At the Intersection of Spin and Saturation Physics
Transverse Spin Asymmetries in p-p and p-A Collisions

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Outline

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   - Definitions and Background
   - Theoretical Tools
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2. Our Calculation
   - Light-Cone Wave Function
   - Interactions

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3. Analysis
   - Preliminary Results
   - Interpretation
Single Transverse Spin Asymmetry - What It Is

An asymmetric distribution is produced when a transversely polarized hadron scatters off an unpolarized target. The asymmetry can be described by the following equation:

$$A_s \equiv \frac{d\sigma (\uparrow) - d\sigma (\downarrow)}{d\sigma (\uparrow) + d\sigma (\downarrow)}$$

Left/right asymmetry and spin up/down asymmetry are equivalent due to rotational invariance.
Single Transverse Spin Asymmetry - What It Is

A\leftarrow\rightarrow\ A

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History and Observation of STSA

Spin effects believed to be negligible at high energies (Kane et al, '78).

STSA first observed in late 70's, interpreted as purely non-perturbative effect.

Fermilab at $\sqrt{s} \approx 20$ GeV (90's) found $A_{NN} \approx 0$ for mid- and backward-rapidities, but large, increasing $A_{NN}$ at forward rapidities.

RHIC at $\sqrt{s} \approx 200$ GeV (00's) confirmed Fermilab's measurements over a wider kinematic range. Observed non-monotonic $p_T$ dependence.
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[D’Alesio and Murgia, ’08]

\[ A_N \times F \rho_T = 1.5 \text{ GeV/c} \]

\[ \pi^+ \quad \pi^0 \quad \pi^- \]

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History and Observation of STSA

[D’Alesio and Murgia, ’08]

[Wei, ’11] - PHENIX

$A_N = 1.5 \text{ GeV/c}$

$\pi^+ - \pi\pi^0 - \pi^-$

$\sqrt{s} = 200 \text{ GeV}$

Vertical Scale Uncertainty: 4.8%
Possible Mechanisms for Generating STSA
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Recently, the STAR Collaboration re-analyzed 2006, 2008 data to isolate Collins effect. Identify jets and plot azimuthal dependence of particles relative to jet thrust axis. Collins contribution proportional to slope of $A_N$ vs $\cos(\gamma)$.

Collins effect is consistent with zero for $\pi^0$ production. [Poljak, '11] - STAR

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$p^+p \rightarrow \text{jet}(\pi^0) + X$ at $\sqrt{s}=200$ GeV

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$A_N f(\gamma) = \frac{p^{+}p \rightarrow \text{jet}(\pi^0) + X \text{ at } \sqrt{s} = 200 \text{ GeV}}{p^{+}p \rightarrow X}$

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At The Intersection of Spin and Saturation Physics
representation of this document as if you were reading it naturally:

Collins and Sivers effects: Most analyses use collinear factorization methods, postulating $k_T$-factorization and including spin (the Generalized Parton Model). This has only been proven in restricted cases.

Interactions: initial-state interactions (ISI) and final-state interactions (FSI) can generate an asymmetry at twist-3 in pp collisions. Specifically, 3-gluon exchange contributes to these operators, with the gluons in the $C$-even ($f_{abc}$) or $C$-odd ($d_{abc}$) color states. [Ji, '92], [Koike and Yoshida, '11]
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Use light-cone perturbation theory (instead of collinear factorization) to calculate light-cone wave function of projectile in transverse coordinate space.

Re-sum the parameter $\alpha_s^2 A_1^2/3$, corresponding to 2-gluon exchange (Pomeron-type interactions).

Projectile scatters off of classical gluon field of the target.

Color-charge density fluctuations generate saturation scale $Q_s^2 \sim \alpha_s^2 A_1^2/3$ that acts as an IR cutoff.

At high enough energies that recoil can be neglected, quark and gluon propagators become Wilson lines.

Easy to incorporate small-$x$ evolution into the light-cone wave function.

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The Plan of Attack: Putting Them Together

Calculate one non-eikonal gluon emission in the wave function to capture lowest-order spin-dependence. For eikonal kinematics, use Wilson lines to describe ISI/FSI. Identify the specific coupling of parts of the wave function to parts of the interaction which generate STSA. Comment on generalization to pA scattering (A-dependence).

