Hadron interactions from lattice QCD

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1. Introduction
How can we extract hadronic interaction from lattice QCD?

**Phenomenological NN potential**

(~40 parameters to fit 5000 phase shift data)

1. 
   - One-pion exchange
     - Yukawa (1935)

2. 
   - Multi-pions
     - Jastrow (1951)
     - Taketani et al. (1951)

3. 
   - Repulsive core
     - Jastrow (1951)

Key features of the Nuclear force

Modern high precision NN forces (90's - )

![Graph showing the potential energy of nucleon-nucleon interaction with labels for different models and parameters.](image-url)
Nuclear force is a basis for understanding ...

- Structure of ordinary and hyper nuclei
- Ignition of Type II SuperNova
- Structure of neutron star

Can we extract a nuclear force in (lattice) QCD?
Plan of my talk

1. Introduction
2. Our strategy
3. Nuclear potential
4. Predictions: Hyperon interactions
5. Some applications to nuclear physics
6. Other recent developments
7. Future prospect
2. Our Strategy
Two strategies in lattice QCD

(Equal-time) Nambu-Bethe-Salpeter (NBS) Wave function

\[ \varphi_k(r) = \langle 0 | N(x + r, 0) N(x, 0) | NN, W_k \rangle \]

\[ W_k = 2 \sqrt{k^2 + m_N^2} \]

\[ N(x) = \varepsilon_{abc} q^a(x) q^b(x) q^c(x) \] is the local operator

Asymptotic region

\[ r = |r| \rightarrow \infty \]

Partial wave

\[ \varphi_k^l \rightarrow A_l \sin \left( k r - l \pi / 2 + \delta_l(k) \right) / k r \]

\[ \delta_l(k) \] is the scattering phase shift (phase of the S-matrix) in QCD!

How can we extract it?

cf. Maiani-Testa theorem

time correlation
Let us consider Two particles in the finite box

1st strategy: Luescher’s formula for phase shift

extract energy in the finite box: \[ E = \frac{k^2_n}{m_N} \]

Finite volume allowed value: \( k^2_n \)

\( k_n = \frac{2\pi}{L} n \)

cf. free theory
different from free theory

two particle energy \( E = \frac{k^2_n}{m_N} \)

different from free theory

\[ \delta_l (k_n) \]

Ex. \[ k \cot \delta_0 (k) = \frac{2}{\sqrt{\pi} L} Z_{00} (1; q^2) \]

\[ q = \frac{kL}{2\pi} \]

non-integer

\[ Z_{00} (s; q^2) = \frac{1}{\sqrt{4\pi}} \sum_{\mathbf{n} \in \mathbb{Z}^3} (\mathbf{n}^2 - q^2)^{-s} \]

generalize zeta-function
2nd strategy: HAL QCD method

Extract information inside the interaction range as

\[
[\epsilon_k - H_0] \varphi_k(x) = \int d^3y U(x, y) \varphi_k(y)
\]

\[
\epsilon_k = \frac{k^2}{2\mu}
\]

\[
H_0 = \frac{-\nabla^2}{2\mu}
\]

solve the Schrödinger Eq. in the infinite volume with this “potential”.

correct phase shifts (and binding energy) below inelastic threshold by construction

resonance

\[
W_k < W_{th} = 2m_N + m_\pi
\]

New method to extract phase shift from QCD
(by-pass Maiani-Testa theorem, using space correlation)

HAL QCD method
1. Potential itself is NOT an observable. Using this freedom, we can construct a non-local but energy-independent potential as

\[ U(x, y) = \sum_{k, k'} \left[ \epsilon_k - H_0 \right] \varphi_k(x) \eta_{k,k'}^{-1} \varphi_{k'}(y) \]

\[ \eta_{k,k'}^{-1} \text{: inverse of } \eta_{k,k'} = (\varphi_k, \varphi_{k'}) \text{ in space w/o zero modes} \]

For \( \forall W_p < W_{th} \)

\[ \int d^3y U(x, y) \phi_p(y) = \sum_{k, k'} \left[ \epsilon_k - H_0 \right] \varphi_k(x) \eta_{k,k'}^{-1} \eta_{k',p} = \left[ \epsilon_p - H_0 \right] \varphi_p(x) \]

