Light-quark mass dependence of QCD: Myths and Facts

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photo by Avi Loud

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Light-quark mass dependence of QCD:

- **Myths:** Effective Field Theory
  Chiral Perturbation Theory

- **Facts:** Numerical Lattice QCD results
Light-quark mass dependence of QCD:

- **Myths:** Effective Field Theory, Chiral Perturbation Theory
- **Facts:** Numerical Lattice QCD results

**Caveat:** I am a born and raised Effective Field Theorist
Light-quark mass dependence of QCD:

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  - Chiral Perturbation Theory

- **Facts:** Numerical Lattice QCD results

**Caveat:** I am a born and raised Effective Field Theorist

I am having fun with my own *faith*, while making a serious point
Light quark mass dependence of the baryon spectrum

Hadron Electromagnetic Polarizabilities from Lattice QCD
Light quark mass dependence of $M_B$

This heralded the paradigm change in the relation between lattice QCD and effective field theory at least for simple quantities.

$m_\pi \rightarrow m_l$

$m_K \rightarrow m_s$

$m_\Xi \rightarrow \text{scale}$
Light quark mass dependence of $M_B$

- Chiral perturbation theory ($\chi$PT) provides a complete (but non-predictive) description of low-energy QCD.

- The chiral logarithms (non-analytic dependence upon the light quark masses) are the "predictions" of $\chi$PT as they encode long-range IR physics not contained in local operators.

- For some (small) values of $m_q$, $\chi$PT should provide a precise and accurate description of low energy hadronic phenomena.

- Confidence in our understanding requires evidence of the chiral logarithms from lattice QCD.
Light quark mass dependence of $M_B$

Heavy Baryon Chiral Perturbation Theory ($HB\chi PT$)

E. Jenkins and A. Manohar  PLB 255 (1991)

Expand about the static heavy baryon limit

$$L = \bar{N} iv \cdot \partial N + 2\alpha_M \bar{N}N \text{tr}(\mathcal{M}_+) - \bar{T}^\mu [i v \cdot \partial - \Delta_0] T_\mu - 2\bar{\gamma}_M \bar{T}^\mu T_\mu \text{tr}(\mathcal{M}_+)$$

$$+ 2g_A \bar{N} S \cdot AN + 2g_{\Delta \Delta} \bar{T}^\mu S \cdot A T_\mu + g_{\Delta N} (\bar{T}^\mu A_\mu N + \bar{N} A^\mu T_\mu)$$

$$\Delta_0 = M_\Delta - M_N \bigg|_{m_q=0} \quad \text{phenomenologically} \quad \Delta \sim 290 \text{ MeV}$$
Light quark mass dependence of $M_B$

\[ -i\Sigma = \quad + \quad + \quad + \quad + \ldots \]

\[ M_N = M_0 - 2\alpha_M(\mu) m^2_\pi - \frac{3\pi g_A^2}{(4\pi f_\pi)^2} m^3_\pi - \frac{8g^2_{\Delta N}}{3(4\pi f_\pi)^2} F(m_\pi, \Delta, \mu), \]

\[ F(m, \Delta, \mu) = (\Delta^2 - m^2 + i\epsilon)^{3/2} \ln \left( \frac{\Delta + \sqrt{\Delta^2 - m^2 + i\epsilon}}{\Delta - \sqrt{\Delta^2 - m^2 + i\epsilon}} \right) - \frac{3}{2} \Delta m^2 \ln \left( \frac{m^2}{\mu^2} \right) - \Delta^3 \ln \left( \frac{4\Delta^2}{m^2} \right) \]
Light quark mass dependence of $M_B$

nucleon mass to nlo

$$M_N = M_0 - 2\alpha_M(\mu) m_{\pi}^2 + \frac{3\pi g_A^2}{(4\pi f_\pi)^2} m_{\pi}^3 - \frac{8g_{\Delta N}^2}{3(4\pi f_\pi)^2} F(m_{\pi}, \Delta, \mu),$$

$$F(m, \Delta, \mu) = (\Delta^2 - m^2 + i\epsilon)^{3/2} \ln \left( \frac{\Delta + \sqrt{\Delta^2 - m^2 + i\epsilon}}{\Delta - \sqrt{\Delta^2 - m^2 + i\epsilon}} \right) - \frac{3}{2} \Delta m^2 \ln \left( \frac{m^2}{\mu^2} \right) - 2 \ln \left( \frac{4\Delta^2}{m^2} \right)$$

$$m_{\pi}^3 \sim m_q^{3/2}$$

leading non-analytic chiral behavior

renders the chiral expansion less convergent (than for mesons)
Light quark mass dependence of $M_B$

NNLO – $m_\pi^4$, with $g_A=1.2(1), g_{\Delta N}=1.5(3)$

NNLO Heavy Baryon Fit

$M_N = 954 \pm 42 \pm 20$ MeV

LHP Collaboration arXiv:0806.4549
Light quark mass dependence of $M_B$

NNLO $- m^4$, with $g_A = 1.2(1)$, $g_{A_{N}} = 1.5(3)$

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Ruler Approximation

$M_N = \alpha_0^N + \alpha_1^N m_\pi$

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I am not advocating this as a good model for QCD!

LHP Collaboration arXiv:0806.4549
Light quark mass dependence of $M_B$
Light quark mass dependence of $M_B$

What does this teach us?

