New Developments in Collective Neutrino Oscillations in Supernovae

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Three-Flavor Neutrino Parameters

Three mixing angles $\theta_{12}, \theta_{13}, \theta_{23}$ (Euler angles for 3D rotation), $c_{ij} = \cos \theta_{ij}$, a CP-violating “Dirac phase” $\delta$, and two “Majorana phases” $\alpha_2$ and $\alpha_3$

\[
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix} =
\begin{pmatrix}
1 & 0 & 0 \\
0 & c_{23} & s_{23} \\
0 & -s_{23} & -c_{23}
\end{pmatrix}
\begin{pmatrix}
\begin{pmatrix}
c_{13} & 0 & e^{-i\delta} s_{13}
\end{pmatrix} \\
0 & 1 & 0 \\
-e^{i\delta} s_{13} & 0 & c_{13}
\end{pmatrix}
\begin{pmatrix}
\begin{pmatrix}
c_{12} & s_{12} & 0
\end{pmatrix} \\
-s_{12} & c_{12} & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
1 & 0 & 0 \\
e^{i\alpha_2/2} & 0 & 0 \\
0 & e^{i\alpha_3/2} & 0
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix}
\]

39° $< \theta_{23} < 53°$
Atmospheric/LBL-Beams

7° $< \theta_{13} < 11°$
Reactor

33° $< \theta_{12} < 37°$
Solar/KamLAND

Relevant for 0ν2β decay

Normal

3 $\mu$ $\tau$

2 $e$ $\mu$ $\tau$

Sun

1 $e$ $\mu$ $\tau$

Atmosphere

Inverted

$\Delta m^2$

72–80 meV²

2180–2640 meV²

Tasks and Open Questions

- Precision for all angles
- CP-violating phase $\delta$?
- Mass ordering?
  (normal vs inverted)
- Absolute masses?
  (hierarchical vs degenerate)
- Dirac or Majorana?
Neutrino oscillations in matter

L. Wolfenstein

Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213
(Received 6 October 1977; revised manuscript received 5 December 1977)

The effect of coherent forward scattering must be taken into account when considering the oscillations of neutrinos traveling through matter. In particular, for the case of massless neutrinos for which vacuum oscillations cannot occur, oscillations can occur in matter if the neutral current has an off-diagonal piece connecting different neutrino types. Applications discussed are solar neutrinos and a proposed experiment involving transmission of neutrinos through 1000 km of rock.

Neutrinos in a medium suffer flavor-dependent refraction

\[ V_{\text{weak}} = \sqrt{2} G_F \times \begin{cases} N_e - N_n/2 & \text{for } \nu_e \\ -N_n/2 & \text{for } \nu_\mu \end{cases} \]

Typical density of Earth: 5 g/cm\(^3\)

\[ \Delta V_{\text{weak}} \approx 2 \times 10^{-13} \text{ eV} = 0.2 \text{ peV} \]
Flavor eigenstates are propagation eigenstates

Neutrino flux

Neutrino sphere

MSW region
Three-Flavor Eigenvalue Diagram

Dighe & Smirnov, Identifying the neutrino mass spectrum from a supernova neutrino burst, astro-ph/9907423
### Signature of Flavor Oscillations

<table>
<thead>
<tr>
<th>1-3-mixing scenarios</th>
<th>A</th>
<th>B</th>
<th>C</th>
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<tr>
<td>$\sin^2 \theta_{13}$</td>
<td>$\gtrsim 10^{-3}$</td>
<td></td>
<td>$\lesssim 10^{-5}$</td>
</tr>
<tr>
<td>MSW conversion</td>
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<td>$\bar{\nu}_e$ Earth effects</td>
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May distinguish mass ordering

Assuming collective effects are not important during accretion phase
(Chakraborty et al., arXiv:1105.1130, Sarikas et al. arXiv:1109.3601)
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May distinguish mass ordering

Assuming collective effects are not important during accretion phase (Chakraborty et al., arXiv:1105.1130, Sarikas et al. arXiv:1109.3601)
Neutrino-neutrino refraction causes a flavor instability, flavor exchange between different parts of spectrum.

Flavor eigenstates are propagation eigenstates.

