Dark Matter and Neutrino Responses: Effective Theory Response Functions

The bottom-up DM effective theory

The nuclear embedding

The six responses
I. Dark Matter Basics

• perhaps the most-likely-to-be-resolved new-physics problem

• closely linked to laboratory-based accelerator and underground experiments to probe for new particles beyond the standard model

• discovered in astrophysics, from the flat velocity rotation curves of galaxies

• must be long-lived or stable, cold or warm (so that it is slow enough to seed structure formation), gravitationally active, but without strong couplings to itself or to baryons

• leading candidates are weakly interactive massive particles (WIMPs - our focus) and axions (where the UW has the leading experiment)

• WIMPS connected to generic expectations that new particles might be found at the mass generation scale of the SM of 10 GeV - 10 TeV
The inventory

There is a small, identified component from the standard model, massive neutrinos: using a conservative cosmological bound on the sum over light active species of 1 eV, the active neutrino contribution is less than 2% of the closure density.

Thus the bulk of the DM must reside beyond the standard model.
• “WIMP miracle:” $G_F^2$ annihilation cross sections imply $\Omega_{\text{WIMP}} \sim 0.1$

• their detection channels include
  - role in LSS formation
  - potential to annihilate into SM particles, a potential astrophysical signal
  - accelerator production in the collision of SM particles
  - scattering off SM particles, particularly heavy nuclear targets

• conventionally nuclear description: spin-independent or spin-dependent nuclear scattering cross sections, depending on parameters

• searches focused on WIMP mass bounds of 10 GeV - TeV, with typical recoil momenta $\sim 100$ MeV (so form factors are important)

• detector masses have reached the 100 kg level
controversy over DAMA, CoGeNT events at low energy vs. efficiency of Xenon100, CDMS to exclude such light-mass WIMPS
The field is not one for the timid! Lot’s of infighting

• e.g., the DAMA group’s result for an annual modulation of nuclear recoil rate -- at $8.2 \sigma$ and climbing -- is close in magnitude and similar in phase to the annual variation in neutron backgrounds observed in Gran Sasso: Nygren observed that HE muon reactions in NaI(Tl) should produce delayed pulses similar to those seen

• negative results from CDMS (but two “events”), Xenon100; ambiguous results from Cogent; excess events above background seen by CREST II; Pamela positron/electron ratio growth with $E$ argued to be an WIMP annihilation signal, versus conventional astro explanations; efforts to find an allowed WIMP “phase space” at $< 10$ GeV masses; Xenon100 latest results (July 19) push limits by x3.5

Our bottom line further complicates matters: a great deal more variability in detector responses theoretically than generally realized
II. The probe: nuclear recoil following elastic scattering

Among the experimentally favored isotopes:

$^{19}\text{F}, \quad ^{23}\text{Na}, \quad 70, 72, 73, 74, 76 \quad \text{Ge}, \quad ^{127}\text{I}, \quad 128, 129, 130, 131, 132, 134, 136 \quad \text{Xe}$
Includes targets with vector (J > 1/2) and tensor (J > 1) responses

$^{19}\text{F}(1/2^+), ^{129}\text{Xe}(1/2^+), ^{23}\text{Na}(3/2^+), ^{73}\text{Ge}(9/2^+), ^{127}\text{I}(5/2^+), ^{131}\text{Xe}(3/2^+)$

and thus in principle the WIMP can scatter off any scalar, vector, tensor static moment provided by the nucleus, consistent with angular momentum and with the assumption that the nuclear ground state is effectively parity- and time reversal-even

With few exceptions, the standard approach has been

1) “top-down” in which an ultraviolet theory motivates a specific nuclear coupling
2) which is then embedded in the nucleus assuming the nucleus is point-like and thus described by nuclear charges and spins
3) with possible form-factor corrections to account for the nonnegligible three-momentum transfer $\sim 100$ MeV

But nucleus is composite and the DM particle could also be complex
(So some inconsistencies here: point-like but not point-like, $qR \sim 1$)