For these pion masses, there is a strong cancelation between LO, NLO and NNLO $\chi$PT contributions perhaps should have been expected given poor convergence (but just not a straight line!!!)
Light quark mass dependence of $M_B$

What if we consider the octet and decuplet in the three flavor theory?

\[ M_N = M_0 + \alpha^\pi_N m^2_\pi + \alpha^K_N m^2_K \]
\[ - \frac{1}{16\pi^2 f^2} \left[ 3\pi (D + F)^2 m^3_\pi + \frac{\pi}{3} (D - 3F)^2 m^3_\eta \right. \]
\[ + \frac{2\pi}{3} (5D^2 - 6DF + 9F^2) m^3_K \]
\[ \left. + \frac{8}{3} \mathcal{F}(m_\pi, \Delta, \mu) + \frac{2}{3} \mathcal{F}(m_K, \Delta, \mu) \right] \]

Possible convergence is significantly challenged (fails) by kaon and eta loops

LHP Collaboration arXiv:0806.4549

PACS-CS Collaboration arXiv:0905.0962
Light quark mass dependence of $M_B$

NLO SU(3) chiral fits to spectrum are not consistent with phenomenological values of $D, F$

$$D \sim 0.75, \quad F \sim 0.50$$
Light quark mass dependence of $M_B$

What is the status now (2012)?

$$M_N = \alpha_0 + \alpha_1 m_\pi$$

$$= 938 \pm 9 \text{ MeV}$$

Physical point NOT included in fit
Light quark mass dependence of $M_B$

What is the status now (2012)?

$$M_N = \alpha_0 + \alpha_1 m_\pi$$

$$= 938 \pm 9 \text{ MeV}$$

$$\alpha_0 = 802 \pm 13 \text{ MeV}$$

$$\alpha_1 = 0.99 \pm 0.03$$

Physical point NOT included in fit
Light quark mass dependence of $M_B$

What is the status now (2012)?

$$M_N = \alpha_0 + \alpha_1 m_\pi = 938 \pm 9 \text{ MeV}$$

$\chi_{QCD}$ Collaboration uses Overlap Valence fermions on Domain-Wall (RBC-UKQCD) sea fermions
Light quark mass dependence of $M_B$

What is the status now (2012)?

$$M_N = \alpha_0 + \alpha_1 m_\pi = 938 \pm 9 \text{ MeV}$$

RBC-UKQCD Collaboration uses Domain-Wall valence and sea fermions
Light quark mass dependence of $M_B$

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$$M_N = \alpha_0 + \alpha_1 m_\pi$$

$$= 938 \pm 9 \text{ MeV}$$
Light quark mass dependence of $M_B$

What is the status now (2012)?

$M_N = \alpha_0 + \alpha_1 m_\pi$

$= 938 \pm 9 \text{ MeV}$

$\sigma_{\pi N} = 67 \pm 4 \text{ MeV}$
Light quark mass dependence of $M_B$

What is the status now (2012)?

Taking this seriously yields

\[ M_N = \alpha_0 + \alpha_1 m_\pi = 938 \pm 9 \text{ MeV} \]

\[ \sigma_{\pi N} = 67 \pm 4 \text{ MeV} \]

I am not advocating this as a good model for QCD!
\[ \Sigma_{\pi}(N) = \frac{3g_{\pi N}^2}{M} \int \frac{d^4k F^2(k^2)}{i(2\pi)^4} \]

\[ \times \frac{k \cdot p}{(k^2 - \mu^2 + i\epsilon)[(p - k)^2 - M^2 + i\epsilon]} . \]

To evaluate integral, used light cone coordinates.
The term carrying out the integration over the four-momentum momentum conservation, utilizes Eq. (first integrating over respectively. The quantity Eqs. (shell. This allows the use of the on-mass-shell form factors way that the intermediate baryon is projected onto its mass shell. This allows one to pick the contour such that the intermediate nucleon is on its mass shell. In that case, one may use a dispersion the pionic vertex function appears between two on-mass- 

\[ \Sigma_\pi(N) = \frac{3g^2_{\pi N}}{M} \int dk^+ d^2k_\perp J \]

\[ J = \frac{1}{i(2\pi)^4} \frac{1}{2} \int dk^- F^2(k^2) \]

\[ \times \frac{k \cdot p}{k^+(p-k)^+(k^- - \frac{k_\perp^2 + \mu^2 - i\epsilon}{k^+})[(p-k)^- - \frac{k_\perp^2 + M^2 - i\epsilon}{p^+ - k^+}]} \]

This allows one to pick the contour such that the intermediate nucleon (delta) is on shell - simplifying the numerator structure
The term given by Feynman rules as neutrino-nucleon interactions. The quantity $\pi$-particles has an off-diagonal Goldberger-Treiman relation. We use the notation $\Sigma_\pi(N) = \frac{3g^2_{\pi N}}{M} \int dk^+ d^2 k_\perp J$.

$$J = \frac{1}{i(2\pi)^4} \frac{1}{2} \int dk^- F^2(k^2) \times \frac{k \cdot p}{k^+(p-k)^+(k^- - \frac{k_\perp^2 + \mu^2 - i\epsilon}{k^+})[(p-k)^- - \frac{k_\perp^2 + M^2 - i\epsilon}{p^+ - k^+}]}.$$ 

Expanding for small pion mass ($\mu$) one recovers the HBChiPT expression.
Nucleon and Delta loop contributions set to zero at origin
Large $N_c$ and SU(3) Chiral Perturbation Theory

What can we do?

Consider 2-flavor expansion for hyperons

Beane, Bedaque, Parreno and Savage arXiv:0311027

Tiburzi and AWL arXiv:0808.0482

Jiang and Tiburzi arXiv:0905.0857

Mai, Bruns, Kubis and Meissner arXiv:0905.2810

Jiang, Tiburzi and AWL arXiv:0911.4721

Jiang and Tiburzi arXiv:0912.2077
Large $N_c$ and SU(3) Chiral Perturbation Theory

What can we do?

