Quarkonium (bottomonium)
in a weakly-coupled quark-gluon plasma

Antonio Vairo

Technische Universität München
In 1986, Matsui and Satz suggest quarkonium as an ideal quark-gluon plasma probe.

- Heavy quarks are formed early in heavy-ion collisions: \(1/m \sim 0.1 \text{ fm} \ll 1 \text{ fm}\).
- Heavy quarkonium formation will be sensitive to the medium.
- The dilepton signal makes the quarkonium a clean experimental probe.
Quarkonium in a thermal bath

A first step towards understanding quarkonium in a quark-gluon plasma may consist in studying how a thermal bath modifies spectrum and width of a heavy quarkonium at rest.

We will work in a well definite setting:
- weakly coupled quarkonium;
- $\pi T \gg \Lambda_{\text{QCD}}$ (realized in a weakly-coupled plasma).

This situation may be realized by the bottomonium ground state at LHC.

Under these conditions, we will provide a description in terms of non-relativistic EFTs of QCD at finite temperature.

Note. What is crucial is the hierarchy of energy scales; the weakly-coupled nature allows definite perturbative calculations, but is not necessary for the EFTs set up. Many results will hold also in a strongly coupled framework.
Scales

Quarkonium in a medium is characterized by different energy and momentum scales:

- the scales of a non-relativistic bound state ($v$ is the relative heavy-quark velocity):
  - $m$ (mass),
  - $mv$ (momentum transfer, inverse distance),
  - $mv^2$ (kinetic energy, binding energy, potential $V$), ...

- the thermodynamical scales:
  - $\pi T$ (temperature),
  - $m_D$ (Debye mass, i.e. screening of the chromoelectric interactions), ...

Non-relativistic scales are hierarchically ordered: $m \gg mv \gg mv^2$,
we may assume that also the thermodynamical scales are: $\pi T \gg m_D$. 
Non-relativistic Effective Field Theories at finite $T$

We assume that bound states exist for

- $\pi T \ll m$
- $(1/r) \sim m v \gtrsim m_D$

We neglect smaller thermodynamical scales.
The bottomonium ground state at finite $T$

The relative size of non-relativistic and thermal scales depends on the medium and on the quarkonium state. The bottomonium ground state, which is a weakly coupled non-relativistic bound state: $m \approx m_{\alpha_s}, m \alpha_s \sim \Lambda_{QCD}$, produced in the QCD medium of heavy-ion collisions at the LHC may possibly realize the hierarchy

$$m \approx 5 \text{ GeV} > m_{\alpha_s} \approx 1.5 \text{ GeV} > \pi T \approx 1 \text{ GeV} > m_{\alpha_s}^2 \approx 0.5 \text{ GeV} > m_D, \Lambda_{QCD}$$
γ suppression at CMS

CMS PRL 107 (2011) 052302
NRQCD

NRQCD is obtained by integrating out modes associated with the scale $m$.

- The Lagrangian is organized as an expansion in $1/m$:

$$
\mathcal{L}_{\text{NRQCD}} = \psi^\dagger \left( iD_0 + \frac{D^2}{2m} + \ldots \right) \psi + \chi^\dagger \left( iD_0 - \frac{D^2}{2m} + \ldots \right) \chi + \ldots
- \frac{1}{4} F_{\mu \nu}^a F^{a \mu \nu} + \sum_{i=1}^{n_f} \bar{q}_i i \gamma \psi q_i
$$

$\psi$ ($\chi$) is the field that annihilates (creates) the (anti)fermion.

- The relevant dynamical scales of NRQCD are: $m_{\chi s}, m_{\alpha s}^2, \ldots, T, m_D, \ldots$

- Thermodynamical scales may be set to zero while matching.

