Hydrodynamics of CCSNe at the Transition to Explosion

Rodrigo Fernández
Einstein Fellow - Institute for Advanced Study
Stalled Shock

Result of Core-Collapse and Bounce

Neutrino Mechanism: heating due to neutrino absorption by nucleons

(Bethe & Wilson 1985)
For stars that form iron cores

\[ M \gtrsim 10M_\odot \]

Agreement in Supernova Community


O-Ne-Mg cores do explode in 1D

(Kitaura et al. 2006, Burrows et al. 2007)
Multidimensional Effects

\[ v_r \left[ 10^8 \text{ cm s}^{-1} \right] \]

Buras et al. (2006)
More Efficient with Increasing Dimensionality?

In disagreement with Hanke et al. (2011)

Nordhaus et al. (2010)
Parametric Hydrodynamic Study

• Identify hydrodynamic processes responsible for converting accretion flow into explosion

• Understand their dependence on system parameters and dimensionality

• Assess the effect of hydrodynamic instabilities on explosion mechanism (magnitude and robustness)
Method

- Time-dependent Hydrodynamic Simulations (FLASH3.2, modified grid)
- **Steady-state** initial and boundary conditions (stalled shock)
- Bruenn (1985) weak interaction rates, lightbulb heating
- **Point-mass** gravity (time-independent)
- Begin with **spherical symmetry** (no turbulence)

- Equations solved:

\[
\begin{align*}
\frac{\partial \rho}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \rho \nu_r) &= 0 \\
\frac{\partial \nu_r}{\partial t} + \nu_r \frac{\partial \nu_r}{\partial r} + \frac{1}{\rho} \frac{\partial \rho}{\partial r} + \frac{GM}{r^2} &= 0
\end{align*}
\]

\[
\begin{align*}
\frac{\partial (\rho e)}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \nu_r \rho e) + \nu_r \frac{GM}{r^2} &= L_{\text{net}} \\
\frac{\partial Y_e}{\partial t} + \nu_r \frac{\partial Y_e}{\partial r} &= \Gamma_{\text{net}}.
\end{align*}
\]

RF (2012)
Limiting Cases (1D)

1) **No Heating:** Accretion Shock

\[ L_\nu \left( \frac{\sigma_\nu}{r^2} \right) \left( \frac{M_g}{m_n} \right) t_{\text{dyn}} \gg E_g \]

\[ t_{\text{dyn}} \gg t_{\text{heat}} \]

2) **Strong Heating:** Sedov-like explosion
Limiting Luminosity in Steady-State

(Burrows & Goshy 1993)

**BG 93 conjecture I**: explosion involves global instability of accretion flow

**BG93 conjecture II**: instability threshold lies at the limiting luminosity
Limiting Luminosity in Steady-State

Antesonic condition:

\[
\frac{c_s^2}{v_{esc}^2} \approx 0.2
\]

Assumes fixed boundary conditions at neutrinosphere and upstream flow

\[
x = \frac{r c_T^2}{2GM} \quad \mathcal{M} = \frac{v_r}{c_T}
\]

(Pejcha & T.Thompson 2012)
Limiting Luminosity in Steady-State

\[ \tau_\nu = \int_{R_\nu}^{R_s} (\rho \kappa_\nu) \, dr \]

\[ \frac{\partial \tau_\nu}{\partial R_s} = (\rho \kappa_\nu) \bigg|_{R_s} + \int_{R_\nu}^{R_s} \frac{\partial (\rho \kappa_\nu)}{\partial R_s} \, dr \]
Transition to Explosion

Fiducial model: \( \dot{M} = 0.3 \, M_\odot \, \text{s}^{-1} \)
\[ R_{\nu} = 30 \, \text{km} \]

\[ t_{\text{dyn}} \sim 2 \, \text{ms} \quad t_{\text{heat}} \sim 10 \, \text{ms} \]
Work Integral

From stellar pulsation theory (Eddington 1926):

$$W = \int \frac{dE}{dt} \, dt$$

If $W > 0$, driving

If $W < 0$, damping

Positive work leads to increase in pulsation kinetic energy

(e.g., Cox 1974)
Work Integral in CCSNe

• Include region in sonic contact

• Neither mass nor volume are constant

\[
\frac{\partial E}{\partial t} = \int d^3 x \frac{\partial (\rho e_{\text{tot}})}{\partial t} + 4\pi R_s^2 \dot{R}_s (\rho e_{\text{tot}}) \bigg|_{R_s} \equiv \dot{E}_{\text{tot}}
\]

\[
e_{\text{tot}} = \frac{1}{2} v^2 + e_{\text{int}} - \frac{GM}{r}
\]
Energy equation

\[
\frac{\partial}{\partial t} \left( \frac{1}{2} \rho v_r^2 + \rho e_{\text{int}} \right) + \nabla \cdot \left( \rho v \left[ \frac{1}{2} v_r^2 + e_{\text{int}} + \frac{p}{\rho} \right] \right) + \rho v_r \frac{GM}{r^2} = L_{\text{net}}
\]

If point mass is time-independent:

\[
\rho v_r \frac{GM}{r^2} = \rho v \cdot \nabla \left( -\frac{GM}{r} \right) = \nabla \cdot \left( \rho v \left[ -\frac{GM}{r} \right] \right) - \nabla \cdot (\rho v) \left[ -\frac{GM}{r} \right] + \frac{\partial \rho}{\partial t} \left[ -\frac{GM}{r} \right] = \frac{\partial}{\partial t} \left( -\rho \frac{GM}{r} \right)
\]

to rate of change

\[
\frac{\partial}{\partial t} (\rho e_{\text{tot}}) + \nabla \cdot \left( \rho v \left[ e_{\text{tot}} + \frac{p}{\rho} \right] \right) = L_{\text{net}}
\]

\[
e_{\text{tot}} = \frac{1}{2} v^2 + e_{\text{int}} - \frac{GM}{r}
\]
Driving and Damping: Oscillatory Mode