- Consider 2-flavor expansion for hyperons
  
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  Jiang and Tiburzi \textit{arXiv:0912.2077}

- Read the literature and apply an old idea to our new problem

  combine the constraints of large $N_c$ and SU(3) symmetries
Large $N_c$ and SU(3) Chiral Perturbation Theory

Combined large $N_c$ and SU(3) symmetries

‘t Hooft 1974
Witten 1979
Coleman 1979
Dashen, Jenkins, Manohar 1993

...
Large $N_c$ and SU(3) Chiral Perturbation Theory

- Theory is placed on solid theoretical foundation

$$\lim_{N_c \to \infty} M_B = \infty$$

controlled expansion in $1/N_c$ (at least formally)

- Inclusion of spin 3/2 dof well defined field theoretically

$$M_\Delta - M_N \propto \frac{1}{N_c}$$

- Naturally explains smallness of baryon octet GMO relation

$$N_c m_s^{3/2} \propto \text{flavor-1}$$

$$m_s^{3/2} \propto \text{flavor-8}$$

$$m_s^{3/2} / N_c \propto \text{flavor-27} \quad \text{leading correction to GMO}$$
Large $N_c$ and SU(3) Chiral Perturbation Theory

gives you “smarter” observables to measure/calculate
eg: Spectrum

$$M = M^{1,0} + M^{8,0} + M^{27,0} + M^{64,0}$$

\[
M^{1,0} = c^{1,0}_{(0)} N_c 1 + c^{1,0}_{(2)} \frac{1}{N_c} J^2
\]

\[
M^{8,0} = c^{8,0}_{(1)} T^8 + c^{8,0}_{(2)} \frac{1}{N_c} \{J^i, G^{i8}\} + c^{8,0}_{(3)} \frac{1}{N^2_c} \{J^2, T^8\}
\]

\[
M^{27,0} = c^{27,0}_{(2)} \frac{1}{N_c} \{T^8, T^8\} + c^{27,0}_{(3)} \frac{1}{N^2_c} \{T^8, \{J^i, G^{i8}\}\}
\]

\[
M^{64,0} = c^{64,0}_{(3)} \frac{1}{N^2_c} \{T^8, \{T^8, T^8\}\}
\]

\[
J^i = q^\dagger (J^i \otimes 1) q \quad \text{one-body spin operator}
\]

\[
T^a = q^\dagger (1 \otimes T^a) q \quad \text{one-body flavor operator}
\]

\[
G^{ia} = q^\dagger (J^i \otimes T^a) q \quad \text{one-body spin-flavor operator}
\]
### Table I: Mass combinations

<table>
<thead>
<tr>
<th>Label</th>
<th>Operator</th>
<th>Coefficient</th>
<th>Mass Combination</th>
</tr>
</thead>
<tbody>
<tr>
<td>M₁</td>
<td>1</td>
<td>160 (N_c) (c_{(0)})</td>
<td>25(2(N + \Lambda + 3\Sigma + 2\Xi)) − 4(4(\Delta + 3\Sigma^* + 2\Xi^* + \Omega))</td>
</tr>
<tr>
<td>M₂</td>
<td>(J^2)</td>
<td>−120 (\frac{1}{N_c}) (c_{(2)})</td>
<td>5(2(N + \Lambda + 3\Sigma + 2\Xi)) − 4(4(\Delta + 3\Sigma^* + 2\Xi^* + \Omega))</td>
</tr>
<tr>
<td>M₃</td>
<td>(T^8)</td>
<td>(20\sqrt{3} \epsilon) (c_{(1)})</td>
<td>5(6(N + \Lambda - 3\Sigma - 4\Xi)) − 2(2(\Delta - \Xi^* - \Omega))</td>
</tr>
<tr>
<td>M₄</td>
<td>({J^i, G^{i8}})</td>
<td>−5(\sqrt{3}) (\frac{1}{N_c}) (c_{(2)})</td>
<td>(−2(N + 3\Lambda - 9\Sigma + 8\Xi)) + 2(2(\Delta - \Xi^* - \Omega))</td>
</tr>
<tr>
<td>M₅</td>
<td>({J^2, T^8})</td>
<td>(30\sqrt{3} \frac{1}{N_c}) (c_{(3)})</td>
<td>35(2(N - 3\Lambda - \Sigma + 2\Xi)) − 4(4(\Delta - 5\Sigma^* - 2\Xi^* + 3\Omega))</td>
</tr>
<tr>
<td>M₆</td>
<td>({T^8, T^8})</td>
<td>(126 \frac{1}{N_c}) (\epsilon^2) (c_{(2)})</td>
<td>7(2(N - 3\Lambda - \Sigma + 2\Xi)) − 2(4(\Delta - 5\Sigma^* - 2\Xi^* + 3\Omega))</td>
</tr>
<tr>
<td>M₇</td>
<td>({T^8, J^i G^{i8}})</td>
<td>−63 (\frac{1}{N_c}) (\epsilon^2) (c_{(3)})</td>
<td>(\Delta - 3\Sigma^* + 3\Xi^* - \Omega)</td>
</tr>
<tr>
<td>M₈</td>
<td>({T^8, {T^8, T^8}})</td>
<td>9(\sqrt{3}) (\frac{1}{N_c}) (\epsilon^3) (c_{(3)})</td>
<td>((\Sigma^* - \Sigma) - (\Xi^* - \Xi))</td>
</tr>
</tbody>
</table>

\[ R \equiv \frac{\sum_i c_i M_i}{\sum_i |c_i|} \quad \epsilon \propto m_s - m_l \]
Large $N_c$ and SU(3) Chiral Perturbation Theory

\[ R_1 \sim \mathcal{O}(N_c) \times \mathcal{O}(\epsilon^0) \]

\[ R_2 \sim \mathcal{O}(1/N_c) \times \mathcal{O}(\epsilon^0) \]