- Caswell Lepage PLB 167 (1986) 437
- Bodwin Braaten Lepage PRD 51 (1995) 1125
pNRQCD

pNRQCD is obtained by integrating out modes associated with the scale $\frac{1}{r} \sim m_{\alpha_S}$

- The degrees of freedom of pNRQCD are quark-antiquark states (color singlet S, color octet O), low energy gluons and light quarks.
- The Lagrangian is organized as an expansion in $1/m$ and $r$:

$$L_{pNRQCD} = \int d^3r \ Tr \left\{ S^\dagger \left( i\partial_0 - \frac{p^2}{m} + \cdots - V_s \right) S \\
+ O^\dagger \left( iD_0 - \frac{p^2}{m} + \cdots - V_o \right) O \right\} - \frac{1}{4} F_{\mu\nu}^a F^{\mu\nu a} + \sum_{i=1}^{n_f} \bar{q}_i i\gamma_\mu q_i + \Delta L$$

$$\Delta L = \int d^3r \ V_A Tr \left\{ O^\dagger \cdot gE S + H.c. \right\} + \frac{V_B}{2} Tr \left\{ O^\dagger \cdot gE O + c.c. \right\} + \cdots$$

- At leading order in $r$, the singlet S satisfies the Schrödinger equation.
• Thermodynamical scales may be set to zero while matching.

• The static potential is the Coulomb potential:

\[ V_s(r) = -\frac{4}{3} \frac{\alpha_s}{r} + \ldots, \quad V_o(r) = \frac{\alpha_s}{6r} + \ldots \]

• \( V_A = V_B = 1 + O(\alpha_s^2) \)

• Feynman rules:

\[
\begin{align*}
\begin{array}{c}
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\end{array}
\end{align*}
\]

\[ = \theta(t) e^{-itH_S} \quad = \theta(t) e^{-itH_o} \left(e^{-i \int dt A^{\text{adj}}} \right) \]

\[ = O^\dagger \mathbf{r} \cdot \mathbf{gE} \quad = O^\dagger \{ \mathbf{r} \cdot \mathbf{gE}, O \} \]

Pineda Soto NP PS 64 (1998) 428
Brambilla Pineda Soto Vairo NPB 566 (2000) 275
Cancellation of divergences in the spectrum I

<table>
<thead>
<tr>
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<th>Thermal</th>
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<tbody>
<tr>
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<td></td>
</tr>
<tr>
<td>$T$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$m_\alpha_s^2$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Kniehl Penin NPB 563 (1999) 200
Real time

The contour of the partition function is modified to allow for real time:

In real time, the degrees of freedom double (“1” and “2”), however, the advantages are

- the framework becomes very close to the one for $T = 0$ EFTs;
- in the heavy quark sector, the second degrees of freedom, labeled “2”, decouple from the physical degrees of freedom, labeled “1”.

In the figure, the contour is shown as a line from $t_0$ to $t_f$ in the real time plane, with the imaginary time $t = t_0 + i/T$.
Real-time gluon propagator

- Free gluon propagator in Coulomb gauge:

\[ D^{(0)}_{00}(k) = D^{(0)}_{00}(k)_{T=0} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \]

\[ D^{(0)}_{ij}(k) = D^{(0)}_{ij}(k)_{T=0} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} + \left( \delta_{ij} - \frac{k^i k^j}{k^2} \right) 2\pi \delta(k^2) n_B(|k^0|) \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \]

where

\[ n_B(k^0) = \frac{1}{e^{k^0/T} - 1} \]

In Coulomb gauge, only transverse gluons carry a thermal part.
Real-time potential

- Quark-antiquark potential:

\[
V = \begin{pmatrix}
V & 0 \\
-2i \text{Im } V & -V^*
\end{pmatrix}
\]

- Quark-antiquark propagator:

\[
S_{QQ}^{(0)}(p) = \begin{pmatrix}
\frac{i}{p^0 - p^2/m - V + i\epsilon} & 0 \\
\frac{2 \text{Re } i}{p^0 - p^2/m - V + i\epsilon} & \frac{i}{p^0 - p^2/m - V^* - i\epsilon}
\end{pmatrix}
\]

The vanishing of the “12” component ensures that the “2” component decouples from the physical heavy quarks, i.e. the component “1”.

*Brambilla Ghiglieri Petreczky Vairo PRD 78 (2008) 014017*
Integrating out $T$ from pNRQCD modifies pNRQCD into pNRQCD$_{HTL}$ whose

- Yang–Mills Lagrangian gets the additional hard thermal loop (HTL) part;
  e.g. the longitudinal gluon propagator becomes

$$\frac{i}{k^2} \rightarrow \frac{i}{k^2 + m^2_D} \left(1 - \frac{i}{2k} \ln \frac{k_0 + k \pm i\eta}{k_0 - k \pm i\eta}\right)$$

where “+” identifies the retarded and “−” the advanced propagator;

- potentials get additional thermal corrections $\delta V$.

