Roper Resonance and $1^+\text{ Meson}$

- Roper resonance: lattice calculations and its nature
- Is $1^+$ meson a hybrid?

**QCD Collaboration:**

INT, Aug. 2, 2012
Many Facets of Roper Resonance

- Roper resonance ($P_{11}$ in $\pi N$ scattering, with $I = \frac{1}{2}$, $M = 1440$ MeV, $\Gamma \sim 300$ MeV) was discovered in pion-nucleon partial wave analysis in 1963.
- $\pi N$ and $\gamma N$ has large contamination from $\Delta$.
- Confirmation from ($\alpha$, $\alpha'$) scattering and $J/\psi \rightarrow \bar{NN}\pi$ decay.

(review by L. Alvarez-Ruso)

Many Facets of Roper Resonance

Theory:

- Quark potential model prediction is 100-200 MeV too high
  (Liu and Wong, 1983, Capstick and Isgur, 1986)

- Skyrmion can accommodate it as a radial excitation
  (J. Breit and C. Nappi, 1984, Liu, Zhang, Black, 1984;
  U. Kaulfuss and U. Meissner, 1985)

- Suggestion as a pentaquark (Krewald 2000);
  as a member of the antidecuplet
  (Jaffe, Wilczek, 2003)

- Perhaps a hybrid
  (Barnes, Close, etc. 1983)

- Lattice calculations

Quenched Lattice Calculations of Roper
Roper on the lattice

4 issues about lattice calculations:

• Radial excitation or pentaquark state?
• Dynamical fermions
• Variation vs Bayesian fitting
• Chiral dynamics
Roper is seen on the lattice with three-quark interpolation field.

Weight:

\[ |<0|O_N|R>|^2 > |<0|O_N|N>|^2 > 0 \]

(point source, point sink)

\[ \sum \psi(x) \sum \psi(y) \sum \psi(z) \]

Point sink                                          Wall source

\[ <0|O_N(0)|N> <N| \sum \psi(x) \sum \psi(y) \sum \psi(z)|0 > > 0 \]

However, \[ <0|O_N(0)|R> <R| \sum \psi(x) \sum \psi(y) | \sum \psi(z)|0 > < 0 \]
Bethe-Salpeter Wavefunction

\[
O_{RN} = \int dr\, \Psi_R^*(r)\Psi_N(r) = 0 \quad \text{at non-relativistic limit},
\]

\[
O_{RN} = \int dr\, \Psi_R^*(r)\Psi_N(r) \uparrow \quad \text{as } m_q \downarrow
\]
Nucleon and Roper wavefunctions for $m_n = 633$ MeV

$O_{RN} = 0.30$
Roper and Nucleon Wavefunctions at $m_\pi = 438$ MeV

$O_{RN} = 0.59$
Dynamical Fermions
Dynamical Fermions
(Overlap on DWF Configurations)

- Improvement of nucleon correlator with low-mode substitution

\[
24^3 \times 64 \text{ lattice with } m_\pi = 331 \text{ MeV, } a = 1.73 \text{ GeV}^{-1} \\
47 \text{ configurations}
\]

- Point source: \( m_N = 1.13(14) \text{ GeV} \)
- \( Z_3 \) grid source: \( m_N = 1.08(5) \text{ GeV} \)
- \( Z_3 \) grid smeared source: \( m_N = 1.14(2) \text{ GeV} \)
- Variation: \( m_N = 1.16(1) \text{ GeV} \)
Roper resonance from Coulomb wall source

\[ m_N = m_0 + c_1 m_\pi^2 + c_2 m_\pi^3 \text{(mixed)} + c_3 m_\pi^4 \ln\left(\frac{m_\pi^2}{\mu^2}\right) + \ldots \]

24^3 \times 64 \text{ lattice with } m_\pi = 331 \text{ MeV(sea)}, a = 1.73 \text{ GeV}^{-1}
$a^1 = 1.73\text{GeV}$, $m_a = 0.005$

Graph showing the relationship between $M_H$ (GeV) and $m_x^2$ (GeV$^2$) with various theoretical models and experimental data points.
Roper and Nucleon Wavefunctions at $m_\pi = 438$ MeV

$O_{RN} = 0.59$
Variation with 2 operators

(10 – operator 1, no smearing, 23 – operator 2, 3 smearing)

Variation with wall and point sources?