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Light-Cone Wave Function: Non-Eikonal Emission

Initial state: quark spin $\chi = \pm 1$ polarized along $x$-axis.  

$U\chi \equiv \frac{1}{\sqrt{2}}(U^{+}(z) - \chi U^{-}(z))$  

Defined by the Pauli-Lubanski covariant spin 4-vector $W^{\mu} \equiv \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} \Sigma_{\nu\rho} p_{\sigma}$  

Initial-state spinors are eigenvectors of $W^{(1)}$:  

$W^{(1)} U\chi = \chi m_{2} U\chi$  

Splitting wave function $\Phi_{\lambda\chi} = \int d^{2}k (2\pi)^{2} d^{2}p (2\pi)^{2} e^{ik \cdot (z-x)} e^{ip \cdot (x-u)} g_{T}^{a} p_{\gamma} \bar{U}\chi'(k) \sqrt{k} + \gamma \cdot \epsilon(\lambda) U\chi(p) \sqrt{p}$
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- **Splitting wave function** $\Phi_{\lambda\chi\chi'}$

\[
\Phi_{\lambda\chi\chi'}(z-x) T^a \delta^2 [x - u + \alpha (z - x)] = \\
\int \frac{d^2 k}{(2\pi)^2} \frac{d^2 p}{(2\pi)^2} \ e^{ik \cdot (z-x)} e^{ip \cdot (x-u)} \ gT^a \frac{\bar{U}_{\chi'}(k)}{\sqrt{k^+}} \frac{\sqrt{p^+}}{(\gamma \cdot \epsilon(\lambda))} U_{\chi}(p)
\]
Light-Cone Wave Function: Non-Eikonal Emission

Direct evaluation of splitting wave function gives:

$$\Phi_{\lambda \chi \chi'}(z-x) = i \epsilon(\lambda) \cdot (z-x) |z-x| \tilde{m} K_1(\tilde{m} |z-x|) \left[ (1+\alpha) \delta \chi \chi' - \lambda (1-\alpha) \delta \bar{\chi}, \chi \right] + (1-\alpha) \chi \sqrt{2} \tilde{m} K_0(\tilde{m} |z-x|) \left[ \delta \chi \chi' + \lambda \delta \bar{\chi}, \chi \right]$$

Transverse wave function mixes the longitudinal same-spin ($K_1$) and spin-flip ($K_0$) terms. Note vector structure of the two terms. Entire splitting function is proportional to the quark mass: a consequence of not being in a pure helicity state.

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\[ + \frac{(1 - \alpha)\chi}{\sqrt{2}} \tilde{m} K_0 (\tilde{m}|z - x|) \left[ \delta_{\chi \chi'} + \lambda \delta_{\chi, -\chi'} \right] \]
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\[ + \frac{(1 - \alpha)x}{\sqrt{2}} \tilde{m} K_0 \left( \tilde{m} |z - x| \right) \left[ \delta_{\gamma\gamma'} + \lambda \delta_{\gamma,-\gamma'} \right] \]

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Interactions: Eikonal Rescattering
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Represent eikonal scattering with Wilson lines

$$V_{\mathbf{x}} = \mathcal{P} \exp \left[ -i g \int dx^+ T^a A^a_{-(x, x^+, b)} \right]$$
Interactions: Eikonal Rescattering

\[ \langle \psi_2 \rangle_{\text{int}} = \delta^2 \left[ u - \alpha z - (1 - \alpha) x \right] \delta^2 \left[ w - \alpha y - (1 - \alpha) x \right] \langle \Phi_2 \chi \rangle (z - x, y - x) I(x, y, z, u, w, b) \]

Splitting wave function:

\[ \langle \Phi_2 \chi \rangle = 2 \alpha s \pi \tilde{m}^2 \left[ (1 + \alpha^2) (z - x) \cdot (y - x) \middle| z - x \middle| \right| y - x \middle| K_1(\tilde{m} | z - x |) K_1(\tilde{m} | y - x |) + (1 - \alpha)^2 K_0(\tilde{m} | z - x |) K_0(\tilde{m} | y - x |) - \chi_\alpha (1 - \alpha) (z_2 - x_2) | z - x | K_0(\tilde{m} | y - x |) K_1(\tilde{m} | z - x |) + y_2 | y - x | K_1(\tilde{m} | y - x |) K_0(\tilde{m} | z - x |)) \]
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Splitting + Scattering:
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\langle \psi_{\text{int}}^2 \rangle = \delta^2 [u - \alpha z - (1 - \alpha) x] \delta^2 [w - \alpha y - (1 - \alpha) x] \times \\
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  \]

