Cold Electroweak Baryogenesis and Real-Time Fermions

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Electroweak Baryogenesis

Mechanism for explaining the matter/antimatter asymmetry in the Universe,

\[ \frac{n_B}{n} \approx 6 \times 10^{-10} \]

Required:
- Baryon number violation
- Breaking of C and CP
- Out-of-equilibrium dynamics

Axial Anomaly, Chern-Simons number
Gauge/fermion coupling, CKM matrix
Electroweak symmetry breaking

1. order phase transition
2. order phase transition
... 
Cross-over transition
Spinodal transition

Higgs mass = 125 GeV → Game over!

Kuzmin, Rubakov, Shaposhnikov: 1985
Kajantie, Laine, Rummukainen, Shaposhnikov: 1996
Electroweak baryon number violation

\[ B(t) - B(0) = 3[N_{cs}(t) - N_{cs}(0)] = L(t) - L(0) \]
Baryogenesis from Leptogenesis

- Heavy (Majorana) neutrinos decay out of equilibrium.
- Violation of Lepton number conservation → B-L nonzero.
- Universe thermalizes, cools.
- Equilibrium sphaleron processes convert only L to some B and some L.
- No need for out-of-equilibrium electroweak transition.

Fukugita, Yanagida: 1986
Luty: 1992

Ambjorn, Askgaard, Porter, Shaposhnikov: 1991
Philipsen: 1993, 1995
Ambjorn, Krasnitz: 1993, 1995
Moore: 1996-2000
& Rummukainen: 1999
& Bödeker: 1999
Shanahan, Davis: 1998
Smit, Tang: 1996
D’Onofrio, Rummukainen, AT: 2012
Arnold, Yaffe, Son, Kajantie, Laine, Burnier...
“Hot” Electroweak Baryogenesis

- Enlarge scalar sector to give strong finite T phase transition.
- Bubble nucleation.
- Advancing bubble wall interacts with plasma, breaking CP, net CP asymmetry inside and outside bubble.
- Sphalerons active outside; suppressed inside. Convert to B asymmetry.
- Bubbles eventually cover space.

K. Rummukainen: 2001
“Cold” Electroweak Baryogenesis

- Enlarged scalar sector allows for super-cooling of Universe...
- ...and then rapid quench.
- → low-T spinodal transition.
- → quench speed determines out-of-equilibrium-ness.
- Thermalization to T<Mw.
- = “Tachyonic preheating”. 

\[
\ddot{\phi}_k + (k^2 - m^2 + g^2 \sigma^2(t))\phi_k = 0
\]

\[
\phi_k \rightarrow e^{\sqrt{m^2-k^2}t}, \quad n_k \rightarrow e^{2\sqrt{m^2-k^2}t}
\]

\[
V(0) - V(\nu) = \frac{\pi^2}{30} g^* T_{\text{reh}}^4, \quad T_{\text{reh}} \sim 40 \text{ GeV}
\]
Two Mechanisms

Krauss and Trodden: 1999  
(and Turok and Zadorozny: 1990-1)

- Symmetry breaking $\rightarrow$ Kibble mechanism.
- Net density of localized Higgs field textures.
- Average winding zero, average Ncs zero.
- Asymmetric unwinding under CP-violation.
- $\rightarrow$ Net asymmetry in Nw and Ncs.

Garcia-Bellido, Grigoriev, Kusenko, Shaposhnikov: 1999

- Spinodal transition $\rightarrow$ unstable IR modes in Higgs field.
- Energy driven into gauge field.
- Growth of gauge field under CP-bias $\rightarrow$ Net Chern-Simons number.
- $\langle Ncs^2 \rangle$ $\rightarrow$ non-equilibrium “diffusion” rate.
- $\rightarrow$ Net asymmetry in Nw and Ncs.

Also: Copeland, Lyth, Rajantie, Trodden: 2001

Also: Garcia-Bellido, Gonzalez-Arroyo, Garcia Perez 2002-2003-2004
What actually happens

Krauss and Trodden: 1999
(and Turok and Zadrozny: 1990-1)

- Symmetry breaking → Kibble mechanism.
- Net density of localized Higgs field textures.
- Average winding zero, average Ncs zero.
- Asymmetric unwinding under CP-violation, nonzero Ncs and oscillating Higgs field.
- → Net asymmetry in Nw and Ncs.