Leads to the common terminology of cross sections characterized as

$$S.I. \quad \Rightarrow \quad \langle g.s. \mid \sum_{i=1}^{A} (a_0^F + a_1^F \tau_3(i)) \mid g.s. \rangle$$

$$S.D. \quad \Rightarrow \quad \langle g.s. \mid \sum_{i=1}^{A} \tilde{\sigma}(i) \left( a_0^{GT} + a_1^{GT} \tau_3(i) \right) \mid g.s. \rangle$$

which has been the basis for most comparisons among experiments
Recent efforts to be more systematic:
- “bottom up” effective theory approach involving leading operators of Fan, Reece, and Wang
  
arXiv: 1008:1591

\[
\begin{align*}
S.I \text{ (scalar)} & \Leftrightarrow A \\
\text{WIMP edm} & \Leftrightarrow Z \\
S.D & \Leftrightarrow \text{GT strength} \\
S.D. & \Leftrightarrow \text{Nuclear CP – odd dipole}
\end{align*}
\]

- a more systematic “bottom up” expansion in which the most general Galilean-invariant effective interaction arising from exchange of particles of spin one or less was derived
  
  +

  embedding of that operator within the nucleus to determine the most general CP- and P-conserving scattering moments
  
Fitzpatrick, WH, Katz, Lubbers, Xu
  
arXiv: 1203:3542
The nucleon-level effective interaction that arises from this treatment and that conserves CP has 11 invariants (x 2 for isospin)

\[ \mathcal{L}_{EFT} = a_1 \mathbf{1} + a_2 \mathbf{v}^\perp \cdot \mathbf{v}^\perp + a_3 \mathbf{S}_N \cdot (\mathbf{q} \times \mathbf{v}^\perp) + a_4 \mathbf{S}_x \cdot \mathbf{S}_N + ia_5 \mathbf{S}_x \cdot (\mathbf{q} \times \mathbf{v}^\perp) + a_6 \mathbf{S}_x \cdot \mathbf{q} \mathbf{S}_N \cdot \mathbf{q}^* \\
+ a_7 \mathbf{S}_N \cdot \mathbf{v}^\perp + a_8 \mathbf{S}_x \cdot \mathbf{v}^\perp + ia_9 \mathbf{S}_x \cdot (\mathbf{S}_N \times \mathbf{q}) + ia_{10} \mathbf{S}_N \cdot \mathbf{q} + ia_{11} \mathbf{S}_x \cdot \mathbf{q} \]

to quadratic order.
(Note to weak interaction experts: the Galilean invariance leads to a Hermitian velocity operator that is less easily obtained in standard treatments that begin with covariant interactions)

Forces one to deal correctly with recoil currents

\[ \mathbf{v}^\perp \rightarrow \frac{1}{2} (\mathbf{v}_\text{X,i} - \mathbf{v}_\text{Nuc,i} + \mathbf{v}_\text{X,f} - \mathbf{v}_\text{Nuc,f}) \]

\[ + \frac{1}{2} \left[ \sum_{i=1}^{A} \frac{1}{2iM_N} \left( -\hat{\nabla}(i) \delta(\mathbf{x} - \mathbf{x}_i) + \delta(\mathbf{x} - \mathbf{x}_i) \hat{\nabla} \right) \right] \text{ intrinsic} \]
The WIMP-nucleus Hamiltonian can be constructed (nucleon level)