Hyperfine Interaction of Quarks in Baryons

- **Color-spin**
  \[ \lambda_1^C \cdot \lambda_2^C \rightarrow \sigma_1 \cdot \sigma_2 \]
- **Flavor-spin**
  \[ \lambda_1^F \cdot \lambda_2^F \rightarrow \sigma_1 \cdot \sigma_2 \]
- **One-gluon exchange**
- **Goldstone boson exchange**
Evidence of $\eta'N$ GHOST State in $S_{11} (1535)$ Channel
Dynamical Fermions

\[ \eta + \eta \]

\[ \eta \text{ and } \Pi \]
Quenched Vs Dynamical $N_{1/2}^1^-$ (1535) (Sommer scale)
Lattice N* states ($m_\pi = 396\,\text{MeV}$)

Dynamics of $P_{11}$-states:
The bare state at $\sim 1750\,\text{MeV}$ through coupling to inelastic channels generates 2 poles below $1400\,\text{MeV}$. They are identified with the “Roper” resonance.

Dynamics of the Roper-like states:
The “bare” quark state at $\sim 1750\,\text{MeV}$ through coupling to inelastic channels generates a pole near $1820\,\text{MeV}$ and two poles at $\sim 1360\,\text{MeV}$. The latter may be identified with the “Roper” resonance.

LQCD finds states as predicted in SU(6)xSU(3)

R. Edwards, J. Dudek, D. Richards,
S. Wallace, PRD84, 074508 (2011)

N. Suzuki et al. (JLab/EBAC),
Nature of Roper Resonance
--- current understanding

• Roper is the radial excitation of nucleon with large couplings to $N\eta$ and $N\pi$. The real part of the $N\eta$ and $N\pi$ loops pushes down the pole of radial excitation and the imaginary part gives the width of Roper, much the same way the $N\pi$ coupling changes the $\Delta$ mass and gives rise of its width.

• Issues with lattice calculations:
  - Variation vs Bayesian fitting: the size of the operators.
  - Chiral dynamics of the fermion action.
Is $1^{+}$ Meson a Hybrid?

Y. Yang, Y. Chen, KFL, arXiv:1202.2205

Exotics:

- Glueballs
- Hybrids $(q\bar{q}g)$
- Tetraquark mesonims $(q\bar{q}q\bar{q})$; pentaquark baryons $(q^4\bar{q})$

How to identify glueballs and hybrids in experiments and lattice calculations and distinguish them from ordinary $q\bar{q}$ mesons? exotic quantum numbers, e.g.

$J^{PC} = 0^{+-}, 1^{--}, 2^{++}$
Meson Interpolation Fields

- **dim 3**: \(\bar{\psi} \gamma \psi\), \(\bar{\psi} \Gamma \bar{D} \psi\), GG
- **dim 4**: \(\bar{\psi} \gamma \psi G\), \(\bar{\psi} \Gamma D D \psi\), GDG
- **dim 5**: \(\bar{\psi} \Gamma \psi G\), \(\bar{\psi} \Gamma D D \psi\), GDG
- **dim 6**: \(\bar{\psi} \Gamma \psi \bar{\psi} \Gamma \psi\), GGG, GDDG, etc

Dim 3 operators \(\bar{\psi} \gamma \psi\) do not generate exotic mesons with \(J^{PC} = 1^{--}, 0^{--}, 0^{+-}, 2^{+-}\) but they can be produced with dim 5 \(\bar{\psi} \Gamma \psi G\) ops. They are hybrids.
Experiments and Lattice Results

Expts on $1^{-+}$:
- $\pi_1(1400)$, $M=1376\pm17\text{ MeV}$, $\Gamma=300\pm40\text{ MeV}$
- $\pi_1(1600)$, $M=1653\pm17\text{ MeV}$, $\Gamma=225\pm38\text{ MeV}$ (?)

