$T$ violation in nuclear systems. An effective approach

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Outline

1. Motivations

2. Sources of $T$ violation

3. Chiral Perturbation Theory Lagrangian with $T$ violation

4. Nucleon EDM

5. Deuteron EDM and MQM

6. Triton and Helion EDM

7. Summary & Conclusion
Motivations and Introduction

A permanent Electric Dipole Moment (EDM) of a particle with spin

- signal of $T$ and $P$ violation
- signal $T$ violation in the flavor diagonal sector
- relatively insensitive to the CKM phase

**Standard Model:**

\[ d_n \sim 10^{-32} \text{ e cm} \]

for review: M. Pospelov and A. Ritz, ‘05

**Current bounds:**

- neutron $|d_n| < 2.9 \times 10^{-26} \text{ e cm}$
  
  UltraCold Neutron Experiment @ ILL
  
  C. A. Baker *et al.*, ‘06

- proton $|d_p| < 7.9 \times 10^{-25} \text{ e cm}$
  
  $^{199}$Hg EDM @ Univ. of Washington
  
  W. C. Griffith *et al.*, ‘09

Large window for new physics and intense experimental activity!
$T$-violating observables

1. Neutron EDM

   UltraCold Neutron experiment @ PSI
   - currently taking data
   - 2013: $d_n \sim 5 \times 10^{-27} \text{ e cm}$
   - 2016: $d_n \sim 5 \times 10^{-28} \text{ e cm}$

UCN experiment @ SNS Oak Ridge

   - 2020: $d_n \sim 10^{-28} \text{ e cm}$
$T$-violating observables

2. Proton EDM

Storage Ring Experiment @ BNL

- 2010-2013: R&D
- 2013: start ring construction
- 2016: start physics run
  aim for $d_p \sim 10^{-29}$ $e$ cm

3. Deuteron, Triton, Helion EDM

Storage Ring Experiment @ COSY Jülich Forschungszentrum
- BNL, after completion proton EDM

- same sensitivity as proton EDM experiment, $d_d \sim 10^{-29}$ $e$ cm
- no definitive timeline for EDM experiment
Motivations and Introduction

Can a measurement of nucleon or deuteron EDM pinpoint the microscopic mechanism(s) that generates it?

a. high energy: modelling beyond SM physics leave it to model builders
b. low energy: hadronic or nuclear matrix element non perturbative QCD problem

Strategy: Chiral symmetry of QCD & low energy Effective Field Theories

different properties under $SU_L(2) \times SU_R(2)$

\[\downarrow\]
different relations between low-energy TV observables
Motivations and Introduction

Can a measurement of nucleon or deuteron EDM pinpoint the microscopic mechanism(s) that generates it?

a. high energy: modelling beyond SM physics  
   leave it to model builders
b. low energy: hadronic or nuclear matrix element  
   non perturbative QCD problem

Strategy: Chiral symmetry of QCD & low energy Effective Field Theories

- integrate out all the heavy fields

\[ \mathcal{L} = \mathcal{L}_{QCD} + \mathcal{L}_T = \mathcal{L}_{QCD} + \sum_n \frac{c_n}{M_T^{d_n-4}} \mathcal{O}_{T,n} (A_\mu, G_\mu, u, d) \]

- construct hadronic operators with same chiral properties as \( \mathcal{O}_{T,n} \)
- organize operators in a systematic expansion in \( m_\pi/M_{QCD} \)
- hide non perturbative ignorance in (hopefully few) unknown coefficients
- look for qualitatively different low energy effects of various TV sources
The QCD Theta Term

\[ \mathcal{L}_4 = -\theta \frac{g_s^2}{32\pi^2} \varepsilon^{\mu\nu\alpha\beta} \text{Tr} G_{\mu\nu} G_{\alpha\beta} - \bar{q}_R M q_L - \bar{q}_L M^* q_R, \]

\[ M = \bar{m} e^{i\varphi} \begin{pmatrix} 1 - \varepsilon & 0 \\ 0 & 1 + \varepsilon \end{pmatrix} \quad \bar{m} = (m_u + m_d)/2 \quad \varepsilon = (m_d - m_u)/(m_d + m_u) \]

- \( \theta, \varphi \neq 0 \) break P and T
- \( M \neq 0 \) explicitly breaks chiral symmetry
The QCD Theta Term

\[ \mathcal{L}_4 = \left[ -\theta \frac{g_s^2}{32\pi^2} \varepsilon^{\mu \nu \alpha \beta} \mathrm{Tr} G_{\mu \nu} G_{\alpha \beta} - \bar{q} R M q_L - \bar{q} L M^* q_R, \right. \]

\[ M = m e^{i \varphi} \left( \begin{array}{cc} 1 - \varepsilon & 0 \\ 0 & 1 + \varepsilon \end{array} \right) \]

\[ \bar{m} = \frac{m_u + m_d}{2}, \quad \varepsilon = \frac{m_d - m_u}{m_d + m_u} \]