\[
\mathcal{H}_{ET}(\vec{x}) = \sum_{i=1}^{A} l_0(i) \delta(\vec{x} - \vec{x}_i) + \sum_{i=1}^{A} l^A_0(i) \frac{1}{2M} \left[ -\frac{1}{i} \vec{\nabla}_i \cdot \vec{\sigma}(i) \delta(\vec{x} - \vec{x}_i) + \delta(\vec{x} - \vec{x}_i) \vec{\sigma}(i) \cdot \frac{1}{i} \vec{\nabla}_i \right] \\
+ \sum_{i=1}^{A} \vec{l}_5(i) \cdot \vec{\sigma}(i) \delta(\vec{x} - \vec{x}_i) + \sum_{i=1}^{A} \vec{l}_M(i) \cdot \frac{1}{2M} \left[ -\frac{1}{i} \vec{\nabla}_i \delta(\vec{x} - \vec{x}_i) + \delta(\vec{x} - \vec{x}_i) \frac{1}{i} \vec{\nabla}_i \right] \\
+ \sum_{i=1}^{A} \vec{l}_E(i) \cdot \frac{1}{2M} \left[ \vec{\nabla}_i \times \vec{\sigma}(i) \delta(\vec{x} - \vec{x}_i) + \delta(\vec{x} - \vec{x}_i) \vec{\sigma}(i) \times \vec{\nabla}_i \right]
\]

All elements are familiar from past studies of electroweak interactions (including neutrino scattering)
The WIMP-nucleus Hamiltonian can be constructed

\[ \mathcal{H}_{ET}(x) = \sum_{i=1}^{A} l_0(i) \delta(x - x_i) + \sum_{i=1}^{A} l_0^A(i) \frac{1}{2M} \left[ -\frac{1}{i} \vec{\nabla}_i \cdot \vec{\sigma}(i) \delta(x - x_i) + \delta(x - x_i) \vec{\sigma}(i) \cdot \frac{1}{i} \vec{\nabla}_i \right] \]

\[ + \sum_{i=1}^{A} l_5(i) \cdot \vec{\sigma}(i) \delta(x - x_i) + \sum_{i=1}^{A} l_M^A(i) \cdot \frac{1}{2M} \left[ -\frac{1}{i} \vec{\nabla}_i \delta(x - x_i) + \delta(x - x_i) \frac{1}{i} \vec{\nabla}_i \right] \]

\[ + \sum_{i=1}^{A} \mathcal{E}_i(i) \cdot \frac{1}{2M} \left[ \vec{\nabla}_i \times \vec{\sigma}(i) \delta(x - x_i) + \delta(x - x_i) \vec{\sigma}(i) \times \vec{\nabla}_i \right] \]

We will be interested generically in elastic channels

This is the vector charge density probed in elastic electron scattering or in coherent neutrino scattering
The WIMP-nucleus Hamiltonian can be constructed

\[ H_{ET}(\tilde{x}) = \sum_{i=1}^{A} l_{0}(i) \delta(\tilde{x} - \tilde{x}_{i}) + \sum_{i=1}^{A} l_{0}^{A}(i) \frac{1}{2M} \left[ -\frac{1}{i} \hat{\nabla}_{i} \cdot \vec{\sigma}(i) \delta(\tilde{x} - \tilde{x}_{i}) + \delta(\tilde{x} - \tilde{x}_{i}) \vec{\sigma}(i) \cdot \frac{1}{i} \hat{\nabla}_{i} \right] \]

\[ + \sum_{i=1}^{A} \vec{l}_{S}(i) \cdot \vec{r} + \sum_{i=1}^{A} \vec{l}_{M}(i) \cdot \frac{1}{2M} \left[ -\frac{1}{i} \hat{\nabla}_{i} \delta(\tilde{x} - \tilde{x}_{i}) + \delta(\tilde{x} - \tilde{x}_{i}) \frac{1}{i} \hat{\nabla}_{i} \right] \]

\[ + \sum_{i=1}^{A} \vec{l}_{E}(i) \cdot \frac{1}{2M} \left[ \hat{\nabla}_{i} \times \vec{\sigma}(i) \delta(\tilde{x} - \tilde{x}_{i}) + \delta(\tilde{x} - \tilde{x}_{i}) \vec{\sigma}(i) \times \hat{\nabla}_{i} \right] \]