Proof of existence (cf. Density Functional Theory)

Of course, potential satisfying this is not unique.

cf. Effective field theories and ChPT

\[ L = \frac{1}{f_\pi(m)^2} \text{tr} \partial^\mu U^\dagger \partial_\mu U + \cdots \]

QCD \rightarrow EFTs

We can make some parameter mass-independent.

\[ L = \frac{1}{f_0} \text{tr} \partial^\mu U^\dagger \partial_\mu U + \cdots \]

ChPT
2. In practice, we expand the non-local potential in terms of derivative: (cf: expansion in ChPT)

\[ U(x, y) = V(x, \nabla) \delta^3(x - y) \]

\[ V(x, \nabla) = V_0(r) + V_\sigma(r)(\sigma_1 \cdot \sigma_2) + V_T(r)S_{12} + V_{LS}(r)L \cdot S + O(\nabla^2) \]

The expansion agrees with the form of potential proposed by Okubo-Marshak (1958).

\[ V_A(x) \text{ local and energy independent coefficient function} \]

(cf. Low Energy Constants(LOC) in Chiral Perturbation Theory)

If we truncate the expansion, some systematic errors are introduced.

\[ \varphi_k(x) \rightarrow V_A(x) \rightarrow \delta(p) \text{ not exact} \]

\[ \varphi_p(x) \rightarrow \tilde{V}_A(x) \rightarrow \tilde{\delta}(p) \text{ exact} \]

(cf. Expansion in ChPT)

(N)LO calculation + observables \rightarrow LOCs \rightarrow predict other observables with errors.

It is difficult to estimate the convergence in ChPT.
3. (Scheme) Potential depends on the choice of $N(x)$. (cf: running coupling)

4. Non-relativistic approximation is **NOT** used. We just take the specific (equal-time) flame.

5. Potential $U(x, y)$ can be used at $\forall L$ and $\forall W_k < W_{th}$. *angular momentum*

6. The method can be extended to inelastic region.

Ex. 

$$NN \rightarrow NN, NN\pi$$

$$\begin{pmatrix}
U_{NN,NN}(x; y) & U_{NN,NN\pi}(x; y, w) \\
U_{NN\pi,NN}(x, z; y) & U_{NN\pi,NN\pi}(x, z; y, w)
\end{pmatrix}$$

$$\Lambda\Lambda \rightarrow \Lambda\Lambda, N\Xi$$

$$\begin{pmatrix}
U_{\Lambda\Lambda,\Lambda\Lambda}(x, y) & U_{\Lambda\Lambda,N\Xi}(x, y) \\
U_{N\Xi,\Lambda\Lambda}(x, y) & U_{N\Xi,N\Xi}(x, y)
\end{pmatrix}$$

In principle, we can treat all QCD processes.

QFT(QCD) at given energy. **coupled channel quantum mechanics.**
HAL QCD Collaboration

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Potentials from lattice QCD

Nuclear Physics with these potentials

Neutron stars Supernova explosion

Our strategy
3. Nuclear potential
Extraction of NBS wave function

\[ \varphi_k(r) = \langle 0 | N(x + r, 0) N(x, 0) | NN, W_k \rangle \]

NBS wave function

\[ [\epsilon_k - H_0] \varphi_k(x) = \int d^3 y U(x, y) \varphi_k(y) \]

Potential

4-pt Correlation function

source for NN

\[ F(r, t - t_0) = \langle 0 | T \{N(x + r, t) N(x, t)\} \overline{J}(t_0) | 0 \rangle \]

complete set

\[ F(r, t - t_0) = \langle 0 | T \{N(x + r, t) N(x, t)\} \sum_{n, s_1, s_2} |2N, W_n, s_1, s_2\rangle \langle 2N, W_n, s_1, s_2 | \overline{J}(t_0) | 0 \rangle \]

\[ = \sum_{n, s_1, s_2} A_{n, s_1, s_2} \varphi_W(r) e^{-W_n(t-t_0)}, \quad A_{n, s_1, s_2} = \langle 2N, W_n, s_1, s_2 | \overline{J}(0) | 0 \rangle. \]

ground state saturation at large \( t \)

\[ \lim_{(t-t_0) \to \infty} F(r, t - t_0) = A_0 \varphi_W(r) e^{-W_0(t-t_0)} + O(e^{-W_{\neq 0}(t-t_0)}) \]