Neutrino flux

Neutrino sphere

MSW region
Flavor-Off-Diagonal Refractive Index

2-flavor neutrino evolution as an effective 2-level problem

\[ i \frac{\partial}{\partial t} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = H \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} \]

Effective mixing Hamiltonian

\[ H = \frac{M^2}{2E} + \sqrt{2}G_F \begin{pmatrix} N_e - \frac{N_n}{2} & 0 \\ 0 & -\frac{N_n}{2} \end{pmatrix} + \sqrt{2}G_F \begin{pmatrix} N_{\nu_e} & N_{\langle \nu_e | \nu_\mu \rangle} \\ N_{\langle \nu_\mu | \nu_e \rangle} & N_{\nu_\mu} \end{pmatrix} \]

Mass term in flavor basis: causes vacuum oscillations

Wolfenstein’s weak potential, causes MSW “resonant” conversion together with vacuum term

Flavor-off-diagonal potential, caused by flavor oscillations.
(J.Pantaleone, PLB 287:128, 1992)

Flavor oscillations feed back on the Hamiltonian: Nonlinear effects!
Collective Supernova Nu Oscillations since 2006

Two seminal papers in 2006 triggered a torrent of activities

Duan, Fuller, Qian, astro-ph/0511275, Duan et al. astro-ph/0606616

Initial fluxes at neutrino sphere

After collective transformation


Explanations in Raffelt & Smirnov arXiv:0705.1830 and 0709.4641
Duan, Fuller, Carlson & Qian arXiv:0706.4293 and 0707.0290
Three Ways to Describe Flavor Oscillations

Schrödinger equation in terms of “flavor spinor”

\[ i \partial_t \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = H \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \frac{\Delta m^2}{2E} \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} \]

Neutrino flavor density matrix

\[ \rho = \begin{pmatrix} \langle \nu_e | \nu_e \rangle & \langle \nu_e | \nu_\mu \rangle \\ \langle \nu_\mu | \nu_e \rangle & \langle \nu_\mu | \nu_\mu \rangle \end{pmatrix} \]

Equivalent commutator form of Schrödinger equation

\[ i \partial_t \rho = [H, \rho] \]

Expand 2×2 Hermitean matrices in terms of Pauli matrices

\[ \rho = \frac{1}{2} [\text{Tr}(\rho) + \mathbf{P} \cdot \mathbf{\sigma}] \quad \text{and} \quad H = \frac{\Delta m^2}{2E} \mathbf{B} \cdot \mathbf{\sigma} \quad \text{with} \quad \mathbf{B} = (\sin 2\theta, 0, \cos 2\theta) \]

Equivalent spin-precession form of equation of motion

\[ \dot{\mathbf{P}} = \omega \mathbf{B} \times \mathbf{P} \quad \text{with} \quad \omega = \frac{\Delta m^2}{2E} \]

\( \mathbf{P} \) is “polarization vector” or “Bloch vector” or “flavor isospin vector”
Flavor Oscillation as Spin Precession

Flavor direction

↑ Spin up $\nu_e$
↓ Spin down $\nu_\mu$

Mass direction

$2\theta$

Twice the vacuum mixing angle

Flavor polarization vector precesses around the mass direction with frequency $\omega = \Delta m^2 / 2E$
Collective Nu Oscillations as a Many-Body Problem

Hamiltonian for interacting “flavor spins” (classical in mean-field approach)

\[ H = \sum_{i=1}^{N} \omega_i \mathbf{B} \cdot \mathbf{P}_i + \sqrt{2} G_F N_e \mathbf{L} \cdot \sum_{i=1}^{N} \mathbf{P}_i + \mu \sum_{i,j=1}^{N} (1 - \cos \theta_{ij}) \mathbf{P}_i \cdot \mathbf{P}_j \]

Unit vector in mass direction  
Unit vector in flavor direction  
Multi-angle effects from current-current structure

“Spin-pairing H” for isotropic system (or single angle), ignoring matter effect

\[ H = \sum_{i=1}^{N} \omega_i \mathbf{B} \cdot \mathbf{P}_i + \mu P_{\text{tot}}^2 \]

BCS theory (using Anderson’s pseudo-spin), nuclear physics, ... 
Integrable system (as many “Gaudin invariants” as spins) 
→ Pehlivan, Balantekin, Kajino & Yoshida [arxiv:1105.1182] for introduction

