Chiral effective field theory constraints for the equation of state and for supernova neutrino rates

Achim Schwenk

INT program on core-collapse supernovae

Seattle, July 3, 2012
Main points

Advances in nuclear forces and nuclear matter theory

Impact on neutron stars:

provides strong constraints for the equation of state

neutron star radius 9.9-13.8 km for M=1.4 $M_{\text{sun}}$ ($\pm 15\%$)


Impact on neutrino-matter interactions

S. Bacca, K. Hally, C.J. Pethick, AS, PRC 80, 032802(R) (2009) and
Chiral Effective Field Theory for nuclear forces

Separation of scales: low momenta \( \frac{1}{\lambda} = Q \ll \Lambda_b \) breakdown scale \( \sim 500 \text{ MeV} \)

limited resolution at low energies, can expand in powers \( (Q/\Lambda_b)^n \)

LO, \( n=0 \) - leading order,
NLO, \( n=2 \) - next-to-leading order, …

expansion parameter \( \sim 1/3 \)

(compare to multipole expansion for a charge distribution)

Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Kaiser, Machleidt, Meissner, …
Chiral Effective Field Theory for nuclear forces

Separation of scales: low momenta $\frac{1}{\lambda} = Q \ll \Lambda_b$ breakdown scale $\sim 500$ MeV

- Include long-range pion physics
- Few short-range couplings, fit to experiment once

Systematic: can work to desired accuracy and obtain error estimates

Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Kaiser, Machleidt, Meissner,…
Chiral Effective Field Theory and many-body forces

Separation of scales: low momenta $\frac{1}{\Lambda} = Q \ll \Lambda_b$ breakdown scale $\sim 500$ MeV

consistent $NN$-$3N$ interactions

$3N, 4N$: only 2 new couplings to $N^3LO$

long-range $3N$: $c_i$ from $\pi N$ and NN

$\begin{align*}
c_1 &= -0.9^{+0.2}_{-0.5} , \\
c_3 &= -4.7^{+1.2}_{-1.0} , \\
c_4 &= 3.5^{+0.5}_{-0.2} \end{align*}$

3- and 4-neutron forces are predicted to $N^3LO$ ($c_{D,E}$ don’t contribute)

Hebeler, AS (2010)

Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Kaiser, Machleidt, Meissner,…
new $^{51,52}$Ca TITAN measurements

$^{52}$Ca is 1.75 MeV more bound compared to atomic mass evaluation

behavior of two-neutron separation energy $S_{2n}$ and odd-even staggering $\Delta_n$ agrees with NN+3N predictions
Impact of 3N forces on nuclear matter

chiral 3N forces fit to light nuclei predict nuclear matter saturation with theoretical uncertainties

Hebeler et al. (2011), Bogner et al. (2005)
neutron matter is a simpler system, only long-range parts of 3N forces contribute ($c_1$ and $c_3$).

Hebeler, AS (2010)
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neutron matter uncertainties
dominated by 3N forces ($c_3$ coupling)
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Impact of 3N forces on neutron matter

neutron matter uncertainties dominated by 3N forces (c\textsubscript{3} coupling)
Hebeler, AS (2010)

other microscopic calculations within band (but without uncertainties)

Gezerlis, Carlson (2009)
Neutron skin of $^{208}\text{Pb}$ probes neutron matter energy/pressure, neutron matter band predicts neutron skin of $^{208}\text{Pb}$: $0.17 \pm 0.03$ fm

Hebeler et al. (2010)
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Hebeler et al. (2010) in excellent agreement with extraction from complete E1 response $0.156\pm0.025-0.021$ fm

**PREX:** neutron skin from parity-violating electron-scattering at JLAB electron exchanges Z-boson, couples preferentially to neutrons

goal II: $\pm0.06$ fm
Symmetry energy and pressure of neutron matter band predicts symmetry energy $S_v$ and its density dependence $L$

comparison to experimental and observational constraints
Lattimer, Lim (2012)

neutron matter constraints
H: Hebeler et al. (2010) and in prep.
G: Gandolfi et al. (2011)

predicts correlation but not range of $S_v$ and $L$
Symmetry energy and pressure of neutron matter

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lower limit on crust-core transition density
Chiral Effective Field Theory for nuclear forces

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<table>
<thead>
<tr>
<th></th>
<th>NN</th>
<th>3N</th>
<th>4N</th>
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<tr>
<td>LO</td>
<td>( \mathcal{O} \left( \frac{Q^6}{\Lambda^6} \right) )</td>
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Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Kaiser, Machleidt, Meissner,…
Complete N$^3$LO calculation of neutron matter

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first complete $N^3$LO result

no RG evolution necessary, includes uncertainties from bare NN, 3N, 4N

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N$^3$LO correlation broader because more density dependences

direct measurement of neutron star mass from increase in signal travel time near companion J1614-2230 most edge-on binary pulsar known (89.17°) + massive white dwarf companion (0.5 $M_{\text{sun}}$) heaviest neutron star with 1.97±0.04 $M_{\text{sun}}$
Equation of state/pressure for neutron-star matter (includes small $Y_{e,p}$).