Garcia-Bellido, Grigoriev, Kusenko, Shaposhnikov: 1999

- Spinodal transition → unstable IR modes in Higgs field.
- Energy driven into gauge field.
- Growth of gauge field under CP-bias → Net Chern-Simons number.
- $\langle Ncs^2 \rangle$ → non-equilibrium “diffusion” rate.
- → Net asymmetry in Nw and Ncs.

AT, Smit; Skullerud, Smit, AT; AT, Smit, Hindmarsh; 2003-2006
How do we know?

Lattice simulations:
- Classical dynamics of...
- \( \ldots SU(2) + \text{Higgs} + \text{CP-violation}. \)
- Cold initial conditions.
- Fast quench (flip the mass).
- Average over ensemble.

\[
\frac{3 \, \delta_{\text{cp}}}{16 \pi^2 m_W^2} \phi \phi^\dagger \text{Tr} F_{\mu\nu} \tilde{F}^{\mu\nu}
\]

\[
n_B = 3 \, n_{cs} = 3 \, n_W
\]

\[
\frac{n_B}{n_{\gamma}} = - \left( 0.32 \pm 0.04 \right) \times 10^{-4} \times \delta_{\text{cp}}
\]

AT, Smit: JHEP 0608:012,2006
Sources of CP-violation

Standard Model:
- CKM matrix

Shaposhnikov: 1987

\[ \delta_{cp} \propto J \frac{\Delta}{T^{12}}, \quad T > m_q \]
\[ \delta_{cp} \propto J \frac{\Delta}{\nu^{12}}, \quad T \approx 0 \]
\[ J = 3 \times 10^{-5}, \]
\[ \Delta = \Pi_{(d,s,b),(u,c,t)} (m_i^2 - m_j^2) \]
\[ \delta_{cp} \approx 10^{-20} \]

Standard Model + Higgs:
- CKM matrix
- 2-Higgs potential

\[
V(\phi_1, \phi_2) = \mu_1^2 \phi_1^\dagger \phi_1 + \mu_2^2 \phi_2^\dagger \phi_2 \\
+ \mu_{12}^2 \phi_1^\dagger \phi_2 + \mu_{12}^2 \phi_2^\dagger \phi_1 \\
+ \lambda_1 (\phi_1^\dagger \phi_1)^2 + \lambda_2 (\phi_2^\dagger \phi_2)^2 \\
+ \lambda_3 \phi_1^\dagger \phi_1 \phi_2^\dagger \phi_2 + \lambda_4 \phi_1^\dagger \phi_2 \phi_2^\dagger \phi_1 \\
+ \lambda_5 (\phi_1^\dagger \phi_2)^2 + \lambda_5^* (\phi_2^\dagger \phi_1)^2
\]
CEW BaG in 2HDM

- Ncs is P and CP odd.
- Potential is C and CP odd.
- Fermion-gauge interaction is C and P odd.
- Integrate out fermion $\rightarrow$ C/P odd bosonic terms.

$\frac{\delta_{C/P}}{16\pi^2 m_W^2} i(\phi_2^\dagger \phi_1 - \phi_1^\dagger \phi_2) \text{Tr} F_{\mu\nu} \tilde{F}^{\mu\nu}$

$\frac{n_B}{n_\gamma} = -(0.28 \pm 0.12) \times 10^{-5} \times \delta_{C/P}$

AT, Bin Wu: 1203.5012 (Last Friday)
Sources of CP-violation

Standard Model:
- CKM matrix

Shaposhnikov: 1987

\[ \delta_{cp} \propto J \frac{\Delta}{T^{12}}, \quad T > m_q \]

\[ \delta_{cp} \propto J \frac{\Delta}{v^{12}}, \quad T \approx 0 \]

\[ J = 3 \times 10^{-5}, \]

\[ \Delta = \Pi_{(d,s,b),(u,c,t)} (m_i^2 - m_j^2) \]

\[ \delta_{cp} \approx 10^{-20} \]