This is an operator density studied in beta decay, through \(0^- \leftrightarrow 0^+\) inelastic transitions.
The WIMP-nucleus Hamiltonian can be constructed

\[ \mathcal{H}_{ET}(\vec{x}) = \sum_{i=1}^{A} l_0(i) \delta(\vec{x} - \vec{x}_i) + \sum_{i=1}^{A} l_0^A(i) \frac{1}{2M} \left[ -\frac{1}{i} \hat{\nabla}_i \cdot \vec{\sigma}(i) \delta(\vec{x} - \vec{x}_i) + \delta(\vec{x} - \vec{x}_i) \vec{\sigma}(i) \cdot \frac{1}{i} \hat{\nabla}_i \right] \\
+ \sum_{i=1}^{A} \vec{l}_5(i) \cdot \vec{\sigma}(i) \delta(\vec{x} - \vec{x}_i) + \sum_{i=1}^{A} \vec{l}_M(i) \cdot \frac{1}{2M} \left[ -\frac{1}{i} \hat{\nabla}_i \delta(\vec{x} - \vec{x}_i) + \delta(\vec{x} - \vec{x}_i) \frac{1}{i} \hat{\nabla}_i \right] \\
+ \sum_{i=1}^{A} \vec{l}_E(i) \cdot \frac{1}{2M} \left[ \hat{\nabla}_i \times \vec{\sigma}(i) \delta(\vec{x} - \vec{x}_i) + \delta(\vec{x} - \vec{x}_i) \vec{\sigma}(i) \times \hat{\nabla}_i \right] \]

This spin density dominates neutrino-nucleus inelastic scattering at solar and supernova neutrino energies. We know a lot about its elastic moments due to nuclear magnetic moments, etc.
The WIMP-nucleus Hamiltonian can be constructed

\[
\mathcal{H}_{ET}(\vec{x}) = \sum_{i=1}^{A} l_{0}(i) \delta(\vec{x} - \vec{x}_{i}) + \sum_{i=1}^{A} l_{0}^{A}(i) \frac{1}{2M} \left[ -\frac{1}{i} \vec{\nabla}_{i} \cdot \vec{\sigma}(i) \delta(\vec{x} - \vec{x}_{i}) + \delta(\vec{x} - \vec{x}_{i}) \vec{\sigma}(i) \cdot \frac{1}{i} \vec{\nabla}_{i} \right] \\
+ \sum_{i=1}^{A} \vec{l}_{5}(i) \cdot \vec{\sigma}(i) \delta(\vec{x} - \vec{x}_{i}) + \sum_{i=1}^{A} \vec{l}_{M}(i) \cdot \frac{1}{2M} \left[ -\frac{1}{i} \vec{\nabla}_{i} \delta(\vec{x} - \vec{x}_{i}) + \delta(\vec{x} - \vec{x}_{i}) \frac{1}{i} \vec{\nabla}_{i} \right] \\
+ \sum_{i=1}^{A} \vec{l}_{E}(i) \cdot \frac{1}{2M} \left[ \vec{\nabla}_{i} \times \vec{\sigma}(i) \delta(\vec{x} - \vec{x}_{i}) + \delta(\vec{x} - \vec{x}_{i}) \vec{\sigma}(i) \times \vec{\nabla}_{i} \right]
\]

This is the vector convection-current response familiar from inelastic electron and neutrino scattering; elastic response known from back-angle magnetic electron scattering and from atomic hyperfine interactions.
The WIMP-nucleus Hamiltonian can be constructed