N-mode coherent solutions ("Normal and anomalous solitons")

  Super-conductivity (BCS)

• Georg Raffelt, Phys. Rev. D 83, 105022 (2011)  
  Collective Nus
Synchronized Flavor Oscillations

Precession equation for each $\nu$ mode with energy $E$, i.e. $\omega = \Delta m^2 / 2E$

$$\dot{P}_\omega = \left( \omega B + \lambda L + \mu P \right) \times P_\omega \quad \text{with} \quad \lambda = \sqrt{2}G_FN_e \quad \text{and} \quad \mu = \sqrt{2}G_FN_\nu$$

Total flavor spin of entire ensemble

$$P = \sum_\omega P_\omega \quad \text{normalize} \quad |P_{t=0}| = 1$$

Individual spins do not remain aligned – feel "internal" field $H_{\nu\nu} = \mu P$

Synchronized oscillations for large neutrino density $\mu \gg \delta\omega$

$P$ precesses with $\omega_{\text{sync}}$ for large $\nu$ density

Individual $P_\omega$ "trapped" on precession cones

Precess around $P$ with frequency $\sim \mu$
Instability in Flavor Space

Two-mode example in co-rotating frame, initially $P_1 = \downarrow$, $P_2 = \uparrow$ (flavor basis)

$$\dot{P}_1 = [-\omega B + \mu (P_1 + P_2)] \times P_1$$
$$\dot{P}_2 = [+\omega B + \mu (P_1 + P_2)] \times P_2$$

0 initially

- Initially aligned in flavor direction and $P = 0$
- Free precession $\pm \omega$

After a short time, transverse $P$ develops by free precession

Matter effect transverse to mass and flavor directions
Both $P_1$ and $P_2$ tilt around $P$ if $\mu$ is large
Two Spins with Opposite Initial Orientation

No interaction ($\mu = 0$)  
Free precession in opposite directions

Even for very small mixing angle, large-amplitude flavor oscillations

Strong interaction ($\mu \to \infty$)  
Pendular motion
Inverse-Energy Spectrum

Fermi-Dirac energy spectrum

\[
\frac{dN}{dE} \propto \frac{E^2}{e^{E/T-\eta}+1}
\]

\(\eta\) degeneracy parameter, \(-\eta\) for \(\bar{\nu}\)

Spectrum in terms of \(\omega = T/E\)

- Antineutrinos \(E \rightarrow -E\)
- and \(dN/dE\) negative

(flavor isospin convention)

\(\omega > 0: \; \nu_e = \uparrow \; \text{and} \; \nu_\mu = \downarrow\)
\(\omega < 0: \; \bar{\nu}_e = \downarrow \; \text{and} \; \bar{\nu}_\mu = \uparrow\)
**Flavor Pendulum**

Single “positive” crossing (IH) (potential energy at a maximum)

Single “negative” crossing (NH) (potential energy at a minimum)

Dasgupta, Dighe, Raffelt & Smirnov, arXiv:0904.3542
For movies see http://www.mppmu.mpg.de/supernova/multisplits
Self-Induced Flavor Conversion

• Flavor content exchanged between different momentum modes (or nus and anti-nus changing together)

• No net flavor conversion of ensemble

• Instability required to get started: Exponential growth of the off-diagonal density matrix parts

→ Linearized Stability Analysis (first stressed by Ray Sawyer)

Linearized Stability Analysis

Schrödinger equation for flavor matrices of neutrino fluxes $\Phi_{\omega,u}$

$$\omega = \pm \Delta m^2 / 2E \quad u = \sin^2(\text{emission angle}) \quad v_u = \text{radial velocity at } r$$

$$i\partial_r \Phi_{\omega,u} = \left[ \frac{\omega + \sqrt{2}G_FN_\ell}{v_u} + \frac{\sqrt{2}G_F}{4\pi r^2} \int d\omega' du' \Phi_{\omega',u'} \frac{1 - v_u v_{u'}}{v_u v_{u'}} , \Phi_{\omega,u} \right]$$

Linearize in small off-diagonal flux terms and Fourier transform

$$\Phi_{\omega,u} = \frac{g_{\omega,u}}{2} \begin{pmatrix} 1 & Q_{\omega,u} e^{-i\Omega r} \\ Q_{\omega,u}^* e^{i\Omega r} & -1 \end{pmatrix}$$

