The role of plasma instabilities in thermalization

Aleksi Kurkela
with Guy Moore

Arxiv: \begin{align*}
1107.5050 & \quad \text{thermalization in generic setup, complete treatment of plasma instabilities} \\
1108.4684 & \quad \text{specialized to heavy ion collisions}
\end{align*}

869 order of magnitude estimates in total
Motivation

**In:**
\[ \text{Pb} + \text{Pb} @ \text{few TeV per nucleon} \]

**Out:**
Anisotropic yield of \( \sim 10^4 \) hadrons ("\( v_2 \)")
Motivation

**Model:**
Hydro flow of *nearly thermal* fluid:

- Collective flow turns spatial anisotropy to momentum anisotropy
Motivation

Model:
Hydro flow of nearly thermal fluid:

- Collective flow turns spatial anisotropy to momentum anisotropy

\[ T_{\mu\nu}^{r,f} \approx \text{diag}(e, p, p, p) \]

Assumes:
Local thermal equilibrium

Inputs:
- Equation of state, viscosities...
- Initial geometry
- Thermalization time \( \tau_0 \sim 0.4 \ldots 1.2 \text{fm/c} \)
Motivation

Objective:
- Why is $\tau_0 \lesssim 0.4 \ldots 1.2 \text{fm}/c$ ???
- What happens before? Maybe observable in experiment

Method:
- Extremely big: $N_{\text{nucl}} \to \infty$
- Extremely high energy: $\sqrt{s} \to \infty$
  - weak coupling: $\alpha_s \ll 1$
  - $\Rightarrow$ Separation of scales: Kinetic theory, Hard loops, Vlasov equations, ...
  - Might be still non-perturbative ($\alpha f \gtrsim 1$)
- Purely parametric: counting powers of $\alpha_s$

Warning: all scales logarithmic
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Objective of this talk: What are the relevant physical process that lead to thermalization in a HIC?