- **Splitting wave function:**
  \[
  \langle \Phi^2 \rangle = \frac{2\alpha_s}{\pi} \tilde{m}^2 \left[ (1 + \alpha^2) \frac{(z-x) \cdot (y-x)}{|z-x||y-x|} K_1(\tilde{m}|z - x|) K_1(\tilde{m}|y - x|) + (1 - \alpha)^2 K_0(\tilde{m}|z - x|) K_0(\tilde{m}|y - x|) - \chi \alpha (1 - \alpha) \left( \frac{z^{(2)} - x^{(2)}}{|z-x|} K_0(\tilde{m}|y - x|) K_1(\tilde{m}|z - x|) + \frac{y^{(2)} - x^{(2)}}{|y-x|} K_1(\tilde{m}|y - x|) K_0(\tilde{m}|z - x|) \right) \right]
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\[ \mathcal{I}(x, y, z, u, w, b) = \]

\[ \frac{C_F}{N_c} \text{Tr}(V_z V_y^\dagger + V_u V_w^\dagger) - \frac{1}{2N_c} \left[ \text{Tr}(V_z V_x^\dagger) \text{Tr}(V_x V_w^\dagger) + \text{Tr}(V_u V_x^\dagger) \text{Tr}(V_x V_y^\dagger) \right] + \frac{1}{2N_c^2} \text{Tr}(V_z V_w^\dagger + V_u V_y^\dagger) \]
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Contribution to cross section:

\[ \frac{d\sigma}{d^2k \, dy} = \]
\[ \frac{1}{2(2\pi)^3} \frac{\alpha}{1-\alpha} \int d^2x d^2y d^2z \int d^2u d^2w \, e^{-ik \cdot (z-y)} e^{ip \cdot (u-w)} \langle \psi_{\text{int}}^2 \rangle \]
Interactions: Eikonal Rescattering

**Interaction:**

\[
\mathcal{I}(x, y, z, u, w, b) = C_F \frac{N_c}{N_c} \text{Tr}(V_z V_y^\dagger + V_u V_w^\dagger) - \frac{1}{2N_c} \left[ \text{Tr}(V_z V_x^\dagger) \text{Tr}(V_x V_w^\dagger) + \text{Tr}(V_u V_x^\dagger) \text{Tr}(V_x V_y^\dagger) \right] + \frac{1}{2N_c^2} \text{Tr}(V_z V_w^\dagger + V_u V_y^\dagger)
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\]

**Need to reorganize into manageable pieces.**
Symmetry and Antisymmetry: $k_T$-Parity

Separate the interaction by its $k_T$-parity (left/right asymmetry) and the wave function by its spin dependence:

$$\langle \Phi_2 \chi \rangle = \Phi_2^\text{unp} + \chi \Phi_2^\text{pol}$$

Both parts of the wave function $\Phi_2^\text{unp}$ and $\Phi_2^\text{pol}$ are even under $k \rightarrow -k$.

By rotational invariance, $k \rightarrow -k$ and $\chi \rightarrow -\chi$ should give the same asymmetry. After averaging over impact parameters $d^2 b$, rotationally non-invariant terms vanish (vector structure vs. $k_T$-parity):

$$\Phi_2^\text{pol} I^\text{symm} = 0$$

$$\Phi_2^\text{unp} I^\text{anti} = 0$$
Symmetry and Antisymmetry: $k_T$-Parity

Separate the interaction by its $k_T$ – parity (left/right asymmetry) and the wave function by its spin dependence:

\[ \mathcal{I} = \mathcal{I}_{\text{symm}} + i\mathcal{I}_{\text{anti}} \]

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Contributions to the STSA come from the spin-dependent part of the wave function $\Phi^2_{\text{pol}}$ coupling to the antisymmetric part of the interaction $I_{\text{anti}}$.