Braaten Pisarski PRD 45 (1992) 1827
Brambilla Ghiglieri Petreczky Vairo PRD 78 (2008) 014017
Escobedo Soto PRA 78 (2008) 032520
Vairo PoS CONFINEMENT8 (2008) 002
Integrating out $T$

The relevant diagram is

and radiative corrections. The loop momentum region is $k_0 \sim T$ and $k \sim T$.

- Since $T \gg (E - \frac{p^2}{m} - V_o)$ we may expand the octet propagator
  \[
  \frac{i}{E - \frac{p^2}{m} - V_o - k_0 + i\eta} = \frac{i}{-k_0 + i\eta} - \frac{i}{(-k_0 + i\eta)^2} + \ldots
  \]
Integrating out $T$: real potential

\[ \sim g^2 r^2 T^3 \times E/T \]

\[ \sim g^2 r^2 T^3 \times (m_D/T)^2 \]

\[ \text{Re} \, \delta V_s(r) = \frac{4}{9} \pi \alpha_s^2 r T^2 + \frac{8 \pi}{9m} \alpha_s T^2 \]
\[ + 4 \alpha_s I_T \left[ \frac{9}{8} \alpha_s^2 \frac{r}{T} - \frac{17}{3} \frac{\alpha_s^2}{m_T^2} + \frac{4 \pi \alpha_s}{9 m_T} \delta^3(r) + \frac{\alpha_s}{m_T} \left\{ \nabla_r^2, \frac{1}{r} \right\} \right] \]
\[ - 2 \zeta(3) \frac{\alpha_s}{\pi} r^2 T m_T^2 + \frac{8}{3} \zeta(3) \alpha_s^2 r^2 T^3 \]

\[ I_T = \frac{2}{\zeta} + \ln \frac{T^2}{\mu^2} - \gamma_E + \ln(4\pi) - \frac{5}{3} \]
Integrating out $T$: imaginary potential

\[
\text{Im} \delta V_s(r) = \frac{2}{9} \alpha_s r^2 T m_D^2 \left( -\frac{2}{3} + \gamma_E + \ln \pi - \ln \frac{T^2}{\mu^2} + \frac{2}{3} - 4 \ln 2 - 2 \frac{\zeta'(2)}{\zeta(2)} \right) + \frac{16\pi}{9} \ln 2 \alpha_s^2 r^2 T^3 \\
\sim g^2 r^2 T^3 \times \left( \frac{m_D}{T} \right)^2
\]
Landau damping

The Landau damping phenomenon originates from the scattering of the quarkonium with hard space-like particles in the medium.

\[
\text{Im} V_s(r) |_{\text{Landau-damping}} \sim \text{Re} V_s(r) \sim \alpha_s / r,
\]

the quarkonium dissociates:

\[
\pi T_{\text{dissociation}} \sim mg^{4/3}
\]

When \(1/r \sim m_D\), the interaction is screened; note that

\[
\pi T_{\text{screening}} \sim mg \gg \pi T_{\text{dissociation}}
\]

Laine Philipsen Romatschke Tassler JHEP 0703 (2007) 054
The $\Upsilon(1S)$ dissociation temperature:

<table>
<thead>
<tr>
<th>$m_c$ (MeV)</th>
<th>$T_{\text{dissociation}}$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\infty$</td>
<td>480</td>
</tr>
<tr>
<td>5000</td>
<td>480</td>
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<tr>
<td>2500</td>
<td>460</td>
</tr>
<tr>
<td>1200</td>
<td>440</td>
</tr>
<tr>
<td>0</td>
<td>420</td>
</tr>
</tbody>
</table>

A temperature $T$ about 1 GeV is below the dissociation temperature.

