Neutrino-Driven Convection and Neutrino-Driven Explosions

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1D simulations
(Rad-hydro)

- Wilson ‘85
- Bethe & Wilson ‘85
- Liebendoerfer et al. ‘01
- Rampp & Janka ‘02
- Buras et al. ‘03
- Thompson et al. ‘03
- Liebendoer et al. ‘05
- Kitaura et al. ‘06
- Burrows et al. ‘07

Neutrino mechanism suggested
No Explosions
(Except lowest masses)
Spherical symmetry!
No GW emission?
Fundamental Question of Core-Collapse Theory

Steady-State Accretion

Explosion

?
Relax 1D assumption?
Neutrino Mechanism:
• Neutrino-heated convection
• Standing Accretion Shock Instability (SASI)
• Explosions? Maybe

Acoustic Mechanism:
• Explosions but caveats.

Magnetic Jets:
• Only for very rapid rotations
• Collapsar?
Fundamental Question of Core-Collapse Theory?
Why is it easier to explode in 2D compared to 1D?
Two Paths to the Solution

• Detailed 3D radiation-hydrodynamic simulations ("Accurate" energies, NS masses, nucleo., etc.)

• Parameterizations that capture essential physics (Tease out fundamental mechanisms)
Critical Curve

Steady-state accretion (Solution)

Explosions!
(No Solution)

\[ \dot{L}_{ve} \]

\[ \dot{M} \]

Burrows & Goshy ‘93
Steady-state solution (ODE)
Explosion is a global, boundary-value problem
Explosion is a global, boundary-value problem

In other words:
What neutrino forcing is required to change the global structure between the NS and shock such that explosions occur?
Explosion is a global, boundary-value problem

This also means that one can’t easily and cleanly pick out one simple diagnostic

Hence... “Mazurek’s Law”
Is a critical luminosity relevant in hydrodynamic simulations?

- 1D
- 2D Convection and SASI?
How do the critical luminosities differ between 1D and 2D?
Murphy & Burrows ‘08
Nordhaus et al. 2010
Despite the basic agreement of the outcome of these investigations, there is a plethora of systematic studies that examine the dependence of progenitor mass on the explosion (or the explosion threshold) and on the neutrino luminosity. Since the mass accretion rate and neutrino luminosity is equivalent to the steady-state accretion solution for a given combination of progenitors, however, the question cannot be answered whether such a limiting accretion solution that corresponds to the critical curve of the explosion takes place and the mass accretion rate has the value of the onset of an explosion. The latter requires the persistently strong energy input by neutrino heating for a sufficiently long period of time. This is especially important in the case of the 15\( M_\odot \) progenitors and for the simulations with different dimensions.

Hanke et al 2011
We ran a series of core collapse simulations in 1D and 2D in
the FLASH framework (Hanke et al. 2011). These routines are available for download at stellarcollapse.org.

We have incorporated the finite temperature equation of state
that "outflow" boundary conditions can, in this way, suppress
explosions for neutrino luminosities near critical. For all of
the models of Lattimer & Swesty EOS result in easier explosions in both 1D and 2D than the Shen et al. EOS.

Figure 4. Shock radii as a function of time for STOS, LS220, and LS180 in 1D (left) and 2D (right). Three different neutrino luminosities are plotted for each
EOS in each panel, as labeled.

Figure 5. Neutrino luminosity versus mass accretion rate at time of explosion (left panel) and time of explosion (right panel) for the three EOS considered in this
study. This trend holds in both 1D and 2D simulations. For
being the softest EOS and STOS being the stiffest EOS we
consider. This trend holds in both 1D and 2D simulations. For
stiffer the EOS, the harder it is to drive an explosion (LS180
curves in Fig. 4). The results then follow a basic trend that the
difficulties result in easier explosions for neutrino luminosity.

Dependence on EOS

Of the 180 MeV and 220 MeV and that of

Table 3.

Couch 2012
Why is critical luminosity of multi-D simulations ~70% of 1D?
Comparison of Timescales
(Thompson et al. ‘00, Janka ’01, Thompson et al. ‘03, Murphy et al. ’08, Pejcha ’11, Fernandez ’12)

\[
\tau_{\text{adv}} = \frac{\Delta r_{\text{gain}}}{v_r}
\]

\[
\tau_q = \frac{E}{\dot{Q}}
\]

\[
\frac{\tau_{\text{adv}}}{\tau_q} \gtrsim 1
\]
$1D \rightarrow \text{one time}$

$\text{mulit-D} \rightarrow \text{distribution of times}$

More heating?

\[ \Delta S \propto \frac{Q}{T} \]
2D & 3D critical luminosity lower than 1D

Turbulence plays an important role!
A Theoretical Framework for Successful Explosions

\[ \begin{align*}
\dot{L}_v &+ \text{Turbulence Model} \\
\dot{M} &
\end{align*} \]