$$d(\Delta \sigma) = -\chi_\alpha S_8 \pi^4 \alpha^2 \tilde{m} \int d^2x d^2y d^2z e^{-i(k - \alpha p) \cdot (z - y)} \times \left[ (\partial_\partial z (2) + \partial_\partial y (2)) K_0(\tilde{m} | y - x |) K_0(\tilde{m} | z - x |) \right] i I_{\text{anti}}(x, y, z, b)$$

Explicitly separate each trace into a symmetric piece $S_{xy}$ (the Pomeron) and an antisymmetric piece $O_{xy}$ (the Odderon):

$$i I_{\text{anti}} = C_F (iO_{zy} + iO_{uw}) + N_c (iO_{yx} S_{xu} + iO_{xu} S_{yx} + iO_{wx} S_{xz} + iO_{xz} S_{wx}) + \frac{1}{N_c} (iO_{zw} + iO_{uy})$$
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The Emerging Picture

The transverse wave function has definite $k_T$-parity and happens to be completely even. Consequently, $\Phi_2^{pol}$ couples to $I_{anti}$ to generate the STSA. Couples the Odderon $O_{xy}$ to an experimental observable, potentially allowing its first direct measurement! Nonlinear terms include both Odderon exchange and Pomeron exchange. At minimum, need one non-eikonal vertex (emission here) to generate STSA. Hence $A_N \propto m$. (ISI$^2$) and (FSI$_2$) contribute to $d\sigma_{unp}$. Only (ISI/FSI) interference terms generate the relative phase needed for STSA. M. Sievert and Y. Kovchegov
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How Not to Generate STSA

A first approximation: linearize the interaction, e.g.,

\[ \sigma_x \approx \sigma_y. \]

Compute contributions to \( \Delta \sigma \), integrating over all transverse coordinates. But this gives a STSA that is identically zero! Terms related by \( k_T \)-parity cancel, and the other terms vanish explicitly. Why does this happen? Extending transverse coordinates to infinity effectively makes the transverse size of the target infinite. This introduces translational invariance into the scattering, which automatically kills any asymmetry. To generate any asymmetry from the interaction, finite size effects must be incorporated.
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Sources of STSA (Preliminary Estimates)

Incomplete cancellation of the linear terms due to finite size effects, e.g. a crude cutoff $\Theta(R - |x - b|)$.

Contributions come from exponential tails of the Bessel functions; STSA is highly suppressed as the nuclear radius increases: $A_N \sim \alpha S e^{-mR} \sim \alpha S e^{-\left(A_1/3\right)}$.

For pp collisions where $e^{-mR} \sim O(1)$, $A_N \sim \alpha S$, but exponential suppression rapidly kills edge effects beyond pp.

Nonlinear terms (Odderon + Pomeron) that couple to gradients of the nuclear profile $\nabla T(b)$:

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More suppressed overall, but with weaker dependence on $A$. 

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At The Intersection of Spin and Saturation Physics
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\[ M. \text{Sievert and Y. Kovchegov} \]

\[ \text{At The Intersection of Spin and Saturation Physics} \]
Strengths and Weaknesses of Our Method

Strengths

LCPT allows a direct calculation from first principles, without needing to assume a non-perturbative ansatz.

The kinematic factor \( \alpha_1 - \alpha_2 \) in \( d(\Delta \sigma) g \) gives an asymmetry that increases at forward rapidities, but is small at mid- and backward rapidities.

Compatibility with saturation allows analysis of both pp and pA scattering within the same formalism.

Reveals an experimental connection to the elusive Odderon.

Qualitatively, we expect a crossover between the edge effects and the nonlinear effects generating STSA at some value of \( A \).
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At The Intersection of Spin and Saturation Physics
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It is difficult to compare the magnitudes of multiple sources of STSAs, since some of them are nonperturbative. This method hinges on eikonal kinematics; recoil corrections cannot be incorporated into the Wilson lines. Describing finite-size effects with $\Theta$-functions is very crude. Is that really better than assuming a nonperturbative ansatz?
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Future Work/Improvements (Wishful Thinking)

- Better estimation of the transverse integrals, especially their $k_T$-dependence.
- Clarify the roles and interplay of the symmetries involved: $C$ (Odderon vs Pomeron), $P$ ($k$ vs $-k$), and $T$ (ISI vs FSI).
- Establish relationships between several observables (possible coupling of the Odderon to longitudinal single-spin asymmetries?)
- Include small-$x$ evolution of the Pomeron/Odderon into the wave function.
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Thank You!