NBS wave function

This is a standard method in lattice QCD and was employed for our first calculation.
Improved method

normalized 4-pt Correlation function

\[ R(r, t) \equiv F(r, t)/(e^{-m_N t})^2 = \sum_n A_n \varphi(W_n(r))e^{-\Delta W_n t} \]

\[ \Delta W_n = W_n - 2m_N = \frac{k_n^2}{m_N} - \frac{(\Delta W_n)^2}{4m_N} \]

\[-\frac{\partial}{\partial t} R(r, t) = \left\{ H_0 + U - \frac{1}{4m_N} \frac{\partial^2}{\partial t^2} \right\} R(r, t) \]

Leading Order

\{ -H_0 - \frac{\partial}{\partial t} + \frac{1}{4m_N} \frac{\partial^2}{\partial t^2} \} R(r, t) = \int d^3r' U(r, r')R(r', t) = V_C(r)R(r, t) + \cdots \]

3rd term (relativistic correction) is negligible.

Ground state saturation is no more required! (advantage over Luescher’s method.)
Another construction of energy-independent and non-local potential

generalized 4-pt Correlation function

\[ R(x, y, t) = \frac{1}{e^{-2m_N t}} \int d^3x_1 d^3y_1 \langle 0 | \mathcal{T} \{ N(x_1 + x, t) N(x_1, t) \bar{N}(y_1 + y, 0) \bar{N}(y_1, 0) \} | 0 \rangle \]

\[ R(x, y, t) = \int d^3z U(x, z) R(z, y, t) \]

\[ U(x, y) = \int d^3z \left( \frac{1}{4m_N} \frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} - H_0 \right) R(x, z, t) \cdot \tilde{R}^{-1}(z, y, t) \]

truncated “inverse”

\[ R^{-1}(z, y, t) = \sum_{\lambda_n(t) \neq 0} \frac{1}{\lambda_n(t)} v_n(x, t) v_n^\dagger(y, t) \]

\( \lambda_n(t), v_n(x, t) \): eigenvalue and eigenfunction of hermitian operator \( R(x, y, t) \)

Remark
NN potential

2+1 flavor QCD, spin-singlet potential (in preparation)

\[ a = 0.09 \text{fm}, L = 2.9 \text{fm} \quad m_\pi \simeq 700 \text{ MeV} \]

Qualitative features of NN potential are reproduced!

(1) attractions at medium and long distances
(2) repulsion at short distance (repulsive core)


This paper has been selected as one of 21 papers in Nature Research Highlights 2007.
It has a reasonable shape. The strength is weaker due to the heavier quark mass. Need calculations at physical quark mass.
Convergence of velocity expansion

If the higher order terms are large, LO potentials determined from NBS wave functions at different energy become different (cf. LOC of ChPT).

Numerical check in quenched QCD

\[ m_\pi \simeq 0.53 \text{ GeV} \]

\[ a = 0.137 \text{ fm} \]

K. Murano, N. Ishii, S. Aoki, T. Hatsuda

PTP 125 (2011)1225.

- PBC (E~0 MeV)
- APBC (E~46 MeV)
Higher order terms turn out to be very small at low energy in HAL scheme.

Need to be checked at lighter pion mass in 2+1 flavor QCD.

Note: convergence of the velocity expansion can be checked within this method.

