propto \omega$.
6. For frozen-spin reactions the monopole $R^2$ and the Quadrupole are the leading terms, and $R(\omega) \propto \omega^5$.
7. We have studied Dimer formation and found that the reaction mechanism change from monopole to quadrupole with increasing gas temperature.
Summary and Conclusions

1. EM reaction provides a prism of the nuclear Hamiltonian.
2. The $0_2^+$ poses a problem to our contemporary understanding.
3. Relation to the $A_y$ problem?
4. The new RF experiments in Cold-Atoms systems carry much in common with photo-reactions and charged current reactions in nuclei.
5. For spin-flip reaction a Fermi type operator is the leading contribution to the cross-section, and $R(\omega) \propto \omega$.
6. For frozen-spin reactions the monopole $R^2$ and the Quadrupole are the leading terms, and $R(\omega) \propto \omega^5$.
7. We have studied Dimer formation and found that the reaction mechanism change from monopole to quadrupole with increasing gas temperature.
Few fish (but from first principles)...

Russian river, Alaska