Inferring Core-Collapse Supernova Physics from Gravitational-Wave Observations

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The Advanced GW Detector Network: 2020+

- Advanced LIGO Hanford 2015+
- Advanced LIGO Livingston 2015+
- Advanced Virgo 2015+
- GEO 600 (HF) 2011
- LIGO India 2020+
- KAGRA 2017+
Observing the CCSN Mechanism

Probing the “Supernova Engine”
- Gravitational Waves
- Neutrinos

**EM waves (optical/UV/X/Gamma):** secondary information, late-time probes of engine.

Red Supergiant Betelgeuse
D \(\sim\) 200 pc

Supernova “Central Engine”

HST

800 million km

300 km
Gravitational-Waves from Core-Collapse Supernovae

Recent reviews: Ott ‘09, Kotake ‘11, Fryer & New ‘11

Need:

\[ h_{jk}^{TT}(t, \vec{x}) = \left( \frac{2}{c^4 |\vec{x}|} \right) G \left( t - \frac{|\vec{x}|}{c} \right)^{TT} \quad \text{accelerated aspherical (quadrupolar) mass-energy motions} \]

Candidate Emission Processes:

- Neutrino-driven Convection and SASI
- Prompt convection
- Protoneutron star convection
- Rotating collapse & bounce
- Rotational 3D instabilities
- Black hole formation
- Pulsations of the protoneutron star
- Anisotropic neutrino emission
- Aspherical accelerated outflows
- Magnetic stresses

Tasks:

1. Determine GW signals from these emission processes.
2. Connect GW emission processes to CCSN Mechanism.
3. Detection: How far out can we detect GWs from CCSNe and can we infer the explosion mechanism (and other physics)?
Gravitational-Waves from Core-Collapse Supernovae

Recent reviews: Ott ‘09, Kotake ‘11, Fryer & New ‘11

Need:

\[ h_{jk}^{TT}(t, \vec{x}) = \left[ \frac{2}{c^4} \frac{G}{|\vec{x}|} \ddot{I}_{jk}(t - \frac{|\vec{x}|}{c}) \right]^{TT} \]

accelerated aspherical (quadrupolar) mass-energy motions

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(2) Connecting GW Signals and Explosion Mechanisms

Ott ’09, CQG
see also Kotake ‘11, Kotake et al.’12
Connecting GW Signals and Explosion Mechanisms

Ott ‘09, CQG 26, 063001

**Neutrino Mechanism**
[e.g. Yakunin et al. ‘10, Müller et al. ‘12ab]

**Magnetorotational Mechanism**
[e.g. Burrows et al. ’07, Takiwaki & Kotake ‘11]

**Acoustic Mechanism**
[e.g. Burrows et al. 06, Ott et al. ’06]

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**Dominant GW Emission Processes**

- Turbulent convection, **Standing-Accretion-Shock Instability**
- Rotating core collapse & bounce, rotational 3D instabilities.
- Protoneutron star pulsations.
  (Caveat: Weinberg & Quataert ’08, Marek & Janka ‘09 -> may not occur in nature)

C. D. Ott @ INT, 2012/07/17
Connecting GW Signals and Explosion Mechanisms

Slow rotation, neutrino-driven explosions

Rapid rotation, magnetorotational mechanism

PNS g-modes: Acoustic Mechanism (2D simulation)

Slow rotation, acoustic mechanism.

