The non-linear Glasma

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“Gauge field dynamics in and out of equilibrium”
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Motivation

- understand thermalization and related questions from first principles
- use ab-initio approach to relativistic heavy-ion collisions
- weak coupling and high energies
Outline

- Introduction to CGC framework and the Glasma
- Plasma instabilities and non-linear dynamics
- Summary & Outlook
CGC and the Glasma

- eikonal approximation – problem becomes particle production in the presence of strong sources

\[ J^\mu_a(t, x_\perp, z) = \delta^{\mu+} \varrho^{a}_{(1)}(x_\perp) \delta(x^-) + \delta^{\mu-} \varrho^{a}_{(2)}(x_\perp) \delta(x^+) \]

Initial state right after the collision

McLerran, Venugopalan, Gelis, Dusling ...
CGC and the Glasma

Initial value problem in QFT

\[ J^\mu_a(x) \sim \frac{1}{g} \Rightarrow A^a_\mu(x) \sim \frac{1}{g} \]

\[ F^{ab}_{\mu\nu}(x, y) = \frac{1}{2} \langle \{ \hat{A}^a_\mu(x), \hat{A}^b_\nu(y) \} \rangle - A^a_\mu(x)A^b_\nu(y) \sim \frac{1}{g} \]

\[ \rho^{ab}_{\mu\nu}(x, y) = i\langle \left[ \hat{A}^a_\mu(x), \hat{A}^b_\nu(y) \right] \rangle \sim 1 \]

Quantum evolution equations:

\[ \frac{\delta S[J, A]}{\delta A^a_\mu(x)} = -J^\mu_a(x) + \text{loop corrections} \]

\[ iG^{-1}_0[x; A] \rho(x, y) = 0 + \text{loop corrections} \]

\[ iG^{-1}_0[x; A] F(x, y) = 0 \]

Closed set of coupled integro-differential equations

\[ \sim 1 \text{ (initially)} \]

unstable in forward lightcone
Glasma and Fluctuations

Weak coupling, small fluctuations

\[ \frac{\delta S[J, A]}{\delta A^\mu_\alpha(x)} = -J^\mu_\alpha(x) \]  + loop corrections

=> recovers classical field solution (McLerran, Venugopalan, Fukushima, Gelis, Lappi,...)

\[ iG_0^{-1}[x; A] \rho(x, y) = 0 \]  + loop corrections
\[ iG_0^{-1}[x; A] F(x, y) = 0 \]

equivalent to linearized classical evolution equations
=> spectrum of fluctuations right after the collision

Dusling, Gelis, Venugopalan (2011)
Plasma instabilities

Consider boost non-invariant fluctuations
=> Grow exponentially in the forward light-cone

Initially small fluctuations become large

Classical statistical lattice simulation
- CGC initial conditions (MV model)
- simplified fluctuations

Romastschke, Venugopalan (2006); Fukushima, Gelis (2011); SS, Berges (in preparation)
Non-linear amplification

SU(2) – fixed box

\[ \frac{IA(t,p)}{A(t=0,p)} \]

SU(2) – CGC expanding

\[ g^2 \mu t [g^2 \mu t]^{1/2} \]

Berges, Scheffler, Sexty (2008)

Berges, SS (2012)
Non-linear amplification

Weak coupling, fluctuations grow with time

\[ \frac{\delta S[J, A]}{\delta A_\mu^a(x)} = -J_\mu^a(x) \]

+ loop corrections

\[ iG_0^{-1}[x; A] \rho(x, y) = 0 \]

+ loop corrections

\[ iG_0^{-1}[x; A] F(x, y) = 0 \]

Q: What are the relevant loop corrections?
Power counting

Take into account enhancement due to large fluctuations

Can distinguish different dynamical regimes where higher order corrections are suppressed by at least a fractional power of the coupling constant

Most important diagrams contained in classical statistical lattice gauge theory

power counting
\[ g^2F^2, g^4F^3, \ldots g^2F, \ldots \]
Non-linear amplification

dominated by *soft unstable* modes

\[ \exp[\Gamma(\nu) \sqrt{g^2 \mu \tau}] \]

\[ \exp[\Gamma(\nu') \sqrt{g^2 \mu \tau}] \]

\[ \exp[2 \Gamma \sqrt{g^2 \mu \tau}] \]

relevant time scale *parametrically*

\[ \sqrt{g^2 \mu \tau_{\text{Sec}}} \approx 1 \frac{1}{2\Gamma_0} \ln g^{-2} \]

Can be calculated explicitly when primary instabilities are described analytically (e.g. Berges, Serreau; Berges, Boguslavski , SS (scalar field theory))
Non-linear amplification

MV model, spectrum of fluctuations simplified

SS, Berges (in preparation)
Growth rates & time scales

Growth rates

SS, Berges (in preparation)

Set-in times
Coupling dependence

\[ \sqrt{g^2 \mu T_{\text{Sec}}} \lesssim \frac{1}{2 \Gamma_0} \ln g^{-2} \]

Confirms logarithmic behavior

Subleading corrections from:

- delayed set-in of primary instability
- Spectrum of initial fluctuations

\[ \Delta^2 : \text{size of initial fluctuations} \sim g^2 \]

SS, Berges (in preparation)
Summary

Different dynamical regimes of a system undergoing instabilities:

- linear instability regime
- non-linear amplification regime
- saturation of growth

Non-linear effects occur before saturation and are dominant for high-momentum modes

Relevant time scale depends only logarithmically on $g^2$

$$\sqrt{g^2 \mu \tau_{\text{Sec}}} \lesssim 1 \frac{1}{2 \Gamma_0} \ln g^{-2}.$$