Escobedo Soto PRA 82 (2010) 042506
Energy and thermal width from the scale $T$

$$
\delta E_{1S}^{(T)} = \frac{2\pi}{3} \alpha_s^2 T^2 a_0 + \frac{8\pi}{9m} \alpha_s T^2 + \frac{7225}{324} \frac{E_1 I_2 a^3}{\pi} \frac{\alpha_s^2 T}{\pi} - \zeta(3) \alpha_s T \left( \frac{m_0^2}{\pi} - 8\alpha_s T^2 \right) a_0^2
$$

$$
\Gamma_{1S}^{(T)} = \left[ -\frac{4}{3} \alpha_s T m_0^2 \left( -\frac{2}{\epsilon} + \gamma_E + \ln \pi - \ln \frac{T^2}{\mu^2} + \frac{2}{3} - 4 \ln 2 - \frac{\zeta'(2)}{\zeta(2)} \right) \right. \\
\left. - \frac{32\pi}{3} \ln 2 \alpha_s^2 T^3 \right] a_0^2
$$

where $E_1 = -\frac{4m_0^2}{9}$ and $a_0 = \frac{3}{2m_0^2}$

Brambilla Escobedo Ghiglieri Soto Vairo JHEP 1009 (2010) 038
Cancellation of divergences in the spectrum II

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</tr>
<tr>
<td>$T$</td>
<td>scaleless</td>
<td>$\sim -m_\alpha s \frac{1}{\epsilon_{IR}}$</td>
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### Cancellation of divergences in the spectrum II

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<tr>
<td>( T )</td>
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</tr>
<tr>
<td>( m_\alpha^2 s )</td>
<td>[\quad]</td>
<td>[\quad]</td>
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- Concerning the width, the IR divergence at the scale \( T \) will cancel against an UV divergence at the scale \( m_\alpha^2 s \).
For general $T$, the thermal width due to Landau damping reads

$$\Gamma_{1S} = \sum_p \int_{q_{\text{min}}} \frac{d^3 q}{(2\pi)^3} f_p(|q|)(1 \pm f_p(|q|)) \sigma_{1S}(|q|),$$

where the sum runs over the different incoming light partons and $f_g = n_B$ or $f_q = n_F$.

$\sigma_{1S}$ is known as the quarkonium quasi-free dissociation cross section. The thermal NR EFTs provide analytic expressions of $\sigma_{1S}$ for different temperatures.
In the previous literature, it was assumed

$$\Gamma_{1S} = \sum_p \int_{q_{\text{min}}} d^3q \frac{1}{(2\pi)^3} f_p(|q|) \sigma_{\text{HQ}}(|q|),$$

with $\sigma_{\text{HQ}} = 2\sigma_c$, where $\sigma_c$ is the cross section for the process $pc \rightarrow pc$ at $T = 0$.

- Grandchamp Rapp, PLB 523 (2001) 60, ...

The EFT analysis proves this assumption to be incorrect, because

- the dependence on the thermal distributions of the incoming and outgoing partons is different;
- $\sigma_{1S}$ cannot be identified with $\sigma_{\text{HQ}}$, moreover it is temperature dependent.
Quasi-free dissociation: light-quark contribution

\[ \sigma_{qq} \]

\[ m_D a_0 = 0.001 \]

**blue line:** \( m v \gg T \gg m_D \gg E \)  
(dipole approximation)

**pink line:** \( T \sim m v \gg m_D \)

**yellow line:** \( T \gg m v \sim m_D \)

\[ \sigma_{eq} \equiv 8 \pi C_F n_f a_2^2 a_0^2 \]

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Quasi-free dissociation: gluon contribution

\[ m_D a_0 = 0.001 \]

blue line: \( m v \gg T \gg m_D \gg E \) (dipole approximation)

pink line: \( T \sim m v \gg m_D \)

yellow line: \( T \gg m v \sim m_D \)

\[ \sigma_{eg} \equiv 8 \pi C_F N_c a_s^2 a_0^2 \]

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Integrating out $E$

The relevant diagram is

where the loop momentum region is $k_0 \sim E$ and $k \sim E$.

- Gluons are HTL gluons.
- Since $k \sim E \ll T$ we may expand the Bose–Einstein distribution

$$n_B(k) = \frac{T}{k} - \frac{1}{2} + \frac{k}{12T} + \ldots.$$  

- Since $k \sim E \gg m_D$, the HTL propagators can be expanded in $m_D^2/E^2 \ll 1$.  

Momentum regions

In the loop with transverse gluons, this type of integral appears

$$\int \frac{d^{D-1}k}{(2\pi)^{D-1}} \int_0^{\infty} dk_0 \frac{1}{2\pi k_0^2 - k^2 - m_D^2 + i\eta} \left( \frac{1}{E - H_0 - k_0 + i\eta} + \frac{1}{E - H_0 + k_0 + i\eta} \right)$$

which exhibits two momentum regions

- off-shell region: \(k_0 - k \sim E\), \(k_0 \sim E\), \(k \sim E\);
- collinear region: \(k_0 - k \sim m_D^2 / E\), \(k_0 \sim E\), \(k \sim E\).