Standard Model + Higgs:
- CKM matrix
- 2-Higgs potential

\[ V(\phi_1, \phi_2) = \mu_1^2 \phi_1^\dagger \phi_1 + \mu_2^2 \phi_2^\dagger \phi_2 \]

\[ + \mu_{12}^2 \phi_1^\dagger \phi_2 + \mu_{12}^{2,*} \phi_2^\dagger \phi_1 \]

\[ + \lambda_1 (\phi_1^\dagger \phi_1)^2 + \lambda_2 (\phi_2^\dagger \phi_2)^2 \]

\[ + \lambda_3 \phi_1^\dagger \phi_1 \phi_2^\dagger \phi_2 + \lambda_4 \phi_1^\dagger \phi_2 \phi_2^\dagger \phi_1 \]

\[ + \lambda_5 (\phi_1^\dagger \phi_2)^2 + \lambda_5^* (\phi_2^\dagger \phi_1)^2 \]
Bosonized Standard Model

- Integrate out fermions at finite T from the SM → TrLog.
- Expand in covariant derivatives.
- → resums Higgs field insertions.
- C(P)-violation at order 6, 4 W and 2 $Z/\partial\phi$.
- For bosonic simulations, we also need C/P violating sector.

$$\Gamma_{CP} = -\frac{i}{2} N_c J G_F \kappa_{cp} \int_0^{1/T} dx_0 \int d^3 x \left( \frac{\nu}{\phi} \right)^2 \mathcal{O}[W, Z, \partial\phi, c_2(\nu T/\phi)]$$


And now for something completely different... ...but related.
Bosonic field

\[ \hat{\phi}(\mathbf{x}, t) = \int \frac{d^d k}{(2\pi)^d} \frac{1}{\sqrt{2 \omega_k}} \left( \hat{a}_k f_k(\mathbf{x}, t) + \hat{a}_k^\dagger f_k^*(\mathbf{x}, t) \right) \]

\[ \langle \hat{\phi}(\mathbf{x}, t)^2 \rangle = \int \frac{d^d k}{(2\pi)^d} \frac{\langle a_k a_k^\dagger + a_k^\dagger a_k \rangle}{2 \omega_k} |f_k(\mathbf{x}, t)|^2, \quad \langle a_k a_k^\dagger \rangle \propto \delta(\mathbf{k}, \mathbf{k}') \]

- Numerical effort \( O(nx^{2d}) \!

- Gaussian truncations of Schwinger-Dyson hierarchy (Hartree, “Large N”= 1/N LO, …)

- Large occupation numbers → Effectively classical dynamics of ensemble of random numbers
  \[ \hat{a}_k \to \xi_k, \quad \langle \xi_k \xi_k^\dagger + \xi_k^\dagger \xi_k \rangle = \langle \hat{a}_k \hat{a}_k^\dagger + \hat{a}_k^\dagger \hat{a}_k \rangle \]

- Generalizes to full, interacting, non-perturbative classical approximation to quantum dynamics. Numerical effort \( O(nx^d * Nqb) \).

- Works well except for equilibrium → truncated quantum dynamics, SD/KB/2PI.
Fermionic field

- Fermion fields are always quantum. No classical limit, no large occupation numbers.
- Fermion fields are always bilinear in the action.

\[
\psi = \frac{1}{\sqrt{2}} [\psi_1 - i\psi_2]
\]

\[
\hat{\psi}_i(x, t) = \int \frac{d^dk}{(2\pi)^d} \frac{1}{\sqrt{2\omega_k}} \left( \hat{b}_k U_k f_k(x, t) + \hat{b}^+_k V_k f^*_k(x, t) \right)
\]

\[
D_{\alpha\beta}(x, y) = \frac{1}{2} \left( \langle \hat{\psi}_\alpha(x) \hat{\psi}_\beta(y) - \hat{\psi}_\beta(y) \hat{\psi}_\alpha(x) \rangle \right) =
\]

\[
\frac{1}{2} \int \frac{d^dk}{(2\pi)^d} \frac{1}{2\omega_k} \left[ U_{k\alpha} V_{k\beta} e^{ik(x-y)} - V_{k\alpha} U_{k\beta} e^{-ik(x-y)} \right],
\]

\[
D^*_{\alpha\beta} = -D_{\alpha\beta}
\]