Change of Total Energy (Work Integral)

\[ e_{tot} = \frac{1}{2}v^2 + e_{int} - \frac{GM}{r} \]

\[
\frac{\partial E}{\partial t} = \int d^3x \frac{\partial (\rho e_{tot})}{\partial t} + 4\pi R_s^2 \dot{R}_s (\rho e_{tot}) \bigg|_{R_s} \equiv \dot{E}_{tot}
\]

\[
\dot{E}_{N} = \int_{R_{in}}^{R_s} 4\pi r^2 dr \mathcal{L}_{net}
\]

\[
\dot{E}_{up} = 4\pi R_s^2 \left[ v_r (\rho e_{tot} + p) \right] \bigg|_{R_s}
\]

\[
\dot{E}_{dn} = 4\pi R_{in}^2 \left[ v_r (\rho e_{tot} + p) \right] \bigg|_{R_{in}}
\]

\[
\dot{E}_{s} = 4\pi R_s^2 \dot{R}_s (\rho e_{tot}) \bigg|_{R_s},
\]

Cancel out in steady-state

Damps on expansion

Driving: positive energy generation

Damping: negative energy generation
Driving and Damping: Non-Oscillatory Mode

Change of Total Energy (Work Integral)

\[ e_{\text{tot}} = \frac{1}{2} v^2 + e_{\text{int}} - \frac{GM}{r} \]

\[ \frac{\partial E}{\partial t} = \int d^3x \frac{\partial (\rho e_{\text{tot}})}{\partial t} + 4\pi R_s^2 \dot{R}_s (\rho e_{\text{tot}}) \bigg|_{R_s} \equiv \dot{E}_{\text{tot}} \]

\[ = \dot{E}_N - \dot{E}_{\text{up}} + \dot{E}_{\text{dn}} + \dot{E}_s, \]

\[
\begin{align*}
\dot{E}_N &= \int_{R_{\text{in}}}^{R_s} 4\pi r^2 dr \mathcal{L}_{\text{net}} \\
\dot{E}_{\text{up}} &= 4\pi R_s^2 [v_r (\rho e_{\text{tot}} + p)] \bigg|_{R_s} \\
\dot{E}_{\text{dn}} &= 4\pi R_{\text{in}}^2 [v_r (\rho e_{\text{tot}} + p)] \bigg|_{R_{\text{in}}} \\
\dot{E}_s &= 4\pi R_s^2 \dot{R}_s (\rho e_{\text{tot}}) \bigg|_{R_s},
\end{align*}
\]

Damping on expansion

Driving: positive energy generation

Damping: negative energy generation
Radial instability leads to Explosion (1D)

Positive, and largely exceeds \( \dot{E}_{\text{up}} \),

Does not decrease if heating by accretion neglected and neutrinospheric parameters are constant.

Only source of damping, decreases in magnitude due to nuclear recombination.
3D Hydrodynamic Studies

Nordhaus et al. (2010)

Hanke et al. (2011)
Approximate Instability Criteria (1D)

Oscillatory: \( t_{\text{adv}} - g > t_{\text{adv}} - e \)

Non-Oscillatory: \( t_{\text{adv}} - g > t_{\text{heat}} - e \)

\[
t_{\text{adv}}(r_1, r_2) = \int_{r_1}^{r_2} \frac{dr}{|v_r|} = \frac{M(r_1, r_2)}{\dot{M}}
\]

\[
t_{\text{adv}} - g = \int_{R_g}^{r} \frac{dr}{|v_r|} \quad R_g = \text{gain radius}
\]

\[
t_{\text{adv}} - e = \int_{r_e}^{R_g} \frac{dr}{|v_r|} \quad r_e = \text{“phase shift” radius}
\]

\[
t_{\text{heat}} - g = \frac{\int_{R_g}^{R_g} d^3x (\rho e_{\text{tot}})}{\int_{R_g}^{R_g} d^3x \mathcal{L}_{\text{net}}}
\]
Threshold Luminosities for Instability (1D)
Implications for Multi-Dimensional Case

1. Unstable oscillatory modes in multi-D (SASI) do not explode the system by themselves. (e.g., Iwakami et al. 2008)

2. Non-oscillatory modes of high angular degree associated with convection \((\ell \gtrsim 5)\) (Yamasaki & Yamada 2007)
3. Hence, multi-D explosion mediated by either:

   a) Unstable spherical mode (oscillatory and/or non-oscillatory), and possibly

   b) Unstable non-oscillatory mode of low angular degree (\( \ell = 1, 2 \))

   Will couple non-linearly to \( \ell = 0 \)

4. Background flow is turbulent in multi-D, thus instability thresholds will change.

   A semi-analytic study would require perturbing time-averaged quantities.
3D Hydrodynamic Studies

Nordhaus et al. (2010)

Hanke et al. (2011)
Summary

1. Radial instability of the shock is a **sufficient** condition for explosion
   \[
   \text{IF} \quad \begin{align*}
   & \text{a) Neutrinospheric parameters are constant in time} \\
   & \text{b) Heating from accretion luminosity is neglected} \\
   & \text{c) Mass accretion rate is constant or decreases with time}
   \end{align*}
   \]

2. Radial **instability thresholds** can be approximately (~5% in $L_\nu$) described by global properties of the flow

3. Instability thresholds are different from **limiting luminosity** for a steady-state configuration (BG93 limit)

4. Multi-dimensional explosions involve growth of spherical and/or non-spherical modes in turbulent background flow