In our energy scale hierarchy, the collinear scale is \(m_g^4 \gg m_D^2 / E \gg m_g^6\), i.e. it is smaller than \(m_D\) by a factor of \(m_D / E \ll 1\) and still larger than the non-perturbative magnetic mass, which is of order \(g^2 T\), by a factor \(T / E \gg 1\).
Integrating out $E$: energy

\[
\delta E_{13}^E = -\frac{4\pi\alpha_s T m^2 T}{3} a_0^2
\]

where \( a_0 = \frac{3}{2m\alpha_s} \).

- Note the complete cancellation of the vacuum contribution to the energy at the scale $E$ (which includes the Bethe logarithm). This comes from the “$-1/2$” in the expansion of the Bose–Einstein distribution.
Cancellation of divergences in the spectrum III

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<td>$\sim m\alpha_s^5 \frac{1}{\epsilon_{UV}}$</td>
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</table>
Integrating out $E$: thermal width

$$g^{(E)}_{1S} = 4\alpha_s^3 T - \frac{64}{9m} \alpha_s T E_1 + \frac{32}{3} \alpha_s^2 T \frac{1}{m a_0} + \frac{7225}{162} E_1 \alpha_s^3$$

$$- \frac{4\alpha_s T m_D^2}{3} \left( \frac{2}{c} + \ln \frac{E_1^2}{\mu^2} + \gamma_E - \frac{11}{3} - \ln \pi + \ln 4 \right) a_0^2$$

$$+ \frac{128\alpha_s T m_D^2}{81} \frac{\alpha_s^2}{E_1^2} I_{1,0}$$

where $E_1 = -\frac{4ma_0^2}{9}$ and $a_0 = \frac{3}{2ma_s}$ and $I_{1,0} = -0.49673$ (similar to the Bethe log).

- The UV divergence at the scale $ma_s^2$ cancels against the IR divergence identified at the scale $T$.

Brambilla Escobedo Ghiglieri Soto Vairo JHEP 1009 (2010) 038
Singlet to octet break up

The thermal width at the scale $E$, which is of order $\alpha^3 T$, is generated by the break up of a quark-antiquark color-singlet state into an unbound quark-antiquark color-octet state: a process that is kinematically allowed only in a medium.

- The singlet to octet break up is a different phenomenon with respect to the Landau damping, the relative size of which is $(E/m_D)^2$. In the situation $m_\alpha^2 \gg m_D$, the first dominates over the second by a factor $(m_\alpha^2/m_D)^2$.

○ Brambilla Ghiglieri Petreczky Vairo PRD 78 (2008) 014017
Gluodissociation

For general $T$, the thermal width due to $S \rightarrow O + g$ break up in a medium reads

$$\Gamma_{1S} = \int_{|q| \geq |E_{1S}|} \frac{d^3q}{(2\pi)^3} n_B(|q|) \sigma_{1S}(|q|) \quad \text{at } T \gg E$$

with

$$\sigma_{1S}(|q|) = \frac{\alpha_s C_F}{3} 2^{10} \pi^2 \rho (\rho + 2) \frac{E_1^4}{m|q|^3} \left( \rho \left( t(|q|)^2 + \rho^2 \right) \exp \left( \frac{4\rho}{t(|q|)} \arctan \left( t(|q|) \right) \right) \right) \frac{e^{2\pi \rho}}{e^{2\pi \rho} - 1}$$

where $\rho \equiv 1/(N_c^2 - 1)$ and $t(|q|) \equiv \sqrt{|q|/|E_1| - 1}$.

$\sigma_{1S}$, which is the cross section of the process $S \rightarrow O + g$ in the vacuum, is known as the quarkonium gluodissociation cross section.