\[ \mathcal{H}_{ET}(\vec{x}) = \sum_{i=1}^{A} \mathcal{L}_0^{(i)} \delta(\vec{x} - \vec{x}_i) + \sum_{i=1}^{A} \mathcal{L}_0^{A(i)} \frac{1}{2M} \left[ -\frac{1}{i} \vec{\nabla}_i \cdot \vec{\sigma}(i) \delta(\vec{x} - \vec{x}_i) + \delta(\vec{x} - \vec{x}_i) \vec{\sigma}(i) \cdot \frac{1}{i} \vec{\nabla}_i \right] 
+ \sum_{i=1}^{A} \mathcal{L}_5^{(i)} \cdot \vec{\sigma}(i) \delta(\vec{x} - \vec{x}_i) + \sum_{i=1}^{A} \mathcal{L}_M^{(i)} \cdot \frac{1}{2M} \left[ -\frac{1}{i} \vec{\nabla}_i \delta(\vec{x} - \vec{x}_i) + \delta(\vec{x} - \vec{x}_i) \frac{1}{i} \vec{\nabla}_i \right] 
+ \sum_{i=1}^{A} \mathcal{L}_E^{(i)} \cdot \frac{1}{2M} \left[ \vec{\nabla}_i \times \vec{\sigma}(i) \delta(\vec{x} - \vec{x}_i) + \delta(\vec{x} - \vec{x}_i) \vec{\sigma}(i) \times \vec{\nabla}_i \right] \]

vector spin-velocity current

The most exotic of the contributing densities: does not contribute in order(1/M) in neutrino physics due to the time-reversal properties of weak currents, but the associated inelastic response was discussed by Serot, who showed that this density arises in 1/M^2, but accompanied by a q_0. Thus we have no elastic probe of this operator.
The WIMP-nucleus Hamiltonian can be constructed

\[ H_{ET}(\vec{x}) = \sum_{i=1}^{A} l_0(i) \delta(\vec{x} - \vec{x}_i) + \sum_{i=1}^{A} l_0^A(i) \frac{1}{2M} \left[ -\frac{1}{i} \vec{\nabla}_i \cdot \vec{\sigma}(i) \delta(\vec{x} - \vec{x}_i) + \delta(\vec{x} - \vec{x}_i) \vec{\sigma}(i) \cdot \frac{1}{i} \vec{\nabla}_i \right] \]

\[ + \sum_{i=1}^{A} l_5(i) \cdot \vec{\sigma}(i) \delta(\vec{x} - \vec{x}_i) + \sum_{i=1}^{A} l_M(i) \cdot \frac{1}{2M} \left[ -\frac{1}{i} \vec{\nabla}_i \delta(\vec{x} - \vec{x}_i) + \delta(\vec{x} - \vec{x}_i) \frac{1}{i} \vec{\nabla}_i \right] \]

\[ + \sum_{i=1}^{A} l_E(i) \cdot \frac{1}{2M} \left[ \vec{\nabla}_i \times \vec{\sigma}(i) \delta(\vec{x} - \vec{x}_i) + \delta(\vec{x} - \vec{x}_i) \vec{\sigma}(i) \times \vec{\nabla}_i \right] \]

We see the nucleon-level effective theory has naturally mapped on to all of the possible charge and three-vector densities one can construct from \{ 1(i), \vec{\sigma}(i), \vec{\nabla}(i) \} consistent with our exchange assumption and hermiticity.
The WIMP-nucleus Hamiltonian can be constructed

\[ \mathcal{H}_{ET}(\vec{x}) = \sum_{i=1}^{A} l_0(i) \delta(\vec{x} - \vec{x}_i) + \sum_{i=1}^{A} l_0^A(i) \frac{1}{2M} \left[ -\frac{1}{i} \nabla_i \cdot \vec{\sigma}(i) \delta(\vec{x} - \vec{x}_i) + \delta(\vec{x} - \vec{x}_i) \vec{\sigma}(i) \cdot \frac{1}{i} \nabla_i \right] \]

\[ + \sum_{i=1}^{A} \vec{l}_5(i) \cdot \vec{\sigma}(i) \delta(\vec{x} - \vec{x}_i) + \sum_{i=1}^{A} \vec{l}_M(i) \cdot \frac{1}{2M} \left[ -\frac{1}{i} \nabla_i \delta(\vec{x} - \vec{x}_i) + \delta(\vec{x} - \vec{x}_i) \frac{1}{i} \nabla_i \right] \]

\[ + \sum_{i=1}^{A} \vec{l}_E(i) \cdot \frac{1}{2M} \left[ \nabla_i \times \vec{\sigma}(i) \delta(\vec{x} - \vec{x}_i) + \delta(\vec{x} - \vec{x}_i) \vec{\sigma}(i) \times \nabla_i \right] \]