• \( \theta, \varphi \neq 0 \) break P and T
• \( M \neq 0 \) explicitly breaks chiral symmetry

• eliminate \( \theta \) with (anomalous) \( SU_A(2) \times U_A(1) \) axial rotation

\[ \mathcal{L}_4 = -\bar{m} \, r(\bar{\theta}) \bar{q} q + \varepsilon \bar{m} \, r^{-1}(\bar{\theta}) \bar{q} \tau_3 q + m^*_\star \sin \bar{\theta} \, r^{-1}(\bar{\theta}) i \bar{q} \gamma^5 q, \]

with

\[ \bar{\theta} = 2\varphi - \theta, \quad m^*_\star = \frac{m_u m_d}{m_u + m_d} = \frac{\bar{m}}{2} \left( 1 - \varepsilon^2 \right) \]
The QCD Theta Term

\[ \mathcal{L}_4 = -\theta \frac{g_s^2}{32\pi^2} \varepsilon^{\mu\nu\alpha\beta} \text{Tr} G_{\mu\nu} G_{\alpha\beta} - \bar{q} R M q_L - \bar{q} L M^* q_R, \]

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- \( \theta, \phi \neq 0 \) break P and T
- \( M \neq 0 \) explicitly breaks chiral symmetry

- eliminate \( \theta \) with (anomalous) \( SU_A(2) \times U_A(1) \) axial rotation

\[ \mathcal{L}_4 = -\bar{m} r(\bar{\theta}) S_4 + \varepsilon \bar{m} r^{-1}(\bar{\theta}) P_3 + m_\ast \sin \bar{\theta} r^{-1}(\bar{\theta}) P_4, \]

- \( \bar{\theta} \) and \( m \) break chiral symmetry in a very specific way

\[ S = \begin{pmatrix} -i \bar{q} \gamma^5 \tau q \\ \bar{q} q \end{pmatrix} \quad P = \begin{pmatrix} \bar{q} \tau q \\ i \bar{q} \gamma^5 q \end{pmatrix} \]

- \( SO(4) \) vector
- \( SO(4) \) vector
Dimension 6 TV sources

- no dimension 5 operator with quarks/gluons
- several **dimension 6** operators

\[
\mathcal{L}_{6, \nu
\nu
\nu} = \frac{d_{W}}{6} f^{abc} \varepsilon_{\mu \nu \alpha \beta} G^a_{\alpha \beta} G^{b}_{\mu \rho} G^{c}_{\nu \rho} + \ldots
\]

\[
\mathcal{L}_{6, \nu \nu \nu} = -\frac{1}{\sqrt{2}} \bar{q}_{L} \sigma^{\mu \nu} \left\{ \tilde{\Gamma}^{u} \lambda^{a} G_{\mu \nu} + \Gamma^{u}_{B} B_{\mu \nu} + \Gamma^{u}_{W} \tau \cdot W_{\mu \nu} \right\} \frac{\tilde{\varphi}}{v} u_{R} + \ldots
\]

\[
\mathcal{L}_{6, \nu \nu \nu} = \Sigma_{1} \left( \bar{q}_{L}^{J} u_{R} \right) \varepsilon_{JK} \left( \bar{q}_{L}^{K} d_{R} \right) + \Sigma_{8} \left( \bar{q}_{L}^{J} \lambda^{a} u_{R} \right) \varepsilon_{JK} \left( \bar{q}_{L}^{K} \lambda^{a} d_{R} \right)
\]

Buchmuller & Wyler ‘86, Weinberg ‘89, de Rujula et al. ‘91, \ldots

- \( \Gamma \) and \( \Sigma \) complex-valued matrices in flavor space

\[
d_{W} = \mathcal{O} \left( 4\pi \frac{w}{M_{T}^{2}} \right), \quad \tilde{\Gamma}^{u,d} = \mathcal{O} \left( 4\pi \delta_{u,d} \frac{v \lambda_{u,d}}{M_{T}^{2}} \right), \quad \Sigma_{1,8} = \mathcal{O} \left( (4\pi)^{2} \frac{\sigma_{1,8}}{M_{T}^{2}} \right)
\]
Dimension 6 TV sources