[Ott ’09; Logue et al. ’12]
Connecting GW Signals and Explosion Mechanisms

Caveats:
• GW signal predictions still mostly based on 2D simulations.
• Advanced LIGO sensitivity:
  Need core-collapse supernova in the Milky Way.
(3) Inferring Physics from GW Observations of Core-Collapse Supernovae

Inferring Core-Collapse Supernova Physics with Gravitational Waves

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(Dated: February 24, 2012)

Stellar collapse and the subsequent development of a core-collapse supernova explosion emit bursts of gravitational waves (GWs) that might be detected by the advanced generation of laser interferometer gravitational-wave observatories such as Advanced LIGO, Advanced Virgo, and LCGT. GW bursts from core-collapse supernovae encode information on the intricate multi-dimensional dynamics at work at the core of a dying massive star and may provide direct evidence for the yet uncertain mechanism driving supernovae in massive stars. Recent multi-dimensional simulations of core-collapse supernovae exploding via the neutrino, magnetorotational, and acoustic explosion mechanisms have predicted GW signals which have distinct structure in both the time and frequency domains. Motivated by this, we describe a promising method for determining the most likely explosion mechanism underlying a hypothetical GW signal, based on Principal Component Analysis and Bayesian model selection. Using simulated Advanced LIGO noise and assuming a single detector and linear waveform polarization for simplicity, we demonstrate that our method can distinguish magnetorotational explosions throughout the Milky Way (D ≲ 10 kpc) and explosions driven by the neutrino and acoustic mechanisms to D ≲ 2 kpc. Furthermore, we show that we can differentiate between models for rotating accretion-induced collapse of massive white dwarfs and models of rotating iron core collapse with high reliability out to several kpc.
Supernova Model Evidence Extractor (SMEE)

- Can we really tell these signals apart in a noisy detector?
- Approach: Bayesian Model Selection

Bayes Theorem:

$$P(M|D, I) = \frac{P(M|I)P(D|M, I)}{P(D|I)}$$

"Posterior Probability"  "Prior Probability"  Normalization  "Evidence (Likelihood)"

- For model selection: When comparing two models, odds ratio is sufficient:

$$O_{ij} = \frac{P(M_i|I)P(D|M_i, I)}{P(M_j|I)P(D|M_j, I)}$$

"Marginal Likelihood"

$$P(D|M, I) = \int_\theta p(\theta|M, I)p(D|\theta, M)d\theta$$

$$\theta: \text{Model Parameters}$$

Ratio of Priors  Bayes Factor

$$\log B_{ij} = \log P(D|M_i, I) - \log P(D|M_j, I)$$

M: Model  D: Data  I: Prior information
SMEE: Signal Models
Consider waveforms representative for:

Neutrino Mechanism
Magnetorotational Mechanism
Acoustic Mechanism
SMEE: Signal Models
Consider waveforms representative for:

**Neutrino Mechanism**

**Magnetorotational Mechanism**

**Acoustic Mechanism**

**What are the parameters to marginalize over?**

- Waveforms impossible to predict exactly.
- Parameter studies of GW emission in CCSNe provide waveform catalogs.
- Approach: Try to isolate robust features present in waveform catalogs and parameterize waveforms according to these.
**Principal Component Analysis**

Logue et al. ‘12, arXiv:1202.3256, PRD in press, previously applied to GWs by Heng ’09, Roever+ ‘09

**Assumption:**

Gravitational wave signals have certain robust features in their time series or power spectra that can be isolated.

**Example:**

Rotating core collapse waveforms from Dimmelmeier+ ’08 (128 waveforms total)
**Principal Component Analysis**

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**Assumption:**

Gravitational wave signals have certain robust features in their time series or power spectra that can be isolated.

**Procedure for feature isolation:** [en.wikipedia.org/wiki/Principal_component_analysis](en.wikipedia.org/wiki/Principal_component_analysis)

- Take m waveforms of length n that span the m-dimensional parameter space and construct n x m matrix A.
- One can show (linear algebra) that the eigenvectors $U_i$ of $A A^T$ are orthogonal basis vectors of the m-dimensional parameter space. They are the **principal components (PCs)**.
- The PCs are ordered according to the values of their eigenvalues $\lambda_i$, which indicate the importance of any given PC in the parameter space spanned by m waveforms.
- If the PCA works efficiently, only $k << m$ PCs are needed to reconstruct any waveform of the catalog with good accuracy.