Lattice calculations (Quenched):
- UKQCD (1997): $2.0(2)\text{ GeV}$
- MILC (1997): $2.0(1)\text{ GeV}$, $2.1(1)\text{ GeV}$
- Lacock and Schilling (1998): $1.9(2)\text{ GeV}$
- MILC (2003): $\sim 1.6 \text{ GeV}$
- Adelaide (2004): $\sim 2.4 \text{ GeV}$; $\sim 1.6 \text{ GeV}$
- HSC (2010)
- J. Dudek (2011)
\[ \vec{J} = \vec{L} + \vec{S}, \quad P = (-)^{L+1}, \quad C = (-)^{L+S} \]

are non-relativistic definitions!

For example:

<table>
<thead>
<tr>
<th>( L )</th>
<th>( S )</th>
<th>( J )</th>
<th>Pauli Spinor</th>
<th>Dirac Spinor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>( \bar{\psi} \vec{\sigma} \cdot \vec{\Delta} \psi )</td>
<td>( \bar{\Psi} \Psi )</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>( \bar{\psi} \vec{\sigma} \times \vec{\Delta} \psi )</td>
<td>( \bar{\Psi} \gamma_i \gamma_5 \psi )</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>( \bar{\psi} \vec{\sigma} \otimes \vec{\Delta} \psi )</td>
<td>( \bar{\Psi} \gamma \otimes \vec{D} \psi )</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>( \bar{\psi} \vec{\Delta} \psi )</td>
<td>( \bar{\Psi} \sigma_{ij} \psi )</td>
</tr>
</tbody>
</table>

Lower component of Dirac spinor has a different parity from the upper one.

\[
\Psi_{\text{free}} \propto \begin{pmatrix}
1 \\
\vec{\sigma} \cdot \vec{p} \\
\frac{E + m}{E + m}
\end{pmatrix} \chi e^{ip \cdot x}
\]
1-+ Meson

Exotic:

- $\vec{J} = \vec{L} + \vec{S}$, $P = (-)^{L+1}$, $C = (-)^{L+S}$
- $P \rightarrow L = \text{even}$
- $J \rightarrow S = 1$
- $C = -$?
- Cannot be $q\bar{q}$ meson

Yet dim 4, $\Psi \gamma_{4} \bar{D}_{i} \Psi$ is 1-+ (B.A. Li, 1975)

- $P: \Psi(\bar{x}) \rightarrow \gamma_{4} \Psi(-\bar{x})$
- $C: \Psi \rightarrow \Psi$
- Exotic $q\bar{q}$?
- Note: there is no dim 3 op. for 2++

It is produced with $\Psi \gamma_{i} \bar{D}_{j} \Psi$. 
Content and Interpolation Field

- One cannot always judge the content of a hadronic state by its interpolation field. For example
  - the lowest state from $\bar{\Psi}\Psi$ is $\eta\pi$ and $\pi\pi$, not $a_0$ and $f_0$.
  - the lowest P-wave state from $\chi_N$ is $\pi N$ not $S_{11}$.
- The lowest pseudoscalars from $G\tilde{G}$ is $\eta$ and $\eta'$ not glueball.
  
  $$<0|G\tilde{G}|\eta>,<0|G\tilde{G}|\eta> \geq <0|G\tilde{G}|G>$$

- will need $$<0|\bar{\Psi}\gamma_5\Psi|\eta>,<0|\bar{\Psi}\gamma_5\Psi|\eta>> <0|\bar{\Psi}\gamma_5\Psi|G>$$
  to help decide which one is a glueball.