Murphy & Meakin 2011
A Theoretical Framework for Successful Explosions

$$L_v + \dot{M}$$

Turbulence Model

Calibrate with 3D Simulations

Murphy et al. 2012, in prep
A Theoretical Framework for Successful Explosions

$L_v + \dot{M}$

Turbulence Model
A Theoretical Framework for Successful Explosions

\[ L_v \quad \dot{M} \]

+ Turbulence Model

1D Rad-hydro simulations
Realistic and quantitative explosions
Systematic exploration
$N^2 < 0, \nabla s < 0$

Convectively unstable

$N^2 > 0$

Stably stratified

(gravitational waves)

$N^2 > 0$
\[ b(r) = \int N^2 dr = \text{buoyant accel.} \]
\( N^2 < 0 \)

Convectively unstable

\[ F = F_{\text{rad}} + F_{\text{conv}} \]

\[ F_{\text{conv}} = C_p Q(v'T') \]

\( N^2 > 0 \)

Stably stratified (gravity waves)

\[ F = F_{\text{rad}} \]

\( b(r) = \int N^2 \, dr = \text{buoyant accel.} \)
\[ F = F_{\text{rad}} + F_{\text{conv}} \]

\[ F_{\text{conv}} = C_p Q (v'T') \]
Need a More General Turbulence Model (Reynolds Decomposition)
Back to the Beginning

\[ \partial_t \rho + \nabla \cdot (\rho \mathbf{v}) = 0 \]

\[ \partial_t (\rho \mathbf{v}) + \nabla \cdot (\rho \mathbf{v} \otimes \mathbf{v}) = -\nabla P + \rho \mathbf{g} + \nabla \cdot \sigma \]

\[ \partial_t (\rho s) + \nabla \cdot (\rho \mathbf{v}s) = \frac{\dot{Q} + \varepsilon}{T} \]

\[ P = P(\rho, s, X_i) \]
Reynolds Decomposition

\[ \phi = \phi_0 + \phi' \]

\[ \phi_0 = \langle \phi \rangle \]

\[ \downarrow \]

Hydro Equations

\[ \downarrow \]

Mean-Field Equations

\[ \downarrow \]

New steady-state solutions & Critical Curve
Reynolds-Averaged Equations

\[ \dot{M} = 4\pi r^2 \left( \rho_0 v_0 + \langle \rho' v' \rangle \right) \]

\[ \langle \rho v \rangle \cdot \nabla v_0 = -\nabla P_0 + \rho_0 g - \nabla \cdot \langle R \rangle \]

\[ \langle \rho v \rangle \cdot \nabla s_0 = \frac{\dot{Q} + \varepsilon}{T} - \nabla \cdot \langle F_s \rangle \]

Murphy & Meakin 2011
\[ \rho \mathbf{v} \cdot \nabla s = \frac{\dot{Q}}{T} \]
\[ \langle \rho v \rangle \cdot \nabla s_0 = \frac{\dot{Q} + \varepsilon}{T} - \nabla \cdot \langle F_s \rangle \]
Reynolds-Averaged Equations

\[ \dot{M} = 4\pi r^2 \left( \rho_0 v_0 + \langle \rho' v' \rangle \right) \]

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Murphy & Meakin 2011
Turbulent Moment Equations

Equations for 2nd order moments

\[ \partial_t \langle \rho K \rangle + \nabla \cdot (\nu_0 \langle \rho K \rangle) \approx \langle \rho' v' \rangle g - \varepsilon - \nabla \cdot \langle F_K \rangle \]

and more ...
Turbulent Moment Equations

Equations for 2nd order moments

\[ \frac{\partial}{\partial t} \langle \rho K \rangle + \nabla \cdot (\nu_0 \langle \rho K \rangle) \]

\[ \approx \langle \rho' \nu' \rangle g - \varepsilon - \nabla \cdot \langle F_K \rangle \]

\[ \sim \frac{\nu'^3}{L} \]

\[ F_K = \rho K \nu' \]

Depends upon higher order moments
Turbulent Moment Equations

Equations for 2nd order moments

\[ \partial_t \langle \rho K \rangle + \nabla \cdot (v_0 \langle \rho K \rangle) \approx \langle \rho' \nu' \rangle g - \varepsilon - \nabla \cdot \langle F_K \rangle \]

A Closure Problem!
# Closure Strategies

<table>
<thead>
<tr>
<th>Local Algebraic</th>
<th>Local Single-Point</th>
<th>Global</th>
</tr>
</thead>
</table>
Closure Strategies