Eigenvalue equation for $Q_{\omega,u}$ in terms of eigenfrequency $\Omega = \gamma + i \kappa$, where $\kappa$ is the exponential growth rate

$$\left[ \omega + u \left( \lambda + \int d\omega' du' g_{\omega',u'} \right) - \Omega \right] Q_{\omega,u} = \mu \int d\omega' du' (u + u') g_{\omega',u} Q_{\omega,u}$$

Straightforward to solve for eigenvalue $\Omega$ and eigenfunction $Q_{\omega,u}$

Banerjee, Dighe & Raffelt, arXiv:1107.2308
Stability Analysis for Simple SN Example

Sarikas, Seixas, Tamborra & Raffelt, work in progress (2012)

Onset radius
Normal vs Inverted Hierarchy

Growth rate $\kappa [\text{km}^{-1}]$

Radius [km]

Normal hierarchy

Inverted hierarchy

Sarikas, Seixas, Tamborra & Raffelt, work in progress (2012)
Multi-Angle Paradox of Numerical Solutions

Represent neutrino field by discrete energy and angle modes

- Number of energy modes chosen to fit desired precision
- \( N_\alpha \gg 1 \) of angle modes required

\( N_\alpha \) too small: Unphysical solutions
New Instabilities for Discrete Angle Modes

Sarikas, Seixas, Tamborra & Raffelt, work in progress (2012)
New Instabilities for Discrete Angle Modes

\[ N_a = 1 \text{ (Single angle)} \]

Continuous angle distribution

True onset radius

Sarikas, Seixas, Tamborra & Raffelt, work in progress (2012)
New Instabilities for Discrete Angle Modes

\[ N_a = 2 \text{ (Two angle bins)} \]

Continuous angle distribution

Numerical onset radius

Sarikas, Seixas, Tamborra & Raffelt, work in progress (2012)
New Instabilities for Discrete Angle Modes

\[
N_a = 3
\]

Continuous angle distribution

Numerical onset radius

Sarikas, Seixas, Tamborra & Raffelt, work in progress (2012)
New Instabilities for Discrete Angle Modes

Sarikas, Seixas, Tamborra & Raffelt, work in progress (2012)

Continuous angle distribution

Numerical onset radius

$N_a = 5$
New Instabilities for Discrete Angle Modes

Sarikas, Seixas, Tamborra & Raffelt, work in progress (2012)
New Instabilities for Discrete Angle Modes

Sarikas, Seixas, Tamborra & Raffelt, work in progress (2012)

$Na = 50$

Continuous angle distribution

Numerical onset radius

Sarikas, Seixas, Tamborra & Raffelt, work in progress (2012)
New Instabilities for Discrete Angle Modes

\[ G(\kappa) \text{ vs. } R \]

- \( N_a = 100 \)
- Continuous angle distribution
- Numerical onset radius (start integration not too deep)

Sarikas, Seixas, Tamborra & Raffelt, work in progress (2012)
New Instabilities for Discrete Angle Modes

Sarikas, Seixas, Tamborra & Raffelt, work in progress (2012)

Numerical onset radius (start integration at nu sphere?)

Na = 500

Continuous angle distribution

Growth rate $\kappa [\text{km}^{-1}]$

Radius [km]

approx nu sphere
Sarikas, Seixas, Tamborra & Raffelt, work in progress (2012)

Continuous angle distribution

Numerical onset radius (start integration at nu sphere?)

$N_a = 1000$
Multi-Angle Matter Effect

Liouville form of oscillation equation

\[ \dot{P}_{\omega,v} + (v \cdot \nabla_r) P_{\omega,v} = (\omega B + \lambda L + \mu P) \times P_{\omega,v} \]

\[ \uparrow \uparrow \uparrow \sqrt{2G_F N_e} \quad \sqrt{2G_F N_\nu} \]

Drops out for stationary solutions

\[ \partial_r P_{\omega,v} = \frac{\omega B + \lambda L + \mu P}{v_r} \times P_{\omega,v} \]

Self-induced conversion suppressed for \( N_e \gtrsim N_\nu \)