(Cheng, Li and Liu, PRD 79, 014024 (2009))
Criteria for a meson to be a hybrid:

Compare the matrix elements of both the dim 4 and dim 5 operators of $1^{-+}$ against other ordinary mesons, particularly the $2^{++}$

- Dim 4 m.e. $\langle 0 | \bar{\Psi} \gamma_4 \tilde{D}_i \Psi | 1^{-+} \rangle << \langle 0 | O_4 | 0^{-+},1^{--},0^{++},1^{\pm},2^{++} \rangle$

- Dim 5 m.e. $\langle 0 | \bar{\Psi} \varepsilon_{ijk} \gamma_j B_k \Psi | 1^{-+} \rangle >> \langle 0 | O_5 | 0^{-+},1^{--},0^{++},1^{\pm},2^{++} \rangle$
Dim 3, 4 (D-type) and dim 5 (B-type) operators

Table 1: Interpolation operators $\bar{\psi} \Gamma \psi$ (dimension 3, $\Gamma$-type), $\bar{\psi} \Gamma \times \vec{D} \psi$ (dimension 4, D-type), and $\bar{\psi} \Gamma \times B \psi$ (dimension 5, B-type). $\Sigma_i \equiv \frac{1}{2} \varepsilon_{ijk} \sigma_{jk}$ and repeated indices are summed over.

<table>
<thead>
<tr>
<th></th>
<th>$\Gamma$</th>
<th>$D$</th>
<th>$B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0--</td>
<td>$\gamma_5$</td>
<td>$\Sigma_i \vec{D}_i$</td>
<td>$\gamma_i B_i$</td>
</tr>
<tr>
<td>1--</td>
<td>$\gamma_i$</td>
<td>$\vec{D}_i$</td>
<td>$\gamma_5 B_i$</td>
</tr>
<tr>
<td>0++</td>
<td>$\mathbb{I}$</td>
<td>$\gamma_i \vec{D}_i$</td>
<td>$\Sigma_i B_i$</td>
</tr>
<tr>
<td>1++</td>
<td>$\gamma_5 \gamma_i$</td>
<td>$\varepsilon_{ijk} \gamma_j \vec{D}_k$</td>
<td>$\varepsilon_{ijk} \Sigma_j B_k$</td>
</tr>
<tr>
<td>1+--</td>
<td>$\Sigma_i$</td>
<td>$\gamma_5 \vec{D}_i$</td>
<td>$B_i$</td>
</tr>
<tr>
<td>2++</td>
<td>$</td>
<td>\varepsilon_{ijk}</td>
<td>\gamma_j \vec{D}_k$</td>
</tr>
<tr>
<td>1--</td>
<td>$\varepsilon_{ijk} \Sigma_j \vec{D}_k$</td>
<td>$\varepsilon_{ijk} \gamma_j B_k$</td>
<td></td>
</tr>
</tbody>
</table>
Non-relativistic Reduction
- why no such operators in quark model

Table 2: Non-relativistic form for the three kinds of operators (Γ, D and B) as shown in Table 1. Here we list the operators $O$ in the interpolation field $\chi^\dagger O \phi$. Repeated indices are summed over.