These 5 nuclear densities are coupled to WIMP tensors that can project out in principle a total of 8 nuclear responses. These 8 responses should be viewed as one’s available probes of DM. The WIMP tensors are themselves functions of 11 DM EFT couplings

\[
\begin{align*}
    l_0(i) & \quad l_0^A(i) & \quad \vec{l}_5(i) \cdot \vec{q} & \quad \vec{l}_5(i) \times \vec{q} \\
    \vec{l}_M(i) \cdot \vec{q} & \quad \vec{l}_M(i) \times \vec{q} & \quad \vec{l}_E(i) \cdot \vec{q} & \quad \vec{l}_E(i) \times \vec{q}
\end{align*}
\]
The Galilean effective theory defines the candidate nuclear densities
Response constrained by good parity and time reversal of nuclear g.s.

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where we list only the leading multipoles in $J$ above
Response constrained by good **parity** and time reversal of nuclear g.s.

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The Galilean effective theory defines the candidate densities

Response constrained by good parity and time reversal of nuclear g.s.

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6 (not 2!) independent responses based on symmetry of 4-current densities
<table>
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<th>Response × $\left[ \frac{4\pi}{2J_i+1} \right]^{-1}$</th>
<th>Leading Multipole</th>
<th>Long-wavelength Limit</th>
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<td>$\sum_{J=1,3,...}^{\infty}</td>
<td>\langle J_i</td>
<td></td>
<td>\Sigma''_{JM}</td>
</tr>
<tr>
<td>$\sum_{J=1,3,...}^{\infty}</td>
<td>\langle J_i</td>
<td></td>
<td>\Sigma'_{JM}</td>
</tr>
<tr>
<td>$\sum_{J=1,3,...}^{\infty}</td>
<td>\langle J_i</td>
<td></td>
<td>\Delta_{JM}</td>
</tr>
<tr>
<td>$\sum_{J=2,4,...}^{\infty}</td>
<td>\langle J_i</td>
<td></td>
<td>\Phi''_{JM}</td>
</tr>
<tr>
<td>$\sum_{J=2,4,...}^{\infty}</td>
<td>\langle J_i</td>
<td></td>
<td>\Phi'_{JM}</td>
</tr>
</tbody>
</table>

Two scalar (one scalar/tensor), three vector, one tensor

Calculate in SM the responses for the key isotopes...
theorist’s analog of the experimentalist’s photo of equipment...

\[
\frac{1}{2J_i + 1} \sum_{M_i, M_f} |\langle J_i M_f | H | J_i M_i \rangle|^2 = \frac{4\pi}{2J_i + 1} \left[ \sum_{J=1,3,\ldots}^{\infty} |\langle J_i \parallel \vec{l}_5 \cdot \hat{q} \sum''_J(q) \parallel J_i \rangle|^2 \right.
\]

\[
+ \sum_{J=0,2,\ldots}^{\infty} \left\{ \langle J_i \parallel l_0 M_J(q) \parallel J_i \rangle^2 + |\langle J_i \parallel \vec{l}_E \cdot \hat{q} \frac{q}{m_N} \Phi''(q) \parallel J_i \rangle|^2 \right. 
\]

\[
+ \left. 2\text{Re} \left[ \langle J_i \parallel \vec{l}_E \cdot \hat{q} \frac{q}{m_N} \Phi''(q) \parallel J_i \rangle \langle J_i \parallel l_0 M_J(q) \parallel J_i \rangle^* \right] \right\}
\]