(cf. convergence of ChPT, convergence of perturbative QCD)
4. Predictions: Hyperon interactions

\[ p = (uud), n = (udd) \]
\[ \Lambda = (uds)_{I=0} \]
\[ \Sigma^+ = (uus), \Sigma^0 = (uds)_{I=1}, \Sigma^- = (dds) \]
\[ \Xi^0 = (uss), \Xi^- = (dss) \]
Octet Baryon interactions

- no phase shift available for YN and YY scattering
- plenty of hyper-nucleus data will be soon available at J-PARC

• prediction from lattice QCD
• difference between NN and YN ?
Baryon Potentials in the flavor SU(3) symmetric limit

1. First setup to predict YN, YY interactions not accessible in exp.
2. Origin of the repulsive core (universal or not)

\[ 8 \times 8 = 27 + 8s + 1 + 10^* + 10 + 8a \]

Symmetric \hspace{2cm} Anti-symmetric

6 independent potentials in flavor-basis

\[ V^{(27)}(r), \ V^{(8s)}(r), \ V^{(1)}(r) \]
\[ V^{(10^*)}(r), \ V^{(10)}(r), \ V^{(8a)}(r) \]

3-flavor QCD \hspace{2cm} a=0.12 \text{ fm}

Inoue et al. (HAL QCD Coll.), PTP124(2010)591 \hspace{2cm} L=2 \text{ fm}

Inoue et al. (HAL QCD Coll.), arXiv:1112.5926[hep-lat] \hspace{2cm} L=2-4 \text{ fm}
\[ L \approx 4 \text{ fm}, \quad m_\pi \approx 470 \text{ MeV} \]

\[ V^{(27)} \]

same as NN

\[ V^{(8s)} \]

8s: strong repulsive core. repulsion only.

\[ V^{(1)} \]

1: attractive instead of repulsive core! attraction only.

\[ V^{(10^s)} \]

same as NN

\[ V^{(10)} \]

10: strong repulsive core. weak attraction.

\[ V^{(8a)} \]

8a: weak repulsive core. strong attraction.

\[ L \approx 4 \text{ fm}, \quad m_\pi \approx 470 \text{ MeV} \]

Flavor dependences of BB interactions become manifest in SU(3) limit!
H-dibaryon:
a possible six quark state (uuddss)
predicted by the model but not observed yet.


Binding baryons on the lattice

April 26, 2011
H-dibaryon in the flavor SU(3) symmetric limit

Attractive potential in the flavor singlet channel

possibility of a bound state (H-dibaryon)

\[ \Lambda\Lambda - N\Xi - \Sigma\Sigma \]

Inoue et al. (HAL QCD Coll.), PRL106(2011)162002

volume dependence

pion mass dependence

L=3 fm is enough for the potential.

lighter the pion mass, stronger the attraction

fit potentials at L=4 fm by

\[ V(r) = a_1 e^{-a_2 r^2} + a_3 \left( 1 - e^{-a_4 r^2} \right)^2 \left( \frac{e^{-a_5 r}}{r} \right)^2 \]
An $H$-dibaryon exists in the flavor SU(3) limit. Binding energy = 25-50 MeV at this range of quark mass. A mild quark mass dependence.

Real world?
5. Some applications to nuclear physics
H-dibaryon with the flavor SU(3) breaking

\[ m_u = m_d \neq m_s \]

SU(3) limit

Real world

\[ \Sigma \Sigma \]

2386 MeV

129 MeV

2257 MeV

25 MeV

2232 MeV

25-50 MeV

\[ \Lambda \Lambda - N \Xi - \Sigma \Sigma \]

Our approximation for SU(3) breaking

1. Linear interpolation of octet baryon masses

\[ M_Y(x) = (1 - x)M_Y^{SU(3)} + xM_Y^{Phys} \]

2. Potentials in particle basis in SU(3) limit

\[ m_\pi \simeq 470 \text{ MeV} \quad m_\pi \simeq 135 \text{ MeV} \]
Potentials in particle basis in SU(3) limit (S=-2, I=0) \( m_\pi \simeq 470 \text{ MeV} \)

\[ V(r) \text{ [MeV]} \]

\[ r \text{ [fm]} \]

This part needs to be improved.

The direct calculation of potentials in 2+1 flavor QCD is in progress.

K. Sasaki et al. (HAL QCD Coll.), Lat 2012
Energy eigenvalues

complex scaling method

Inoue et al. (HAL QCD Coll.), arXiv:1112.5926[hep-lat]

H-dibaryon seems to become resonance at physical point.