<table>
<thead>
<tr>
<th></th>
<th>$\Gamma$</th>
<th>$D$</th>
<th>$B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0--</td>
<td>$\mathbb{1}$</td>
<td>$\frac{1}{2m_c} \vec{D}_i \vec{D}_i$</td>
<td>$i\vec{\sigma}_i B_i$</td>
</tr>
<tr>
<td>1--</td>
<td>$\vec{\sigma}_i$</td>
<td>$\frac{1}{2m_c} \vec{D}_i \vec{D}_i \vec{D}_i$</td>
<td>$\vec{B}_i$</td>
</tr>
<tr>
<td>0++</td>
<td>$\frac{1}{2m_c} \vec{D}_i \vec{D}_i \vec{D}_i$</td>
<td>$\vec{\varepsilon}_{ijk} \vec{D}_j \vec{D}_k$</td>
<td>$\frac{1}{2m_c} (\varepsilon_{ijk} \vec{D}_j B_k + i \vec{\partial}_i (\vec{\sigma}_j B_j))$</td>
</tr>
<tr>
<td>1++</td>
<td>$\frac{1}{2m_c} \varepsilon_{ijk} \vec{D}_j \vec{D}_k$</td>
<td>$\frac{1}{2m_c} \vec{D}_i \vec{D}_i \vec{D}_i$</td>
<td>$\frac{1}{2m_c} (\varepsilon_{ijk} \vec{D}<em>j B_k + i \varepsilon</em>{jmn} \vec{\sigma}_m \partial_n (B_k))$</td>
</tr>
<tr>
<td>1--</td>
<td>$\frac{1}{2m_c} (\sigma \cdot \vec{D} \vec{D} + \vec{D} \cdot \vec{D})$</td>
<td>$\frac{1}{2m_c} (\vec{D}_i \vec{D}_j \vec{D}_i + \vec{D}_j \vec{D}_j \vec{D}_i)$</td>
<td>$\varepsilon_{ijk} \vec{\sigma}_j B_k$</td>
</tr>
</tbody>
</table>

Note: $\vec{D} + \vec{\bar{D}}$ is the $q \bar{q}$ center of mass momentum which is not a dynamical d.o.f. in the constituent quark model.
Charmoniums

Anisotropic $12^3 \times 96$ lattice with Wilson action, $\beta = 2.8$, $\zeta = 5$, $a_s = 0.138$ fm.
Table 2: Non-relativistic form for the three kinds of operators (Γ, D and B) as shown in Table 1. Here we list the operators $O$ in the interpolation field $\chi^\dagger O\phi$. Repeated indices are summed over.

<table>
<thead>
<tr>
<th>$\Gamma$</th>
<th>$D$</th>
<th>$B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0--</td>
<td>$\frac{1}{2m_c} \frac{\sigma_i}{\frac{1}{2m_c} D_i D_j D_l}$</td>
<td>$i\sigma_i B_i$</td>
</tr>
<tr>
<td>1--</td>
<td>$\frac{1}{2m_c} \frac{\sigma_j}{\frac{1}{2m_c} D_j D_l} D_i$</td>
<td>$\frac{1}{2m_c} D_i B_i$</td>
</tr>
<tr>
<td>0++</td>
<td>$\frac{1}{2m_c} \frac{\varepsilon_{ijk} \sigma_i D_j}{\frac{1}{2m_c} D_i}$</td>
<td>$\frac{1}{2m_c} [\varepsilon_{ijk} D_j B_k + \frac{i}{\hbar}(\sigma_j B_j)]$</td>
</tr>
<tr>
<td>1++</td>
<td>$\frac{1}{2m_c} \frac{</td>
<td>\varepsilon_{ijk} \sigma_j D_k</td>
</tr>
<tr>
<td>1--</td>
<td>$\frac{1}{2m_c} (\sigma \cdot \frac{D_i D_j}{D_j D_l} + \frac{D_j D_l}{D_i} \sigma \cdot \frac{D_l}{D_i})$</td>
<td>$\varepsilon_{ijk} \sigma_j B_k$</td>
</tr>
</tbody>
</table>

\[
\int d^3x \bar{\psi} \gamma_4 \gamma_i \frac{\vec{D}_i}{N.R.} \psi \rightarrow \int d^3x \frac{-\varepsilon_{ijk}}{m} \chi^\dagger \sigma_j B_k \phi;
\]

\[
\int d^3x \varepsilon_{ijk} \bar{\psi} \gamma_j \frac{\vec{D}_k}{N.R.} \psi \rightarrow \int d^3x \frac{\varepsilon_{ijk}}{m} \chi^\dagger \sigma_j B_k \phi
\]

Table 4: Matrix elements $<0|O_p|J^{PC}>$ for charmoniums.