\[
h_i \approx \sum_{j=1}^{k} U_j \beta_j
\]
Principal Components

- Dim Catalog (Magnetorotational Mechanism)
- Mur Catalog (Neutrino Mechanism)
- Ott Catalog (Acoustic Mechanism)

Graphs showing time [s] on the x-axis and principal components on the y-axis.
Supernova Model Evidence Extractor (SMEE)


Must compute the likelihood that the data is consistent with model M.

\[
P(D|M, I) = \int_{\theta} p(\theta|M, I)p(D|\theta, M)d\theta
\]

\[h_i \approx \sum_{j=1}^{k} U_j \beta_j\]

\(\theta\): Set of Model Parameters \(\Rightarrow\) coefficients of the PCs

In SMEE:

- Uniform prior on the \(\beta_i\).
- Ranges set by ranges found for each signal catalog.
- Usually use 3 or 7 PCs \(\Rightarrow\) marginalization integral is multi-dimensional.
- Efficient integration technique: “Nested Sampling” (Skilling ‘04)

http://en.wikipedia.org/wiki/Nested_sampling_algorithm
(similar to Markov-Chain Monte Carlo)
Supernova Model Evidence Extractor (SMEE)


\[ P(D|M, I) = \int_\theta p(\theta|M, I)p(D|\theta, M)d\theta \]

\[ \log B_{ij} = \log P(D|M_i, I) - \log P(D|M_j, I) \]

Yes

injected signal from catalog \( M_1 \)

No

injected signal from catalog \( M_2 \)
Supernova Model Evidence Extractor (SMEE)


Must consider two cases:

(1) Is signal different from noise?

$$\log B_{SN} = \log P(D|M_S, I) - \log P(D|M_N, I)$$

$M_S$ signal model $M_N$ noise model (here: Gaussian, stationary)

(note: real detector noise: non-Gaussian, non-stationary)

(2) Comparison of signal models

$$\log B_{ij} = \log P(D|M_i, I) - \log P(D|M_j, I)$$

Problem: How to describe signal mode? GW signals cannot be predicted exactly (turbulence! + unknown physics).
First SMEE Study


Magnetorotational Mechanism

- 128 waveforms from Dimmelmeier et al. 2008

Neutrino Mechanism

- 16 waveforms from Murphy et al. 2009

Acoustic Mechanism

- 7 waveforms from Ott 2009

Simplifications:

- Single detector.
- Gaussian noise.
- Linearly polarized waves.
- Optimally oriented source.
Results: Pure Noise


Agrees with analytic calculation based on noise model.
Results: Signal vs. Noise


Note: Real SNR for detection will need to be > 8-10.
**Results: Ideal Case**


Injected “known” waveforms from catalogs that were used to generate principle components (PCs); use first 7 PCs.

\[
\log B_{ij} = \log P(D|M_i, I) - \log P(D|M_j, I)
\]
Use **unknown waveforms** from different studies modeling the same physics.
Scheidegger et al. ‘10: magnetorotational mechanism.
Yakunin et al. ‘10: neutrino mechanism

\[ \text{Fig. 9: Mean and 1-} \]

- Method robust for magnetorotational mechanism out to 10 kpc.
- Can identify neutrino mechanism out to \(~2\) kpc (using Murphy+ 09 PCs).
Summary

- Magnetorotational, neutrino-driven, and acoustically-driven CCSN explosions are likely to have distinct GW signatures.

- Provided (a) that this is the case, and, (b) robust large catalogs of waveform predictions are available, The Supernova Model Evidence Extractor (SMEE) can determine the core-collapse supernova explosion mechanism based on the GW signal alone.

- Need nearby event (< 2 – 10 kpc).

- Neutrinos will provide additional information (to be explored).

- Many Limitations: PCA not good for some signal types, so far only considered ideal case of Gaussian noise, single detector, optimal orientation, linear polarization, catalogs with limited predictive power.