Jenkins, Manohar, Negele + AWL arXiv:0907.0529
Large $N_c$ and SU(3) Chiral Perturbation Theory

\[ R_3 \sim \mathcal{O}(N_c^0) \times \mathcal{O}(\epsilon) \]

\[ R_4 \sim \mathcal{O}(1/N_c) \times \mathcal{O}(\epsilon) \]

Jenkins, Manohar, Negele + AWL  arXiv:0907.0529
Large $N_c$ and SU(3) Chiral Perturbation Theory

\[ R_5 \sim \mathcal{O}(1/N_c^2) \times \mathcal{O}(\epsilon) \]

\[ R_6 \sim \mathcal{O}(1/N_c) \times \mathcal{O}(\epsilon^2) \]

Jenkins, Manohar, Negele + AWL arXiv:0907.0529
\[ \mathcal{L} = i \text{Tr} \, \bar{B}_v (v \cdot D) B_v - i \, \bar{T}_v^\mu \, (v \cdot D) \, T_v^\mu - \frac{1}{4} \Delta_0 \, \text{Tr} \, \bar{B}_v B_v + \frac{5}{4} \Delta_0 \, \bar{T}_v^\mu T_v^\mu \\
+ 2D \, \text{Tr} \left( \bar{B}_v S_v^\mu \{ A_\mu, B_v \} \right) + 2F \, \text{Tr} \left( \bar{B}_v S_v^\mu [ A_\mu, B_v ] \right) \\
+ \mathcal{C} \left( \bar{T}_v^\mu A_\mu B_v + \bar{B}_v A_\mu T_v^\mu \right) + 2\mathcal{H} \, \bar{T}_v^\mu S_v^\nu A_\nu T_v^\mu \\
+ 2\sigma_B \, \text{Tr} \left( \bar{B}_v B_v \right) \text{Tr} M_+ - 2\sigma_T \, \bar{T}_v^\mu T_v^\mu \text{Tr} M_+ \\
+ 2b_D \, \text{Tr} \left( \bar{B}_v \{ M_+, B_v \} \right) + 2b_F \, \text{Tr} \left( \bar{B}_v [ M_+, B_v ] \right) + 2b_T \, \bar{T}_v^\mu M_+ T_v^\mu \]

**Large N_c expansion simplifies operators:**

\[ b_D = \frac{1}{4} b_{(2)} , \quad b_F = \frac{1}{2} b_{(1)} + \frac{1}{6} b_{(2)} , \quad b_T = - \frac{3}{2} b_{(1)} - \frac{5}{4} b_{(2)} \]

\[ \sigma_B = \frac{1}{2} b_{(1)} + \frac{1}{12} b_{(2)} , \quad \sigma_T = \frac{1}{2} b_{(1)} + \frac{5}{12} b_{(2)} . \]

\[ D = \frac{1}{2} a_{(1)} , \quad F = \frac{1}{3} a_{(1)} + \frac{1}{6} a_{(2)} , \]

\[ \mathcal{C} = - a_{(1)} , \quad \mathcal{H} = - \frac{3}{2} a_{(1)} - \frac{3}{2} a_{(2)} , \quad \mathcal{H} = 3D - F. \]
Evidence for non-analytic light quark mass dependence arXiv:1112.2658

\[ \frac{3}{2} R_1(m_l, m_s) = M_0 - \left( \frac{3}{4} b_{(1)} + \frac{5}{24} b_{(2)} \right) (2m_l + m_s) \]

\[ - \frac{1}{12} \left( 35a_{(1)}^2 - 5a_{(2)}^2 \right) \left( \frac{3F(m_\pi, 0, \mu) + 4F(m_K, 0, \mu) + F(m_\eta, 0, \mu)}{8(4\pi f)^2} \right) \]

\[ - \frac{1}{12} a_{(1)}^2 \left[ 50 \left( \frac{3F(m_\pi, \Delta, \mu) + 4F(m_K, \Delta, \mu) + F(m_\eta, \Delta, \mu)}{8(4\pi f)^2} \right) \right. \]

\[ \left. - 4 \left( \frac{3F(m_\pi, -\Delta, \mu) + 4F(m_K, -\Delta, \mu) + F(m_\eta, -\Delta, \mu)}{8(4\pi f)^2} \right) \right] \]

\[ a_{(1)} = 0.2(5) \]

\[ D = 0.10(25) \]
Evidence for non-analytic light quark mass dependence

$m_{\pi}L = 7.7, 5.8, 4.8, 3.9$

While SU(3) HBChPT fails to converge with acceptable values of $D, F, H, C$, provides a quantitatively accurate description of finite volume corrections (with acceptable $D, F, H, C$)
Evidence for non-analytic light quark mass dependence

\[ R_3 \propto m_s - m_l \]

\[ R_4 \propto (m_s - m_l) / N_c \]

\[
R_3(m_l, m_s) = \frac{20}{39} b_1 (m_s - m_l) - \frac{20a_1^2 - 5a_2^2}{117} \frac{3F_\pi^0 - 2F_K^0 - F_\eta^0}{(4\pi f)^2} - \frac{a_1^2}{117} \left[ 35 \frac{3F_\pi^\Delta - 2F_K^\Delta - F_\eta^\Delta}{(4\pi f)^2} - \frac{3F_\pi^\Delta - 2F_K^\Delta - F_\eta^\Delta}{(4\pi f)^2} \right],
\]

\[
R_4(m_l, m_s) = -\frac{5}{18} b_2 (m_s - m_l) + \frac{a_1^2 + 4a_1a_2 + a_2^2}{36} \frac{3F_\pi^0 - 2F_K^0 - F_\eta^0}{(4\pi f)^2} - \frac{2a_1^2}{9} \frac{3F_\pi^\Delta - 2F_K^\Delta - F_\eta^\Delta}{(4\pi f)^2}
\]
Evidence for non-analytic light quark mass dependence arXiv:1112.2658