Esteban-Pretel, Mirizzi, Pastor, Tomàs, Raffelt, Serpico & Sigl, arXiv:0807.0659
Accretion-Phase Matter Profiles

Dasgupta, O’Connor & Ott, arXiv:1106.1167
Multi-Angle Matter Suppression

Sarikas, Seixas, Tamborra & Raffelt, work in progress (2012)
Multi-Angle Matter Suppression

Electron density = 0.2 × Neutrino density

Sarikas, Seixas, Tamborra & Raffelt, work in progress (2012)
Multi-Angle Matter Suppression

Electron density = 0.4 × Neutrino density

Sarikas, Seixas, Tamborra & Raffelt, work in progress (2012)
Multi-Angle Matter Suppression

Electron density = $0.7 \times$ Neutrino density

Sarikas, Seixas, Tamborra & Raffelt, work in progress (2012)
Multi-Angle Matter Suppression

Electron density $= 1.0 \times$ Neutrino density

Sarikas, Seixas, Tamborra & Raffelt, work in progress (2012)
Multi-Angle Matter Suppression

Electron density $= 2.0 \times$ Neutrino density

Sarikas, Seixas, Tamborra & Raffelt, work in progress (2012)
Electron density = 5.0 × Neutrino density

Sarikas, Seixas, Tamborra & Raffelt, work in progress (2012)
Multi-Angle Matter Suppression (NH)

No matter

Sarikas, Seixas, Tamborra & Raffelt, work in progress (2012)
Electron density = 0.1 × Neutrino density

Sarikas, Seixas, Tamborra & Raffelt, work in progress (2012)
Multi-Angle Matter Suppression (NH)

Electron density $= 0.2 \times$ Neutrino density

Sarikas, Seixas, Tamborra & Raffelt, work in progress (2012)
Multi-Angle Matter Suppression (NH)

Electron density = 0.3 × Neutrino density

Sarikas, Seixas, Tamborra & Raffelt, work in progress (2012)
Multi-Angle Matter Suppression (NH)

Electron density = 0.4 × Neutrino density

Sarikas, Seixas, Tamborra & Raffelt, work in progress (2012)
Multi-Angle Matter Suppression (NH)

Electron density = 0.5 × Neutrino density

Sarikas, Seixas, Tamborra & Raffelt, work in progress (2012)
Multi-Angle Matter Effect (Basel Model 10.8 $M_{\odot}$)

Schematic single-energy, multi-angle simulations with realistic density profile

Chakraborty, Fischer, Mirizzi, Saviano & Tomàs, arXiv:1105.1130
Multi-Angle Multi-Energy Stability Analysis

Tested 15 $M_\odot$ accretion-phase models (Garching) are stable against collective flavor conversion.

Contours of growth rate $\kappa \, [\text{km}^{-1}]$.

Sarikas, Raffelt, Hüdepohl & Janka, arXiv:1109.3601
To investigate neutrino oscillations in SNe need

- Profile of electron density
- Flavor-dependent neutrino flux spectra
- Flavor-dependent neutrino angular distribution
Small “scattering halo” important for nu-nu refraction? (Cherry et al., arXiv:1203.1607)

Picture from Ott, Burrows, Dessart & Livne, arXiv:0804.0239
Scattered Neutrinos as a Source of Refraction

SN neutrino angle distribution seen at $10^4$, 3000, 1000 and 300 km

Relative importance of halo for nu-nu refractive potential

Cherry, Carlson, Friedland, Fuller & Vlasenko, arXiv:1203.1607
The tested Garching 15 $M_\odot$ accretion-phase model is stable against collective flavor conversion.
Summary

Supernova neutrino flavor evolution remains a complicated subject

- Axial symmetry was always assumed – too symmetric?
- Numerical treatments challenging
- Novel role for neutrino “scattering halo”?
- Simultaneous space and time dependence important?

Theoretical developments

- Analogy to BCS theory
- Linearized stability analysis provides many conceptual insights
- And practical results

Working hypothesis for SN neutrinos

- Multi-angle matter effect can prevent instability
- No collective conversion during early accretion phase
- Can test for nu mass hierarchy (because $\theta_{13}$ is large)
More theory progress is needed to reliably interpret neutrino signal of next galactic supernova!