- Numerical effort $O(nx^{2d})$!
Male and Female

- Complex numbers do not anticommute. Use two ensembles of fields.

\[ \psi^M(x, t) = \frac{1}{\sqrt{2}} \int \frac{d^d k}{(2\pi)^d} \frac{1}{\sqrt{2} \omega_k} \left( \eta_k U_k f_k(x, t) + \eta_k^* V_k f_k^*(x, t) \right), \]

\[ \psi^F(x, t) = \frac{i}{\sqrt{2}} \int \frac{d^d k}{(2\pi)^d} \frac{1}{\sqrt{2} \omega_k} \left( \eta_k U_k f_k(x, t) - \eta_k^* V_k f_k^*(x, t) \right), \]

\[ D_{\alpha\beta}(x, y) \rightarrow i \langle \psi_\alpha(x) \psi_\beta(y) \rangle \quad \hat{b}_k \rightarrow \eta_k, \quad \langle \eta_k \eta_k^\dagger \rangle = \langle \hat{b}_k \hat{b}_k^\dagger \rangle \]

\[ = \frac{1}{2} \int \frac{d^d k}{(2\pi)^d} \frac{1}{2\omega_k} \left[ U_k\alpha V_k\beta e^{ik(x-y)} - V_k\alpha U_k\beta e^{-ik(x-y)} \right], \]

- Male-Female correlators reproduce all fermion bilinears. Effort \( O(2nx^d \cdot Nqf) \).
- Classical, nonlinear bosonic fields. Fermion evolution linear in bosonic background.
- Bilinear quantum fermion sources to boson eoms → M-F correlators.

Hindmarsh, Borsanyi: 2009
1+1D model: U(1)-Higgs+fermions

\[ S = - \int d^2 x \left[ (D_\mu \phi)^\dagger (D^\mu \phi) + \lambda (\phi^\dagger \phi - v^2 / 2)^2 \right] 
- \int d^2 x \, \frac{1}{4e^2} F_{\mu \nu} F^{\mu \nu} 
- \int d^2 x \left[ \bar{\psi} \gamma^\mu (\partial_\mu + iA_\mu \gamma_5) \psi + G \bar{\psi} (\phi^* P_L + \phi P_R) \psi \right] \]

- Anomaly equation:
  \[ N_f = \int dx \, j^0 = - \frac{1}{2\pi} \int dx \, A_1(x) = N_{cs} \]

- Higgs winding:
  \[ N_W = \frac{1}{2\pi} \int dx \, \partial_1 \theta(x), \quad \phi(x) = |\phi(x)| e^{i\theta(x)} \]
Technical stuff

- **Lattice implementation:**
  - 1D spatial lattice.
  - Non-compact U(1) gauge field.
  - Wilson fermions in space.
  - Timelike fermion doublers not initialized → stay unexcited.

\[ \gamma^\mu D_\mu \psi \rightarrow \gamma^\mu D_\mu \psi + \frac{r_\mu}{2} D_\mu D^\mu \psi \]

- **Charge conjugation on upper fermion component:**
  - Axial → vector.
  - Dirac mass → Majorana mass.
  - Fermion current ↔ Axial current.
By-hand sphaleron transitions

- $N_q = 10, 30, \ldots, 2430$
- $N_{qs}$

Saffin, AT, JHEP 1107
(2011) 066
Space-like doublers

Saffin, AT, JHEP 1107 (2011) 066
Spinodal transition

Saffin, AT, JHEP 1107 (2011) 066
Yukawa couplings

Saffin, AT, JHEP 1107 (2011) 066
With CP-violation: Average

\[ S \rightarrow S - \int d^2x \, \frac{\kappa}{4\pi} \phi^\dagger \phi \epsilon_{\mu\nu} F^{\mu\nu} \]
With CP-violation: Average
Baryon asymmetry