This needs a direct confirmation by 2+1 flavor QCD.

H-bound state from ΛΛ

ΛΛ resonance from ΛΛ

NΞ bound state from ΛΛ

H-resonance from ΛΛ
$H$ couples most strongly $N\Xi$.  
$\Lambda\Lambda$ interaction is attractive.  
$H$ has a sizable coupling to $\Lambda\Lambda$ near and above the threshold.
Invariant mass spectrum $\Lambda\Lambda \rightarrow \Lambda\Lambda$

Inoue et al. (HAL QCD Coll.), arXiv:1112.5926[hep-lat]

$x=0$

$x=0.2$

$x=0.4$

$x=0.6$

$x=0.8$

$x=1$

A peak of the resonance H might be observed in experiments !?
Other observables in the flavor SU(3) limit

**NN Phase shift, deuteron and 4N state**

Inoue et al. (HAL QCD Coll.), arXiv:1112.5926[hep-lat]

**NN $^1S_0$**
- $m_{PS} = 1171$ [MeV] (red)
- $m_{PS} = 1015$ [MeV] (green)
- $m_{PS} = 837$ [MeV] (blue)
- $m_{PS} = 672$ [MeV] (cyan)
- $m_{PS} = 469$ [MeV] (pink)

**NN $^3S_1^−3D_1$**
- Attraction is stronger in triplet, but no deuteron so far.
- Also, no 3N state.

**Binding energy by variational method**

$^4\text{He} \quad (L, S)J^P = (0, 0)0^+$

A 4N bound state exists at lightest pion mass.

$m_\pi = 470$ MeV

$E_{4N} = -5.1$ MeV
EoS of nuclear matter

Inoue et al. (HAL QCD Coll.), in preparation

**NN potentials** $m_\pi = 470$ MeV

Energy density of Nuclear matter

Neutron matter

Nuclear matter shows the saturation at the lightest pion mass, but the saturation point deviates from the empirical one obtained by Weizsacker mass formula.

No saturation for Neutron matter.
Pressure of Neutron matter

Our Neutron matter becomes harder as the pion mass decreases, but it is still softer than phenomenological models.
6. Other recent developments
Parity-odd potential and LS force

Murano et al. (HAL QCD), lat2012

2-flavor QCD, a=0.16 fm

\( m_\pi \simeq 1.1 \text{ GeV} \)

LO

\[ \mathcal{V}_C \]

from \( ^3P_0 \)

LO

\[ \mathcal{V}_T \]

from \( ^3P_1 \)

NLO

\[ \mathcal{V}_{LS} \]

from \( ^3P_2 + ^3F_2 \)

We now have all potentials at LO.

Very weak!
(1,2) pair \( \frac{1}{2} S_0, \frac{3}{2} S_1, \frac{1}{2} D_1 \) S-wave only

\[ \psi = \frac{1}{\sqrt{2}} (\psi_0 + \psi_1) \]

\[ \psi = \psi_0 \]

\[ \psi = \psi_1 \]

Triton \( (I = 1/2, J^P = 1/2^+) \)

\[ \mathcal{V}_{\text{TNF}} \] scalar/isoscalar TNF is observed at short distance.

further study is needed to confirm this result.

Analysis by OPE (operator product expansion) in QCD predicts universal short distance repulsions in TNF.

Aoki, Balog and Weisz, NJP14(2012)043046
7. Future prospect

- HAL QCD scheme is shown to be a promising method to extract hadronic interactions in lattice QCD.
  - ground state saturation is not required.
  - Calculate potential (matrix) in lattice QCD on a finite box.
  - Calculate phase shift by solving (coupled channel) Shroedinger equation in infinite volume.
- bound/resonance/scattering

- Future directions
  - calculations at the physical pion mass on “K-computer”
  - hyperon interactions with the SU(3) breaking
  - Baryon-Meson, Meson-Meson
  - Exotic other than H such as penta-quark, X, Y etc.
  - 3 Nucleon forces
  - Other applications? (weak interaction?)

Please join us if you are interested in.