Local Algebraic  Local Single-Point  Global

$$\partial_t \langle \rho K \rangle + \nabla \cdot (v_0 \langle \rho K \rangle)$$
$$\approx \langle \rho' v' \rangle g - \varepsilon - \nabla \cdot \langle F_K \rangle$$
Closure Strategies

Local Algebraic  Local Single-Point  Global

\[
\frac{\partial_t \langle \rho K \rangle}{\tau_0 \langle \rho K \rangle} + \nabla \cdot \langle \rho' v' \rangle g - \varepsilon - \nabla \langle F_K \rangle
\]

\[
\langle \rho' v' \rangle g \approx \varepsilon \sim \frac{\rho v'^3}{L}
\]

MLT is a classic example
Closure Strategies

Local Algebraic  Local Single-Point  Global

\[ \partial_t \langle \rho K \rangle + \nabla \cdot (v_0 \langle \rho K \rangle) \]
\[ \approx \langle \rho' v' \rangle g - \varepsilon - \nabla \cdot \langle F_K \rangle \]

Local models for these
Closure Strategies

Local Algebraic  Local Single-Point  Global

\[ \partial_t \langle \rho K \rangle + \nabla \cdot (v_0 \langle \rho K \rangle) \]
\[ \approx \langle \rho' v' \rangle g - \varepsilon - \nabla \cdot \langle F_K \rangle \]

\[ \int \langle \rho' v' \rangle g \, dV = \int \varepsilon \, dV \]
Global Closure Examples

Earth’s Atmospheric Convective Layer

$$\nabla s_0 = 0$$

$$\rho_0 \left\langle \dot{s} \right\rangle = -\partial_z \left\langle F_s \right\rangle$$

Tennekes 1973
Global Closure Examples

Stellar Convection

\[ \nabla s_0 = 0 \quad \Rightarrow \quad \rho_0 \langle \dot{s} \rangle = -\nabla \cdot \langle F_s \rangle + \frac{\dot{Q} + \varepsilon}{T} \]

Table 1

<table>
<thead>
<tr>
<th>Model ID</th>
<th>(\Delta \theta, \Delta \phi)</th>
<th>(N_r \times N_\theta \times N_\phi)</th>
<th>(t_{avg}) [s]</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>h1</td>
<td>30,30</td>
<td>200 \times 50 \times 50</td>
<td>[300, 500]</td>
<td>narrow, static heating profile</td>
</tr>
<tr>
<td>h1.z2</td>
<td>30,30</td>
<td>400 \times 100 \times 100</td>
<td>- model h1 with moderate resolution increase</td>
<td></td>
</tr>
<tr>
<td>h1.z1</td>
<td>30,30</td>
<td>800 \times 200 \times 200</td>
<td>- model h1 with high resolution</td>
<td></td>
</tr>
<tr>
<td>h3</td>
<td>30,30</td>
<td>200 \times 50 \times 50</td>
<td>[375, 575]</td>
<td>broad, static heating profile</td>
</tr>
<tr>
<td>c1</td>
<td>30,30</td>
<td>200 \times 50 \times 50</td>
<td>[200, 400]</td>
<td>static top cooling profile</td>
</tr>
</tbody>
</table>

a The computational domain is centered on the equator so that the domain extends \(\Delta \theta/2\) degree above and below the equator.
b Provided is the time interval over which averages are performed.

Fig. 1 (left) Heating / cooling profiles for the models listed in Table 1. (right) The time evolution of the total kinetic energy in the simulation domain. The two additional high resolution models \(h1.z1\) and \(h1.z2\) are indicated by the magenta crosses and the orange line, respectively.

Fig. 2 Convective flux: (left) time averaged simulation data and (right) calculated from the background structure as described by Eq. 3.

Meakin & Arnett 2010
Global Closure For CCSNe

\[ \langle \rho v \rangle \cdot \nabla s_0 = \frac{\dot{Q}}{T} + \varepsilon - \nabla \cdot \langle F_s \rangle \]

Use

\[ \int \langle \rho' v' \rangle g \, dV = \int \varepsilon \, dV \]

And simulations to inform assumptions about profiles
Comparison of Timescales

\[
\frac{\tau_{\text{adv}}}{\tau_{\text{heat}}} \geq 1
\]