Saffin, AT, JHEP 1107 (2011) 066
\[ S = S_H + S_W + S_F + S_Y \]

\[ S_H = - \int d^4x \left[ D_\mu \phi^\dagger D^\mu \phi + \lambda (\phi^\dagger \phi - v^2/2)^2 \right], \]

\[ S_W = - \int d^4x \frac{1}{4} W_{\mu \nu}^a W^{a,\mu \nu}, \]

\[ S_F = - \int d^4x \left[ \bar{q}_L \gamma^\mu D_\mu q_L + \bar{u}_R \gamma^\mu D_\mu u_R + \bar{d}_R \gamma^\mu D_\mu d_R \right. \]
\[ \left. + \bar{l}_L \gamma^\mu D_\mu l_L + \bar{\nu}_R \gamma^\mu D_\mu \nu_R + \bar{e}_R \gamma^\mu D_\mu e_R \right], \]

\[ S_Y = - \int d^4x \left[ G^{\mu \nu} \bar{q}_L \phi u_R + G^{d} \bar{q}_L \phi d_R + G^{e} \bar{l}_L \phi e_R + G^{\nu} \bar{L} \phi \nu_R \right. \]
\[ \left. + \hat{G}^{\mu \nu} \bar{q}_L \phi u_R + \hat{G}^{d} \bar{q}_L \phi d_R + \hat{G}^{e} \bar{l}_L \phi e_R + \hat{G}^{\nu} \bar{L} \phi \nu_R \right. \]
\[ \left. + h.c. \right] \]
Technical stuff

• Lattice implementation:
  – 3D spatial lattice.
  – SU(2) gauge Wilson action.
  – Wilson fermions in space.
  – Timelike fermion doublers not initialized → stay unexcited.

\[ \gamma^\mu D_\mu \psi \rightarrow \gamma^\mu D_\mu \psi + \frac{\tau_\mu}{2} D_\mu D^\mu \psi \]

• Charge conjugation on upper fermion component and regrouping:
  – Axial → vector.
  – 2 L-H doublets + 4 R-H singlets → 1 doublet + 2 singlets
  – Fermion current (old fields) ↔ Axial current (new fields).
By-hand transitions
Ensemble size

- $N_q = 20$, $n_x = 32$ fits on 1-2GB memory.
- $N_q = 10240$ on 512 procs, running 8 hours.
- Computer intensive!
- Would like $n_x \rightarrow 64$
- Would like $t \rightarrow 100$
- Would like $N_q \rightarrow 20000$
- Would like 3 colours
- Would like 3 generations
- $\rightarrow$ factor 400(!)

Saffin, AT: JHEP 1202:102, 2012
Spatial doublers

Saffin, AT: JHEP 1202:102, 2012
Spinodal instability

Saffin, AT: JHEP 1202:102, 2012
With Yukawa coupling

- SM CP-violation is encoded in the CKM quark mixing matrix.
- We need Yukawa couplings of all the 3 generations of quarks to generate a baryon asymmetry!
- Need more statistics/smaller timestep.
- Note: top-mass = 173 GeV corresponds to \( l = 1 \).
- Almost there!

Saffin, AT: JHEP 1202:102,2012
Conclusions I

- The SM cannot provide baryogenesis $\rightarrow$ no out-of-equilibrium.
- 3 main contenders based on SM anomaly: Lepto, “Hot”, “Cold”.
- 3 sources of out-of-equilibrium
  - Out-of-equilibrium decay
  - Bubble nucleation
  - Spinodal instability

Simulations of bosonized CEWBaG
- CKM (maybe) enough if effective temperature $\sim 1$ GeV
- Dim-6 operator works if coefficient $\sim 10^{-5}$
- 2HDM with Dim-6 works if coefficient $\sim 10^{-4}$
Conclusions II

- 3 (sofa) proposals to trigger super-cooled, fast spinodal transition
  - Extra scalar field, which is the inflaton \textit{vanTent, Smit, AT: 2004}
  - Extra scalar field, which is not the inflaton \textit{Enqvist, Stephens, Taanila, AT: 2010}
  - First order phase transition in special potential \textit{Konstandin, Servant: 2011}

- Full dynamics with fermions:
  - Anomaly equation holds.
  - Numerically hard.
  - Large Yukawa couplings even harder.
  - In principle possible to do 3 generations with full CKM matrix. Separation of masses? Tune Yukawa couplings to maximize signal? Can it be seen on the lattice?