\[
+ \frac{q^2}{2m_N^2} \sum_{J=2,4,\ldots}^{\infty} \left( \langle J_i \parallel \vec{l}_E \Phi'_J(q) \parallel J_i \rangle \cdot \langle J_i \parallel \vec{l}_E \Phi'_J(q) \parallel J_i \rangle^* - |\langle J_i \parallel \vec{l}_E \cdot \hat{q} \Phi'_J(q) \parallel J_i \rangle|^2 \right)
\]

\[
+ \sum_{J=1,3,\ldots}^{\infty} \left\{ \frac{q^2}{2m_N^2} \left( \langle J_i \parallel \vec{l}_M \Delta_J(q) \parallel J_i \rangle \cdot \langle J_i \parallel \vec{l}_M \Delta_J(q) \parallel J_i \rangle^* - |\langle J_i \parallel \vec{l}_M \cdot \hat{q} \Delta_J(q) \parallel J_i \rangle|^2 \right) \right.
\]

\[
+ \left. \frac{1}{2} \left( \langle J_i \parallel \vec{l}_5 \Sigma'_J(q) \parallel J_i \rangle \cdot \langle J_i \parallel \vec{l}_5 \Sigma'_J(q) \parallel J_i \rangle^* - |\langle J_i \parallel \vec{l}_5 \cdot \hat{q} \Sigma'_J(q) \parallel J_i \rangle|^2 \right) \right\}
\]

\[
+ \left. 2\text{Re} \left[ iq \cdot \langle J_i \parallel \vec{l}_M \frac{q}{m_N} \Delta_J(q) \parallel J_i \rangle \times \langle J_i \parallel \vec{l}_5 \Sigma'_J(q) \parallel J_i \rangle^* \right] \right\}
\]

the general result for scattering probability
Shell model calculations performed (CENPA), modest bases < 0.65M after symmetries ⇒ could be substantially improved (shape transitions)

Purpose: quick survey to access target response variability as well as the systematic of these operators

\[^{19}\text{F}, ^{23}\text{Na}\] standard \(2s_{1/2} 1d_{3/2} 1d_{5/2}\) calculations, BW interaction

\[^{70,72,73,74,76}\text{Ge}\ 1f_{5/2} 2p_{1/2} 2p_{3/2} 1g_{9/2}\] above \(^{56}\text{Ni}\) core, truncated so that occupation of the \(1g_{9/2}\) orbit is no more than minimum occupation + 2; potential from Madrid/Strasbourg group

\[^{127}\text{I}, ^{128,129,130,131,132,134,136}\text{Xe}\ 3s_{1/2} 2d_{3/2} 2d_{5/2} 1g_{7/2} 1h_{11/2}\] above \(^{100}\text{Sn}\) core, for most similar truncations involving \(1h_{11/2}\) occupation; potential based on bare g-matrix as modified by Baldridge, Vary
vector charge (amplitudes!)

\[ |M_p| \]

\[ |M_n| \]

\[ |\Sigma_p'| \]

\[ |\Sigma_n'| \]

transverse electric axial (spin) response
longitudinal electric axial (spin) response

vector transverse magnetic (orbital angular momentum)
semi-coherent isoscalar $\bar{\sigma}(i) \cdot \bar{l}(i)$

note the absence of the nuclear-physics-allowed tensor response

cross sections arranged so the nucleus is the probe: operators map onto the unique nuclear densities with definite behavior under $P,T$ the coefficients map onto the EFT coefficients, and thus make the “hand shake” with ultraviolet theories
Bottom line: Adequate particle physics freedom in a CP-conserving, \( \leq \) (spin-1) ET to turn on or off any of the five nuclear responses

Conclusions

* The elastic response to DM is considerably richer than traditionally described: huge variations among experimental sensitivities possible

* The Galilean invariant ET is an elegant way to factor the nuclear and particle physics: the two communities have a simple meeting point

* Recommend a view of DM where the nuclear densities are viewed as the probe: Our “master formula” was constructed in this way

* Quite surprising to me that the field is this mature, yet previously lacked a straight-forward delineation of the response possibilities: nuclear physics is useful!
Thanks to my collaborators Liam Fitzpatrick and Ami Katz

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