Heuristic & Empirical

See...
Thompson et al. ’00
Janka ’01
Thompson et al. ’03
Murphy et al. ’08
Buras ’06
Pejcha ’11
Fernandez ’12
Comparison of Timescales

\[ \frac{\tau_{\text{adv}}}{\tau_{\text{heat}}} \gtrsim 1 \]

\[ \rho v \cdot \nabla s = \frac{\dot{Q}}{T} \]
Comparison of Timescales

\[
\frac{\tau_{\text{adv}}}{\tau_{\text{heat}}} \sim \frac{1}{\Delta s} \int \frac{\dot{Q}}{T} \frac{dr}{\rho v}
\]

\[
\frac{\tau_{\text{adv}}}{\tau_{\text{heat}}} \geq 1
\]

\[
\rho v \cdot \nabla s = \frac{\dot{Q}}{T}
\]
Comparison of Timescales

\[
\frac{\tau_{\text{adv}}}{\tau_{\text{heat}}} \sim \frac{1}{\Delta s} \int_0^{\Delta s} \frac{\dot{Q}}{T} \frac{dr}{\rho v}
\]

\[
\frac{\tau_{\text{adv}}}{\tau_{\text{heat}}} \gtrsim 1
\]

\[
\Delta s \geq \Delta s_{\text{crit}}
\]

\[
\rho v \cdot \nabla s = \frac{\dot{Q}}{T}
\]
Comparison of Timescales

\[ \Delta s \gtrsim \Delta s_{\text{crit}} \]

\[ \frac{\tau_{\text{adv}}}{\tau_{\text{heat}}} \gtrsim 1 \]

\[ \dot{M} \Delta s = \int \frac{\dot{Q}}{T} \, dV + \int \frac{\varepsilon}{T} \, dV + L_s \]
The luminosity, turbulent dissipation (green dashed line), and the modeled convective entropy (blue line), the modeled integrated shock radius, and turbulent dissipation (green dot-dashed line) are evaluated from Global Model, Model 4. The integrated neutrino heating and cooling (blue line) represent 2D simulation results, and the dashed lines are the results of the global model (red dashed line) and the 2D data (black dashed line). The solid blue line represents the change in entropy (in units of \( L_s^{–1} R_s \)) derived using the entropy equation, \( \frac{\rho}{\rho_0} \), for the rest. The total entropy change, \( \Delta s \), at 404 ms, 518 ms, and 632 ms is only slightly larger than the 2D simulation results (red solid lines in Figures 11 and 12). The Reynolds stress, \( \mathcal{R} \), at 404 ms and only one-third of the entropy change at 632 ms.

In Section 4.5, we compare the terms in the entropy equation, \( \rho \mathcal{R} \), at 404 ms, 518 ms, and 632 ms. We find that convection fills the region where this integral is greater than zero. In the convective region, the neutrino heating and cooling curve accounts for only half of the total entropy change. Moreover, this shape is simply modeled as a piecewise linear, pointed hat. The solid blue line reproduces the general radial profile and temporal evolution.

Radial profiles of entropy change in units of \( L_s^{–1} R_s \) (green dot-dashed line). The entropy difference derived using the algebraic model does not match exactly the 2D profiles. For-
“What about the SASI?”
What dominates the post shock flow? Convection, SASI... both?
Compare nonlinear theories for convection and SASI with post shock flow

SASI nonlinear theory
Compare nonlinear theories for convection and SASI with post shock flow

Convection nonlinear theory
100+ years
In CCSN...Murphy & Meakin 2012
Compare nonlinear theories for convection and SASI with post shock flow

Convection nonlinear theory
100+ years
In CCSN...Murphy & Meakin 2012

We can test this theory with 3D simulations
\[ L_s = \langle \rho v' s' \rangle 4\pi r^2 \]
\[ R_{ij} = \left \langle v'_i v'_j \right \rangle \]

\[ R_{rr} \approx R_{\theta\theta} + R_{\phi\phi} \]

\[ R_{\theta\theta} \approx R_{\phi\phi} \]
\[ \int \left\langle \rho' \nu' \right\rangle g \, dV = \int \frac{\rho (\nu'_r)^3}{L} \, dV \]
Neutrino–Driven Convection

\[ T_0L_s = \alpha L_{\nu_s}\tau \]

\[ \alpha = 0.70 \]
$\langle \rho v^2 \rangle_{u, \text{fit}}$

$(P + \rho v^2 + \rho R_{rr})_{d, \text{fit}}$

$P$: 3D sim

$R_{\text{shock}}$ w/o $R_{rr}$

$R_{\text{shock}}$ w/ $R_{rr}$

$<R_s>$: 3D sim

$L_{\nu} = 2.1$
Nonlinear Convection is Consistent with Post Shock Flow

1. Consistent buoyancy flux profile
2. Consistent Reynolds stresses
3. Buoyant driving balances dissipation
4. Analytic scaling between buoyant flux and neutrino driving
5. Expansion of shock due to turbulent ram pressure
Nonlinear Convection is Consistent with Post Shock Flow

But what about the SASI?
A theory for neutrino-driven explosions

A turbulence model for CCSNe

Post shock flow is consistent with nonlinear convection theory