<table>
<thead>
<tr>
<th>$\Gamma_p$</th>
<th>$D_p$</th>
<th>$B_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0--</td>
<td>$0.0697 \pm 0.0014$</td>
<td>$0.0503 \pm 0.0007$</td>
</tr>
<tr>
<td>1--</td>
<td>$0.0502 \pm 0.0005$</td>
<td>$0.0149 \pm 0.0001$</td>
</tr>
<tr>
<td>0++</td>
<td>$0.035 \pm 0.005$</td>
<td>$0.075 \pm 0.015$</td>
</tr>
<tr>
<td>1++</td>
<td>$0.020 \pm 0.003$</td>
<td>$0.062 \pm 0.005$</td>
</tr>
<tr>
<td>1--</td>
<td>$0.014 \pm 0.002$</td>
<td>$0.045 \pm 0.005$</td>
</tr>
<tr>
<td>2++</td>
<td>$0.044 \pm 0.003$</td>
<td>$0.00980 \pm 0.00008$</td>
</tr>
<tr>
<td>1++</td>
<td>$0.0059 \pm 0.0005$</td>
<td>$0.0082 \pm 0.0006$</td>
</tr>
</tbody>
</table>

\[
\times \frac{1}{\nu_{c\bar{c}}} \sim 0.027(3)
\]

\[
\times \frac{1}{\nu_{rel}} \sim 0.020(2)
\]

\[
\times \frac{1}{\nu_{c\bar{c}}} = \begin{cases} 0.020(2) \\ 0.018(1) \end{cases}
\]

\[
\times \frac{1}{\nu_{rel}} = 0.0058(4)
\]
0^{++}, 1^{++}, 1^{−−} : Γ m.e. $\approx \frac{1}{2m}$ D m.e.

1^{−+} : D m.e. $\approx \frac{1}{m}$ B m.e.

$\epsilon = \frac{1}{2m} \sim 0.352$

- Leading NR reduction is reasonably good.
- Mixing of operators with different dimensions seems to be small.
Strangeness mesons

D and B m.e. of $1^{-+}$ are comparable in size to those of other ordinary mesons.

No evidence for $1^{-+}$ in the charmonium and strangeonium regions to be hybrids.
Exotic Quantum Numbers

• NR reduction shows that $1^{-+}$ involves a center of mass motion of the $q\bar{q}$ pair.

• MIT bag model (Jaffe and Johnson, 1976; DeGrand and Jaffe, 1976)

$$\Psi(2^{++}) = \frac{1}{\sqrt{2}} (S_{1/2} \bar{P}_{3/2} \mp P_{3/2} \bar{S}_{1/2}) \rightarrow \text{cm motion for } 2^{++}$$

\[ \Rightarrow \text{Spectrum is doubled. Similarly (hamonic oscillator wf),} \]

$$\Psi(1^{-+}) = \frac{1}{\sqrt{2}} (1S_{1/2} 2\bar{S}_{1/2} \mp 2S_{1/2} 1\bar{S}_{1/2})$$

• The `exotic’ q.n. can be accommodated by

$$C = (-)^{l+s}$$

$$\bar{J} = \bar{L} + \bar{l} + \bar{s}$$

$$P = L + l + 1$$

$1^{-+} : L = l = 1, s = 1$
Conclusion

By examining m.e. of dim 4 and dim 5 operators of $1^{-+}$ against those of ordinary mesons, we find no evidence for it to be a hybrid in the $c\bar{c}$ and $s\bar{s}$ regions.

NR reduction shows that it involves a center of mass AM of the $q\bar{q}$ pair.

These `exotic' q.n. are accessible in chiral quark models, bag models, flux-tube models, and QCD.

To accommodate these q.n., the parity and AM rules need to be modified to

$$C = \left(-\right)^{l+s}, \quad \vec{J} = \vec{L} + \vec{l} + \vec{s}, \quad P = L + l + 1$$