Pressure below nuclear densities agrees with standard crust equation of state only after 3N forces are included.
Impact on neutron stars \cite{Hebeler2010} and in prep.

Equation of state/pressure for \textbf{neutron-star matter} (includes small $Y_{e,p}$)

\begin{itemize}
\item pressure below nuclear densities agrees with standard crust equation of state only after 3N forces are included
\item extend uncertainty band to higher densities using piecewise polytropes
\item allow for soft regions
\end{itemize}
Pressure of neutron star matter constrain polytropes by causality and require to support $1.97 \, M_{\odot}$ star.

Low-density pressure sets scale, chiral EFT interactions provide strong constraints, ruling out many model equations of state.
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Central densities for $1.4 \, M_{\odot}$ star: $1.7-4.4 \, \rho_0$
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darker blue band for $2.4\,M_{\text{sun}}$ star
Neutron star radius constraints

uncertainty from many-body forces and general extrapolation

constrains neutron star radius: 9.9-13.8 km for $M=1.4 \ M_{\text{sun}}$ ($\pm 15\%$ !)

consistent with extraction from X-ray burst sources Steiner et al. (2010)

provides important constraints for EOS for core-collapse supernovae
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Neutrino rates – Motivation

Neutrino rates important for neutron star crust and core cooling, supernova explosions, neutrino spectra,…

processes involving two nucleons play a special role Friman,… Suzuki, Raffelt,…

neutrino-pair bremsstrahlung and absorption $NN \leftrightarrow NN\nu\bar{\nu}$

standard cooling of low-mass neutron stars

key for production of muon and tau neutrinos in supernovae

and for equilibrating neutrino number densities

at subnuclear densities $\rho < 10^{14} \text{ g cm}^{-3}$ no systematic calculations beyond one-pion exchange (OPE) approximation for nuclear interactions

can calculate systematically using chiral effective field theory, electroweak interactions, many-body theory
Many-body theory: single- and two-nucleon processes

elastic scattering from nucleons (space-like $\omega < q$)

initial and final state interactions, inelastic scattering $\nu nn \leftrightarrow \nu nn$

collisional damping - Landau-Pomeranchuk-Migdal effect

neutrino-pair bremsstrahlung/absorption $nn \leftrightarrow nn\nu\Bar{\nu}$ (time-like $\omega > q$)

need collisions between nucleons for the latter processes

noncentral contributions, due to tensor forces from pion exchanges and spin-orbit forces, are essential for the two-neutron response

follows from direct calculations Friman, Maxwell (1979)

and from conservation laws Olsson, Pethick (2002)

developed a unified treatment that consistently includes one- and two-nucleon response in a strongly-interacting many-body system (Boltzmann eqn for collisions, spin-dependent mean-field effects, ...
Neutrino processes and dynamical structure factors

neutrinos interact weakly \( \rightarrow \) rates for neutrino scattering, emission and absorption determined by dynamical structure factors of nucleon matter
generally axial/spin response most important, \( \sim \) factor 3

neutrino rates \( \Gamma(\omega, \mathbf{q}) = 2\pi n G_F^2 C_A^2 (3 - \cos \theta) S_A(\omega, \mathbf{q}) \)

with spin dynamical structure factor \( S_A(\omega, \mathbf{q}) = \frac{1}{\pi n} \frac{1}{1 - e^{-\omega/T}} \text{Im} \chi_\sigma(\omega, \mathbf{q}) \)

energy, momentum transfers \( \omega, \mathbf{q} \) small compared with Fermi momentum

for low temperatures \( \sim \) Fermi temperature or less \( \rightarrow \) Landau Fermi liquid theory is a reasonable first approximation

problem is to calculate structure factors of nucleon matter

not included: reduction of axial coupling \( g_a \) for nucleon quasiparticles by 5-10% in neutron matter \( \text{Cowell, Pandharipande (2003)} \)