Fit yields

\[ b_1 \text{[NLO]} = -6.6(5), \quad b_2 \text{[NLO]} = 4.3(4), \quad a_1 \text{[NLO]} = 1.4(1). \]

\[ D = 0.70(5), \quad F = 0.47(3), \quad C = -1.4(1), \quad H = -2.1(2). \]

First time axial couplings left as free parameters and:
values consistent with phenomenological determinations
Fit yields

\[ b_1[NLO] = -6.6(5), \quad b_2[NLO] = 4.3(4), \quad a_1[NLO] = 1.4(1). \]

\[ D = 0.70(5), \quad F = 0.47(3), \quad C = -1.4(1), \quad H = -2.1(2) \]

but still observe large cancellations between LO and NLO
Evidence for non-analytic light quark mass dependence arXiv:1112.2658

Work of Mathias Lutz and Alexandre Semke who fit the masses (not mass splittings) of 4 different lattice QCD groups, and obtained similar axial couplings

Fit i: each fit is to set of BMW, LHPC, PACS-CS none of the fits include QCDSF-UKQCD, who computed masses in SU(3) limit as well as SU(3)-broken (with similar agreement) I do not understand - but this agreement is remarkable
Evidence for non-analytic light quark mass dependence \texttt{arXiv:1112.2658}

**Gell-Mann--Okubo Relation**

- \textit{SU(3) \chi PT}
- NLO Fit $a_1 = 0.92(17)$
- NLO Fixed $a_1 = 1.4(1)$

\textit{SU(3) Vector}

\[ d_2(m_s - m_l)^2 + d_3(m_s - m_l)^3 \]

\textit{SU(3) \chi PT}

- NNLO Fit

Only NNLO SU(3) naturally supports strong light quark mass dependence
Combined with R3 and R4 - provides first compelling evidence of non-analytic light quark mass dependence in the baryon spectrum
the more I study baryons, the more confused I get

there now seems to be un-ignorable evidence for entirely unexpected light quark mass dependence in the nucleon (baryon) spectrum, basically down to the physical pion mass

\[ M_N = \alpha_0 + \alpha_1 m_\pi \]

combining large \( N_c \) with SU(2) and SU(3) flavor symmetry is showing promise - at least qualitatively

what is clearly (still) needed is high statistics study of baryons with (with the aim of understanding chiral perturbation theory)

\[ 120 \leq m_\pi \leq 400 \text{ MeV} \]
Hadron Electromagnetic Polarizabilities from Lattice QCD

electric polarizabilities and magnetic moments of the nucleon from lattice QCD

electromagnetic collaboration:
Will Detmold, Brian Tiburzi, AWL
Compass at CERN will measure pion and kaon polarizabilities through Primakoff process.

Compton MAX-lab (Lund) will extract neutron $\varepsilon M$ polarizabilities from Compton scattering on deuterium.

HI$\gamma$S TUNL will make high precision measurements of proton and neutron electromagnetic and spin polarizabilities.
Hadron Electromagnetic Polarizabilities from Lattice QCD

comparison of experiment and phenomenological prediction

pion
two-loop ChPT prediction

\[ \alpha^\pi_E = 2.4 \pm 0.5 \]
\[ \beta^\pi_E = -2.1 \pm 0.5 \]

experimental determination

\[ \alpha^\pi_E = -\beta^\pi_M = 6.8 \pm 1.4 \pm 1.2 \]
assumed \( (\alpha^\pi_E = -\beta^\pi_M) \)

nucleon

<table>
<thead>
<tr>
<th>Polarizability</th>
<th>Proton</th>
<th>Neutron</th>
</tr>
</thead>
<tbody>
<tr>
<td>\alpha [10^{-4} fm^3]</td>
<td>11.9 \pm 1.4</td>
<td>12.5 \pm 1.7</td>
</tr>
<tr>
<td>\beta [10^{-4} fm^3]</td>
<td>1.2 \pm 0.9</td>
<td>2.7 \pm 1.8</td>
</tr>
<tr>
<td>\gamma_1 [10^{-4} fm^4]</td>
<td>1.1 \pm 0.25</td>
<td>3.7 \pm 0.4</td>
</tr>
<tr>
<td>\gamma_2 [10^{-4} fm^4]</td>
<td>-1.5 \pm 0.36</td>
<td>-0.1 \pm 0.5</td>
</tr>
<tr>
<td>\gamma_3 [10^{-4} fm^4]</td>
<td>0.2 \pm 0.24</td>
<td>0.4 \pm 0.5</td>
</tr>
<tr>
<td>\gamma_4 [10^{-4} fm^4]</td>
<td>3.3 \pm 0.11</td>
<td>2.3 \pm 0.35</td>
</tr>
<tr>
<td>\gamma_\pi [10^{-4} fm^4]</td>
<td>-38.7 \pm 1.8</td>
<td>58.6 \pm 4.0</td>
</tr>
</tbody>
</table>

measured
(expected (theoretical disagreements)
Prediction from Chiral Perturbation Theory ($\chi$PT):
Non-analytic dependence on the light quark masses

$$m_\pi^2 = 2Bm_q \left[ 1 + \frac{2Bm_q}{(4\pi f)^2} \ln \left( \frac{2Bm_q}{\mu^2} \right) + 4\frac{2Bm_q}{f^2} l_3(\mu) \right] + \ldots$$