- Brambilla Escobedo Ghiglieri Vairo JHEP 1112 (2011) 116
- Brezinski Wolschin PLB 707 (2012) 534
Gluodissociation: the Bhanot–Peskin approximation

In the large $N_c$ limit:

\[
\sigma_{1S}(|q|) \xrightarrow[N_c \to \infty]{} 16 \frac{2^9 \pi \alpha_s}{9} \frac{|E_1|^{5/2}}{m} \frac{(|q| + E_1)^{3/2}}{|q|^5} = 16 \sigma_{1S, BP}(|q|)
\]

\[
\Gamma_{1S} \xrightarrow[N_c \to \infty]{} \int_{|q| \geq |E_1|} \frac{d^3q}{(2\pi)^3} n_B(|q|) 16 \sigma_{1S, BP}(|q|) = \Gamma_{1S, BP}
\]

The Bhanot–Peskin (BP) approximation corresponds to neglecting final state interactions, i.e. the rescattering of a $Q\bar{Q}$ pair in a color octet configuration.
Gluodissociation: full cross section vs BP cross section

\[ \approx 1.13 \]

Brambilla Escobedo Ghiglieri Vairo JHEP 1112 (2011) 116
Gluodissociation: full width vs BP width

Lines on the left correspond to the $T \gg |E_1|$ analytic results of the previous slides.

*Brambilla Escobedo Ghiglieri Vairo JHEP 1112 (2011) 116*
Integrating out $m_D$

The relevant diagram is (gluons are HTL gluons)

where the loop momentum region is $k_0 \sim m_D$ and $k \sim m_D$. This contribution is negligible with respect to the other terms calculated.
The complete mass and width up to $\mathcal{O}(m_\alpha^5)$

$$\delta E_{1S}^{\text{(thermal)}} = \frac{34\pi}{27} \alpha_s^2 T^2 a_0 + \frac{7225}{324} E_1 \alpha_s^3 \left( \ln \left( \frac{2\pi T}{E_1} \right)^2 - 2\gamma_E \right)$$
$$+ \frac{128 E_1 \alpha_s^3}{81\pi} L_{1,0} - 3a_0 \left\{ \frac{6}{\pi} \zeta(3) + \frac{4\pi}{3} \alpha_s T m_D^2 - \frac{8}{3} \zeta(3) \alpha_s^2 T^3 \right\}$$

$$\Gamma_{1S}^{\text{(thermal)}} = \frac{1156}{81} \alpha_s^2 T + \frac{7225}{162} E_1 \alpha_s^3 + \frac{32}{9} \alpha_s T m_D^2 a_0^2 I_{1,0}$$
$$- \left[ \frac{4}{3} \alpha_s T m_D^2 \left( \ln \frac{E_1^2}{T^2} + 2\gamma_E - 3 - \ln 4 - 2 \frac{\zeta'(2)}{\zeta(2)} \right) + \frac{32\pi}{3} \ln 2 \alpha_s^2 T^3 \right] a_0^2$$

where $E_1 = -\frac{4m_\alpha^2}{9}$, $a_0 = \frac{3}{2m_\alpha^2}$ and $L_{1,0}$ (similar $I_{1,0}$) is the Bethe logarithm.

○ Brambilla Escobedo Ghiglieri Soto Vairo JHEP 1009 (2010) 038
Lattice width

Consistent with \[ \Gamma^{(\text{thermal})}_{1S} = \frac{1156}{81} \alpha_s^3 T \Rightarrow \alpha_s \approx 0.4. \]

Aarts Allton Kim Lombardo Oktay Ryan Sinclair Skullerud
JHEP 1111 (2011) 103
Consistent with \( \delta E_{12}^{(\text{thermal})} = \frac{17\pi}{9} \alpha_s \frac{T^2}{m} \) using \( \alpha_s = 0.4 \) and \( m = 5 \text{ GeV} \).
Conclusions

In a framework that makes close contact with modern effective field theories for non relativistic bound states at zero temperature, we have studied the real-time evolution of a heavy quarkonium (specifically the $\Upsilon(1S)$) in a thermal bath of gluons and light quarks.

- For $T < E$ the potential coincides with the $T = 0$ one.
- For $T > E$ the potential gets thermal contributions.
- Two mechanisms contribute to the thermal decay width: the imaginary part of the gluon self energy induced by the Landau damping phenomenon (aka quasi-free dissociation), and the quark-antiquark color singlet to color octet thermal breakup (aka gluodissociation).
- Quarkonium dissociates at a temperature $\pi T_{\text{dissociation}}$, which may be lower than the screening one.