beyond quasiparticle contributions (incoherent parts)
Unified approach to structure factors

solve linearized quasiparticle transport equation for the spin response

\[(\omega - \varepsilon_{p+q/2} + \varepsilon_{p-q/2}) \delta s_p + (n_{p+q/2} - n_{p-q/2}) \delta h_p = i I_\sigma[s_{p'}]\]

includes one-pair states through perturbation of the quasiparticle energy, spin-dependent mean-field

\[\delta h_p = U_\sigma + 2 \int \frac{dp'}{(2\pi)^3} g_{pp'} \delta s_{p'}\]

two-nucleon contributions through collision term \(I_\sigma[s_{p'}] = -\frac{\delta s_p - \delta s_{p'}}{\tau_\sigma}\)
in relaxation time approximation

spin relaxation rate \(\frac{1}{\tau_\sigma} = C_\sigma [T^2 + (\omega/2\pi)^2]\)

from quasiparticle scattering amplitude averaged over the Fermi surface

solution to qp transport equation includes multiple-scattering effects, Landau-Pomeranchuk-Migdal effect
Spin dynamical structure factor

solution to quasiparticle transport equation \( \text{Im} \chi_{\sigma} = N(0) \frac{\text{Im} \tilde{X}_{\sigma}}{|1 + G_0 \tilde{X}_{\sigma}|^2} \)

isotropic Landau interaction \( G_0 \) dominates in neutron matter

in relaxation time approximation \( \tilde{X}_{\sigma} = 1 - \frac{\omega}{2v_F q} \ln \left( \frac{\omega + i/\tau_{\sigma} + v_F q}{\omega + i/\tau_{\sigma} - v_F q} \right) \)
Neutrino rates from chiral EFT S. Bacca et al. (2009)

neutrino rates in 2N processes determined by spin relaxation time
= rate of change of nucleon spin through collisions with other nucleons

shorter-range interactions significantly reduce neutrino rates (compared to OPE) in neutron matter for all relevant densities

first calculation of neutrino processes in dense matter from chiral EFT
Chiral Effective Field Theory for nuclear forces

Separation of scales: low momenta \( \frac{1}{\Lambda} = Q < \Lambda_b \) breakdown scale \( \sim 500 \text{ MeV} \)

- \( \mathcal{O} \left( \frac{Q^0}{\Lambda^0} \right) \)
- \( \mathcal{O} \left( \frac{Q^2}{\Lambda^2} \right) \)
- \( \mathcal{O} \left( \frac{Q^3}{\Lambda^3} \right) \)
- \( \mathcal{O} \left( \frac{Q^4}{\Lambda^4} \right) \)

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systematic: can work to desired accuracy and obtain \textbf{error estimates}

Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Kaiser, Machleidt, Meissner,…
Relevant conditions in core-collapse supernovae

15 M⊙ progenitor

S. Bacca et al., arXiv:1112.5185
simulations by M. Liebendörfer et al.

partially degenerate

neutron-rich

crucial densities below nuclear matter density \( \sim 10^{13}-10^{14} \text{ g/cm}^3 \)
(high densities: neutrinos trap; low densities: few interactions)
Neutrino rates from chiral EFT

S. Bacca et al., arXiv:1112.5185

neutrons only, arbitrary degeneracy

similar reduction along SN conditions

towards chiral EFT rates in supernova simulations

with A. Bartl et al.
Energy transfer in neutrino scattering from nucleons

mean-square neutrino energy transfer in $\nu nn \leftrightarrow \nu nn$

$$\langle \Delta E \rangle^2 = \frac{\int d\mathbf{p}'_\nu (E_\nu - E'_\nu)^2 \Gamma(E_\nu - E'_\nu, p_\nu - p'_\nu)}{\int d\mathbf{p}'_\nu \Gamma(E_\nu - E'_\nu, p_\nu - p'_\nu)}$$

leads to heating,

NN analogue of inelastic excitations of nuclei (but post-collapse)

energy transfer significant, dominates over recoil effects

not included in simulations
Main points and summary

Chiral EFT interactions provide strong constraints for EOS, 3N forces are a frontier for neutron-rich nuclei/matter

dominant uncertainty of neutron (star) matter below nuclear densities also key to explain neutron-rich nuclei

neutron star radius 9.9-13.8 km for M=1.4 $M_{\text{sun}}$ (±15%) towards chiral EFT rates in supernova simulations, will provide simple structure factors to use