Polarizabilities:

$$\alpha_{E}^{\pi \pm} = \frac{8\alpha_{f.s.}}{f_\pi^2} \frac{L_9 + L_{10}}{m_\pi}$$  \hspace{1cm} \text{LO $\chi$PT}

$$\alpha_{E}^{N} = \frac{5\alpha_{f.s.} g_A^2}{192\pi f_\pi^2} \frac{1}{m_\pi} + \Delta\text{-contributions}$$  \hspace{1cm} \text{NLO $\chi$PT (leading loop)}

$$\beta_{B}^{N} = \frac{\alpha_{f.s.} g_A^2}{384\pi f_\pi^2} \frac{1}{m_\pi} + \Delta\text{-contributions}$$  \hspace{1cm} \text{NLO $\chi$PT (leading loop)}

$$\gamma_{E_1 E_1}^{N} = -\frac{5\alpha_{f.s.} g_A^2}{192\pi^2 f_\pi^2} \frac{1}{m_\pi^2} + \Delta\text{-contributions}$$  \hspace{1cm} \text{NLO $\chi$PT (leading loop)}

Evidence for this non-analytic light quark mass dependence is smoking gun for being in the chiral regime.
For sufficiently low energy ($\omega << m_\pi$), a spin 1/2 baryon has the effective Hamiltonian

$$H_{\text{eff}} = \frac{(\vec{p} - Q\vec{A})^2}{2M} + Q\phi - \frac{1}{2} 4\pi \left( \alpha \vec{E}^2 + \beta \vec{B}^2 \right. \left. + \gamma_{E_1 E_1} \vec{\sigma} \cdot \vec{E} \times \dot{\vec{E}} + \gamma_{M_1 M_1} \vec{\sigma} \cdot \vec{B} \times \dot{\vec{B}} + \gamma_{M_1 E_2} \sigma_i \epsilon_{ij} B_j + \gamma_{E_1 M_2} \sigma_i B_{ij} \epsilon_j \right)$$

where

$$\epsilon_{ij} = \frac{1}{2} (\nabla_i \epsilon_j + \nabla_j \epsilon_i) \quad \beta_{ij} = \frac{1}{2} (\nabla_i B_j + \nabla_j B_i)$$

$$\gamma_{E_1 E_1} = -\gamma_1 - \gamma_3 \quad \gamma_{M_1 M_1} = \gamma_4$$

$$\gamma_{E_1 M_2} = \gamma_3 \quad \gamma_{M_1 E_2} = \gamma_2 + \gamma_4$$

For specific choices of $A_\mu$, one can isolate the various (spin) polarizabilities W. Detmold, B.C. Tiburzi, AWL PRD 73 (2006).
For our calculation, we want Euclidean action which respects periodic boundary conditions (hyper-torus)

\[
e^{-i \int d^4 x M \frac{1}{4} F_{\mu \nu} F^{\mu \nu}} = e^{i \int d^4 x M \frac{1}{2} (\mathcal{E}_M^2 - B_M^2)}
\]

\[
\longrightarrow e^{-\int d^4 x_E \frac{1}{4} F_{\mu \nu} F_{\mu \nu}} = e^{-\int d^4 x_E \frac{1}{2} (\mathcal{E}_E^2 + B_E^2)}
\]

In this way, the $U(1)$ gauge links are given by a phase

\[
U_{\mu}(x) = e^{iaqA_{\mu}(x)}
\]

Consequences:

\[
M(\mathcal{E}_M) = M_0 - 2\pi \alpha \mathcal{E}_M^2 + \ldots \longrightarrow M(\mathcal{E}_E) = M_0 + 2\pi \alpha \mathcal{E}_E^2 + \ldots
\]
On a compact torus, not all values of the field strength are allowed:
G. ‘t Hooft NPB 153 (1979)

\[ 0 = \Phi = \Phi_1 + \Phi_2 \quad \quad A_1 = TL_z - A_2 \]

\[ \rightarrow \exp \{ iq \varepsilon A_1 \} = \exp \{ iq \varepsilon (TL_z - A_2) \} \rightarrow 1 = \exp \{ iq \varepsilon TL_z \} \]

\[ q \varepsilon = \frac{2\pi}{TL_z} n \quad \text{for } n = 1, 2, \ldots \]
Non-Quantized

\[ \pi^0: \text{Eq. (2), } n=3 \]

Quantized

\[ \pi^0: \text{Eq. (4), } t_{\text{src}}=52 \]

- \( n = 3, \ t_{\text{src}} = 0 \)
- \( n = 3, \ t_{\text{src}} = 52 \)
- \( n = e, \ t_{\text{src}} = 52 \)

\[ aM_{\text{eff}}(t) = \ln \left( \frac{C(t)}{C(t+1)} \right) \]
In a background field, what do we expect the correlation functions to look like?

- For $J = 0$, $Q = 0$:
  \[ C(t, \mathcal{E}) = \sum_n Z_n(\mathcal{E}) e^{-E_n(\mathcal{E})t} \]

- For $J = 1/2$, $Q = 0$:
  \[ C(t, \mathcal{E}) = \sum_n Z_n(\mathcal{E}, \mu_n) e^{-E_n(\mathcal{E}, \mu_n)t} \]

- For $J = 0$, $Q = 1$:
  \[ C(t, \mathcal{E}) = \sum_n Z_n(\mathcal{E}) G(E_n, \mathcal{E}, t) \]

- For $J = 1/2$, $Q = 1$:
  \[ C(t, \mathcal{E}) = \sum_n Z_n(\mathcal{E}, \mu_n) G(E_n, \mathcal{E}, \mu_n, t) \]
Consider spin-less, relativistic particle of unit charge coupled to an electric field

$$\mathcal{L} = D_\mu \pi^\dagger D_\mu \pi + m_{\text{eff}}^2 \pi^\dagger \pi, \quad D_\mu = \partial_\mu + iA_\mu, \quad A_\mu = (0, 0, -E_t, 0)$$

integrating by parts and changing variables

$$D^{-1} = p_\tau^2 + \mathcal{E}^2 \tau^2 + E_{k_\perp}^2 \equiv 2 \left( \mathcal{H} + \frac{1}{2} E_{k_\perp}^2 \right),$$