Warning: all scales logarithmic
Initial condition: $t \sim Q_s^{-1}$

At weak coupling, well understood: Color Glass Condensate

- Characteristic, saturation scale: $Q_s$
- High occupancy: $f(p < Q_s) \sim 1/\alpha$ (need something to cancel $\alpha$ in $\sigma_{\text{large angle}}$)
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Distribution of gluons: $f(p) \sim \alpha^{-c} \theta(Q_s - p) \theta(\underbrace{\alpha^d}_{\delta} Q_s - p_z)$
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```
\begin{align*}
\text{f}(p) & \quad p_t \\
\text{Q} & \quad p_z \\
\delta Q & \quad \text{f} \sim \alpha \\
\text{c} & \quad \text{f} \sim 1 \\
\text{d} & \quad \text{f} \sim \alpha \\
\text{Initial condition} & \quad \text{d} = \ln(\delta)/\ln(\alpha) \\
\\end{align*}
```
Out of equilibrium systems: descriptors

High anisotropy:

$$f(p) \sim \alpha^{-c} \theta(Q_s - p) \theta(\alpha^d Q_s - p_z)$$

Small anisotropy:

$$f(p) \sim \alpha^{-c} (1 + \alpha^{-d} F(\hat{p}))$$
Longitudinal expansion

Spatial expansion translates into redshift in $p_z \sim \delta Q_s$

- Changes only $p_z$, $Q_s$ stays constant
- Changes $\delta = \alpha^d$
- If $p_z \ll p_t$, $\varepsilon(t) \sim \alpha^{-1} Q_s^4/(Q_s t)$

![Diagram showing longitudinal expansion]
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![Graph showing initial condition and varying time scales](image)
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Scattering: Elastic

Elastic scattering makes the distribution fluffier

\[ f(p) \]

Along the attractive solution, scattering and expansion compete.

When typical occupancies \( f \ll 1 \): loss of Bose enhancement.

At late times: Fixed anisotropy, dilute away.
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Elastic scattering not enough for thermalization!
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Elastic scattering not enough for thermalization!
Scattering: Inelastic

Inelastic scattering plays two significant roles (Baier, Mueller, Schiff & Son 2000)

1. soft splitting: creation of a soft thermal bath
2. hard splitting: breaking of the hard particles
Scattering: Inelastic

Soft splitting: Creation of soft thermal bath

- Soft modes quick to emit
  \[ n_s \sim \alpha n_{\text{col}} \]

- Low \( p \): easy to bend
  \[ \Rightarrow \text{thermalize quickly} \]

- Can dominate dynamics!
  (i.e. scattering, screening, ...)

\[ \delta Q \]

\[ p_t \]

\[ p_z \]

\[ Q \]

\[ T \]
Scattering: Inelastic

Hard splitting: $Q_s$ modes break before they bend!

$Q$ \hspace{2cm} $Q/2$ \hspace{2cm} $Q/4$ \hspace{2cm} $T$

$\Delta p^2_{\perp} \sim \hat{q} t$, \hspace{1cm} $t_{\text{split}}(k) \sim \alpha \sqrt{\hat{q}/k}$ (LPM)

- In vacuum: on-shell particles, no splitting
- In medium: Particles receive small kicks frequently
  - For stochastic uncorrelated kicks: Brownian motion in $p$-space
    - Momentum diffusion coefficient $\hat{q}$ describes how the medium wiggles a hard parton
Thermal bath eats the hard particles away:

- Scales below $k_{\text{split}}$ have cascaded down to $T$-bath

$$t_{\text{split}}(k_{\text{split}}) \sim t \Rightarrow k_{\text{split}} \sim \alpha^2 \hat{q} t^2$$

- "Falling" particles heat up the thermal bath

$$T^4 \sim k_{\text{split}} \int d^3 p f(p)$$

- Thermalization when $Q_s$ gets eaten

$$k_{\text{split}} \sim Q_s$$

Needs $\hat{q}$ as an input
\( \hat{q} \) dominated by elastic scattering with thermal bath?

BMSS assumed: \( \hat{q} \sim \hat{q}_{\text{elastic}} \sim \alpha^2 T^3 \)

Solve self-consistently:

\[
\begin{align*}
  k_{\text{split}} & \sim \alpha^2 \hat{q} t^2 \\
  T^4 & \sim k_{\text{split}} (Q_s^3 / (Q_s t)) \\
  \hat{q} & \sim \alpha^2 T^3
\end{align*}
\]

for:

\[
\begin{align*}
  T & \sim \alpha^3 Q_s (Q_s t) \\
  k_{\text{split}} & \sim \alpha^{13} (Q_s t)^5 Q \\
  \tau_0 & \sim \alpha^{-13/5} Q_s^{-1}
\end{align*}
\]

But: Is this all there is?
\( \hat{q} \) dominated (always!) by plasma instabilities.
Plasma instabilities: Idea

Exponential growth of (chromo)-magnetic fields in anisotropic plasmas

How do particles deflect?
Plasma instabilities: Idea

Exponential growth of (chromo)-magnetic fields in anisotropic plasmas

Induced current feeds the magnetic field
Plasma instabilities: Idea

When chromo-magnetic field becomes strong enough to mix colors, currents no longer "feed" the magnetic field. Growth is cut off.
Plasma instabilities: Slightly more quantitative

- **Lorentz force:** $F \sim gB$
- **Displacement:** $\delta z \sim gB t^2 / p$

![Diagram showing plasma instabilities with Lorentz force and displacement arrows]
Plasma instabilities: Slightly more quantitative

- Lorentz force: $F \sim gB$
- Dislocation: $\delta z \sim gBt^2/p$
- Current:

$$J \sim g \underbrace{\int d^3pf(p)}_{\text{charge}} \underbrace{[\delta z \, k]}_{\text{# of part.s disp. fraction}} \sim kBt^2 \alpha \underbrace{\int \frac{d^3p}{p}f(p)}_{m^2} \sim kBm^2t^2$$
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- Competes in Maxwell’s equation with $\nabla \times B \sim kB$

  $\Rightarrow$ Exponential growth if $m^2t^2 > 1$

  $\Rightarrow \left\{ \begin{array}{l}
  k_{z}^{\text{inst}} \sim \text{any} \\
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- Saturation when competition in \( D_{\mu} = k_{\text{inst}} + igA_{\mu} \)

\[\Rightarrow A \sim k_{\text{inst}}/g, \text{ or } B \sim k_{\text{inst}}A \sim k_{\text{inst}}^2/g, \text{ or } f(k_{\text{inst}}) \sim 1/\alpha\]
Don’t take my word for it.

Bödeker and Rummukainen (0705.0180) and many others...
Plasma instabilities: More complicated distributions

Strong anisotropy:

\[ f(\vec{p}) \sim \alpha^{-c} \Theta(Q_s - p)\Theta(\delta Q_s - p_z) \]

Weak anisotropy:

\[ f(\vec{p}) \sim f_0(|\vec{p}|)(1 + \epsilon F(\vec{p})) \]
Plasma instabilities: Momentum transfer

Hard parton traveling through magnetic fields receives coherent kicks from patches of same-sign magnetic fields

\[ \Delta p_{\text{kick}} \sim g B l_{\text{coh}} \]
\[ \hat{q} t \sim N_{\text{kick}} (\Delta p_{\text{kick}})^2 \]
\[ \hat{q} \sim \alpha l_{\text{coh}} B^2 \]

Weak anisotropy: \( \hat{q}_\epsilon \sim \epsilon^{3/2} m^3 \)
Strong anisotropy: \( \hat{q}_\delta \sim m^3 / \delta^2 \)
The new bottom-up

Important physics:

1. soft splitting: creation of a soft thermal bath
2. hard splitting: breaking of the hard particles
3. Plasma instabilities, screening

In particular, elastic scattering irrelevant
The new bottom-up: Early stages: $Q_s t < \alpha^{-12/5}$

Broadening of the hard particle distribution dominated by the plasma instabilities ($\hat{q}_{el} \ll \hat{q}_{inst}$) originating from the scale $Q_s$
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Even the instabilities are not strong enough to thermalize when $f \gg 1$ ⇒ classical theory does not thermalize under longitudinal expansion!
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Broadening of the hard particle distribution dominated by the plasma instabilities ($\hat{q}_{el} \ll \hat{q}_{inst}$) originating from the scale $Q_s$

$$\hat{q}_\delta \sim m^3/\delta^2, \quad m^2 \sim \alpha \delta f Q_s^2, \quad \delta \sim p_z/Q_s \sim \sqrt{\hat{q}_\delta t}/Q_s, \quad f \sim 1/(\alpha \delta Q_s t)$$

$$\Rightarrow \delta \sim (Q_s t)^{-1/8}, \quad f(Q_s) \sim (Q_s t)^{-7/8}, \quad \hat{q}_\delta \sim Q_s^3 (Q_s t)^{-5/4}$$
The new bottom-up: Late stages: $\alpha^{-12/5} < Qt < \alpha^{-5/2}$:

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\end{align*}

- At $(Q_s t) \sim \alpha^{-5/2}$, $k_{split} \sim Q_s$.
  Plasma instabilities continue to dominate until $(Q_s t) \sim \alpha^{-45/16}$. 
We identified the relevant physics in occupancy-anisotropy plane...
Summary, complete (parametric) description of HIC:

Featuring:
- Obscure powers: $\alpha^{\frac{184+15a}{554}}$
- Redshifted relics
- Synchrotron absorption
- Complicated angle dependence
- …and many more

…applied to the case of longitudinal expansion.
Conclusions:

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  - Numerical treatment underway $\tau_0 \sim \#\alpha^{-5/2}$
    - Three-scale problem: Three different treatments of d.o.f.’s on lattice
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  - Thermalization as quenching of jets with $p \sim Q_s$.
    Hard particles perturbative, medium non-perturbative?
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- Apply same methods to
  - Cosmology, reheating
  - Neutron stars, anomalous viscosities
Second attractor?

Plasma Instabilities

$\alpha^d = \ln(\delta)/\ln(\alpha)$

Occupancy
$c = -\ln(f)/\ln(\alpha)$

Initial Condition

Attractor 1
$c = -1/3$
$d = -1/7$

Attractor 2

Soft Particle

Bath Forms

Scattering

$f(p) \sim \alpha^{-c}(1 + \alpha^{-d}F(\hat{p}))$
Second attractor?

Assume nearly isotropic distribution:

- Longitudinal expansion reduces energy: $\varepsilon \propto t^{-4/3}$
  - $p_z$ reduces.
- Plasma-instabilities keep system nearly isotropic $\varepsilon \sim t_{iso}/t \sim \frac{Q^2}{q_\varepsilon t}$
  - $Q$ reduces
- Instability induced particle joining at hard scale
  - $Q$ increases

$$\varepsilon \sim \alpha^{-d} \sim (Q_s t)^{-\frac{8}{135}}, \quad f(Q) \sim \alpha^{-1}(Q_s t)^{-\frac{56}{135}}, \quad Q \sim Q_s (Q_s t)^{-\frac{31}{135}}$$
Why do we think the first attractor is the relevant one?

Initially, unstable modes in unpopulated part of phase space

Growth from vacuum fluctuations, slowed by a log:

\[ t_{growth} \sim Q_s^{-1} \ln^2 \left( \frac{1}{\alpha} \right) \]