$$\tau = t - \frac{k_z}{\mathcal{E}}, \quad E_{k_\perp}^2 = E_k^2 - k_z^2$$


$$D(\tau', \tau) = \frac{1}{2} \int_0^\infty ds \langle \tau', s|\tau, 0 \rangle e^{-sE_{k_\perp}^2/2}$$

$$\langle \tau', s|\tau, 0 \rangle = \sqrt{\frac{\mathcal{E}}{2\pi \sinh \mathcal{E}s}} \exp \left\{ -\frac{\mathcal{E}}{2\sinh \mathcal{E}s} \left[ (\tau'^2 + \tau^2) \cosh \mathcal{E}s - 2\tau' \tau \right] \right\}$$
Take \( \tau = 0, \, \vec{k} = 0 \):

\[
C(\tau, \mathcal{E}) = \sum_n Z_n(\mathcal{E}) G(\tau, \mathcal{E})
\]

\[
G(\tau, \mathcal{E}) = \frac{1}{2} \int_0^\infty ds \sqrt{\frac{\mathcal{E}}{2\pi \sinh \mathcal{E} s}} \exp \left\{ -\frac{1}{2} \left( \mathcal{E} \tau^2 \coth \mathcal{E} s + s m^2_{\text{eff}} \right) \right\}
\]

in the weak field limit

\[
C(\tau, \mathcal{E}) = Z(\mathcal{E}) \exp \left\{ -M(\mathcal{E}) \tau - \frac{\mathcal{E}^2}{M(\mathcal{E})^4} \left( \frac{1}{6} (M(\mathcal{E}) \tau)^3 + \frac{1}{4} (M(\mathcal{E}) \tau)^2 + \frac{1}{4} (M(\mathcal{E}) \tau) \right) \right\}
\]

\[M(\mathcal{E}) = M_0 + 2\pi \alpha \mathcal{E}^2 + \mathcal{O}(\mathcal{E}^4)\]

computing hadron deformations in background \( \mathcal{E} M \) fields amounts to spectroscopy.
neutron in background electric field: W. Detmold, B.C. Tiburzi, AWL PRD 81 (2010)

\[
S = \int d^4x \bar{\psi}(x) \left[ \partial + E(\mathcal{E}) - \frac{\mu(\mathcal{E})}{4M} \sigma_{\mu\nu} F_{\mu\nu} \right] \psi(x),
\]

\[
F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu},
\]

\[
\sigma_{\mu\nu} F_{\mu\nu} = 2 \vec{K} \cdot \mathcal{E}, \quad \text{for background } \mathcal{E}-\text{field and } \vec{K} = i\vec{\gamma}_4
\]

\[
\mu(\mathcal{E}) = \mu + \mu'' \mathcal{E}^2 + \ldots \quad \text{anomalous magnetic coupling}
\]

motion of the quarks in the \( \mathcal{E} \)-field gives rise to the magnetic coupling

with \( \vec{\mathcal{E}} = \mathcal{E} \hat{z} \), construct

\[
G_{\pm}(t, \mathcal{E}) \equiv \text{tr}[\mathcal{P}_{\pm} G(t, \mathcal{E})] = Z(\mathcal{E}) \left( 1 \pm \frac{\mathcal{E} \mu(\mathcal{E})}{2M^2} \right) \exp \left[ -t E_{\text{eff}}(\mathcal{E}) \right],
\]

\[
\mathcal{P}_{\pm} = \frac{1}{2} [1 \pm K_3] \quad E_{\text{eff}} = E(\mathcal{E}) - \frac{\mu(\mathcal{E})^2 \mathcal{E}^2}{8M^3}
\]

\[
= M + \frac{1}{2} \mathcal{E}^2 \left( 4\pi \alpha_E - \frac{\mu^2}{4M^3} \right) + \ldots
\]
proton in background electric field: W. Detmold, B.C. Tiburzi, AWL PRD 81 (2010)

\[ S = \int d^4x \overline{\psi}(x) \left[ \mathcal{D} + E(\mathcal{E}) - \frac{\tilde{\mu}(\mathcal{E})}{4M} \sigma_{\mu\nu} F_{\mu\nu} \right] \psi(x), \]

\[ D_\mu = \partial_\mu + iQA_\mu \quad \mu = Q + \tilde{\mu}(0) \]

boost projected correlation functions

\[ G_\pm(t, \mathcal{E}) = Z(\mathcal{E}) \left( 1 \pm \frac{\tilde{\mu}\mathcal{E}}{2M^2} \right) D \left( t, E_{\text{eff}}(\mathcal{E})^2 \mp Q\mathcal{E}, \mathcal{E} \right) \]

\[ D(t, E^2, \mathcal{E}) = \int_0^\infty ds \sqrt{\frac{Q\mathcal{E}}{2\pi \sinh(Q\mathcal{E}s)}} \exp \left[ -\frac{Q\mathcal{E}t^2}{2} \coth(Q\mathcal{E}s) - \frac{E^2s}{2} \right] \]
Hadron Electromagnetic Polarizabilities from Lattice QCD

Results I am going to present are from

- **mesons**: W. Detmold, B.C. Tiburzi, AWL PRD 79 (2009)
- **proton and neutron**: W. Detmold, B.C. Tiburzi, AWL PRD 81 (2010)

To date, we have set \( q_{\text{sea}} = 0 \) (Quenched \( \mathcal{E}M \))

\[
m_\pi \sim 390 \text{ MeV} \quad L = 2.5 \text{ fm}
\]

<table>
<thead>
<tr>
<th>( V )</th>
<th>( a_s )</th>
<th>( a_s/a_t )</th>
<th>( a_t m_u^0 )</th>
<th>( a_t m_s^0 )</th>
<th>( m_\pi )</th>
<th>( m_K )</th>
<th>Field</th>
<th>( N_{\text{src}} \times N_{\text{cfg}} )</th>
<th>total # of props ( (u, d, s) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 20^3 \times 128 )</td>
<td>0.123</td>
<td>3.5</td>
<td>-0.0840</td>
<td>-0.0743</td>
<td>390</td>
<td>546</td>
<td>0 ( \times 15 )</td>
<td>6,000</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( \pm 1 )</td>
<td></td>
<td>15 × 200</td>
<td></td>
<td>9,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( \pm 2 )</td>
<td></td>
<td>10 × 200</td>
<td></td>
<td>6,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( \pm 3 )</td>
<td></td>
<td>10 × 200</td>
<td></td>
<td>6,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( \pm 4 )</td>
<td></td>
<td>10 × 200</td>
<td></td>
<td>6,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( 24^3 \times 128 )</td>
<td>0.123</td>
<td>3.5</td>
<td>-0.0840</td>
<td>-0.0743</td>
<td>390</td>
<td>546</td>
<td>0 ( \times 10 )</td>
<td>3,900</td>
<td></td>
</tr>
<tr>
<td></td>
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<td></td>
<td>( \pm 1 )</td>
<td></td>
<td>10 × 195</td>
<td></td>
<td>5,850</td>
<td></td>
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</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( \pm 2 )</td>
<td></td>
<td>10 × 195</td>
<td></td>
<td>5,850</td>
<td></td>
<td></td>
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<tr>
<td></td>
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<tr>
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<td></td>
<td>( \pm 4 )</td>
<td></td>
<td>10 × 195</td>
<td></td>
<td>5,850</td>
<td></td>
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</tr>
<tr>
<td>( 32^3 \times 256 )</td>
<td>0.123</td>
<td>3.5</td>
<td>-0.0860</td>
<td>-0.0743</td>
<td>225</td>
<td>467</td>
<td>0 ( \times 7 )</td>
<td>2,226</td>
<td></td>
</tr>
</tbody>
</table>

**TABLE I**: Propagators generated to date with our 2008-09 and 2009-10 USQCD allocations.
$\pi^0$ Mass Shift:
$\pi^+$ Effective Mass

\[ C(\tau, \mathcal{E}) = \sum_n Z_n(\mathcal{E}) G(\tau, \mathcal{E}) \]

\[ G(\tau, \mathcal{E}) = \frac{1}{2} \int_0^\infty ds \sqrt{\frac{\mathcal{E}}{2\pi \sinh \mathcal{E}s}} \exp \left\{ -\frac{1}{2} \left( \mathcal{E} \tau^2 \coth \mathcal{E}s + s m_{\text{eff}}^2 \right) \right\} \]
Hadron Electromagnetic Polarizabilities from Lattice QCD

\[ n = 0 \]

\[ n = 1 \]

\[ n = 2 \]

\[ n = 3 \]

\[ n = 4 \]

\[ m(\mathcal{E}) \begin{array}{cccccc}
0 & 1 & 2 & 3 & 4 \\
0.0691(4) & 0.0702(6) & 0.0718(8) & 0.0733(16) & 0.0497(129)
\end{array} \]
Hadron Electromagnetic Polarizabilities from Lattice QCD

\[ m(\mathcal{E}) = m_0 + \alpha^{\text{latt}}_E \mathcal{E}^2 + \bar{\alpha}^{\text{latt}}_{E\mathcal{E}} \mathcal{E}^4 \]

<table>
<thead>
<tr>
<th></th>
<th>$\pi^0$</th>
<th>$\pi^+$</th>
<th>$K^0$</th>
<th>$K^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha^{\text{latt}}_E$</td>
<td>-2.6(5)(9)</td>
<td>18(4)(6)</td>
<td>1.5(4)(7)</td>
<td>8(3)(1)</td>
</tr>
<tr>
<td>$\bar{\alpha}^{\text{latt}}_E$</td>
<td>1.8(5)</td>
<td>24(10)</td>
<td>0.6(5)</td>
<td>17(5)</td>
</tr>
</tbody>
</table>
Hadron Electromagnetic Polarizabilities from Lattice QCD

\[ G_{\pm}(t, \mathcal{E}) = Z(\mathcal{E}) \left( 1 \pm \frac{\hat{\mu} \mathcal{E}}{2M^2} \right) D \left( t, E_{\text{eff}}(\mathcal{E})^2 \mp Q\mathcal{E}, \mathcal{E} \right) \]

\[ D(t, E^2, \mathcal{E}) = \int_0^\infty ds \sqrt{\frac{Q\mathcal{E}}{2\pi \sinh(Q\mathcal{E}s)}} \exp \left[ -\frac{Q\mathcal{E}t^2}{2} \coth(Q\mathcal{E}s) - \frac{E^2s}{2} \right] \]
Hadron Electromagnetic Polarizabilities from Lattice QCD

\[ \alpha_E^V(m_\pi = 390 \text{ MeV}) = -0.9(2.5)(.3)(.4) \times 10^{-4} \text{ fm}^3 \quad \mu^V(m_\pi = 390 \text{ MeV}) = 4.3(2)(.1)(.1)[\mu_N] \]
over the last few years, we have established a program to compute polarizabilities of hadrons as well as magnetic moments, utilizing background electromagnetic fields

we now have to address several systematics (which need more computing time)

- sea quark electric charges need to be “turned on”
- light quark mass extrapolation - do we see $\frac{1}{m_\pi}$ behavior?
- nucleon spin polarizabilities (need field gradients - more difficult quantization condition if any)
- explicit magnetic background fields
the era of physical quark mass lattice QCD calculations is just around the corner - exciting time

care must be taken to understand the light quark mass dependence of observables - unique predictions from effective field theory - are these predictions verified in the numerical simulations?

effective field theory provides us with a deeper understanding of the underlying physics

(I realize here I am preaching to the choir)
Fin