• spontaneous symmetry breaking: \( \varphi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \)

• integrate out heavy stuff \((c, b, t, W, Z, \text{Higgs})\)

• gluon chromo-EDM (gCEDM)

\[
\mathcal{L}_{6, vvv} = \frac{d_W}{6} f_{abc} \varepsilon^{\mu\nu\alpha\beta} G^a_{\alpha\beta} G^b_{\mu\rho} G^c_{\nu\rho}
\]

• quark EDM (qEDM) and chromo-EDM (qCEDM)

\[
\mathcal{L}_{6, qq\varphi v} = -\frac{1}{2} \bar{q} i\sigma^{\mu\nu} \gamma^5 \left( d_0 + d_3 \tau_3 \right) q F_{\mu\nu} - \frac{1}{2} \bar{q} i\sigma^{\mu\nu} \gamma^5 \left( \tilde{d}_0 + \tilde{d}_3 \tau_3 \right) \lambda^a q G^a_{\mu\nu}
\]

• TV 4-quark operators

\[
\mathcal{L}_{6, qqqq} = \frac{1}{4} \text{Im} \Sigma_1 \left( \bar{q} q \bar{i} \gamma^5 q - \bar{q} \tau q \cdot \bar{q} \tau \gamma^5 q \right) + \ldots
\]
Chiral properties of dimension 6 sources

1. qCEDM & qEDM

\[ \mathcal{L}_{qq\varphi} = -\tilde{d}_0\tilde{V}_4 + \tilde{d}_3\tilde{W}_3 - d_0V_4 + d_3W_3 \]

- \(\tilde{V}, \tilde{W}\) and \(V, W\) are \(SO(4)\) vectors

\[ \tilde{W} = \frac{1}{2} \begin{pmatrix} -i\bar{q}\sigma^{\mu\nu}\gamma^5\tau^a\lambda^aq \\ \bar{q}\sigma^{\mu\nu}\lambda^aq \end{pmatrix} G_{\mu\nu}^a, \quad \tilde{V} = \frac{1}{2} \begin{pmatrix} \bar{q}\sigma^{\mu\nu}\tau^a\lambda^aq \\ i\bar{q}\sigma^{\mu\nu}\gamma^5\lambda^aq \end{pmatrix} G_{\mu\nu}^a. \]

2. gCEDM & TV 4-quark operators

\[ \mathcal{L}_{\nu\nu\nu} + \mathcal{L}_{qqqq} = d_W I_W + \text{Im}\Sigma_1 I_{qq}^{(1)} + \text{Im}\Sigma_8 I_{qq}^{(8)} \]

- \(I_W, I_{qq}^{(1)}, I_{qq}^{(8)}\) are chiral invariant (\(\chi I\))
Dimension 6 TV sources

\[ d_{0,3} = \mathcal{O} \left( e\delta \frac{\bar{m}}{M_T^2} \right), \quad \tilde{d}_{0,3} = \mathcal{O} \left( 4\pi \tilde{\delta} \frac{\bar{m}}{M_T^2} \right), \]
\[ d_w = \mathcal{O} \left( 4\pi \frac{w}{M_T^2} \right), \quad \Sigma_{1,8} = \mathcal{O} \left( (4\pi)^2 \frac{\sigma}{M_T^2} \right) \]

• dimensionless factor \( \delta, \tilde{\delta}, w \) and \( \sigma \) depend on details of TV mechanism

1. Naturalness

\[ \delta = \mathcal{O}(1), \quad \tilde{\delta} = \mathcal{O} \left( \frac{g_s}{4\pi} \right), \quad w = \mathcal{O} \left( \frac{g_s^3}{(4\pi)^3} \right), \quad \sigma = \mathcal{O}(1) \]

2. Standard Model

• \( M_T = M_W \)

\[ \delta \sim eJ_{\text{CP}} \frac{m_{c,s}^2}{M_W^2} \quad w \sim \frac{g_s^3}{(4\pi)^3} J_{\text{CP}} \frac{m_b^2 m_c^2 m_s^2}{M_W^6} \]

M. Pospelov and A. Ritz, ‘05

• suppressed by extra powers of \( M_W \)!
Dimension 6 TV sources

3. MSSM

- $M_T = \tilde{m} \sim \text{TeV}$
- gluino contribution (under various simplifying assumptions)

$$
\tilde{\delta} \sim \frac{g_s}{4\pi} \frac{\alpha_s(\tilde{m})}{4\pi} \text{Im} \frac{X_q}{\tilde{m}}
\quad \delta \sim \frac{4}{3} e \frac{\alpha_s(\tilde{m})}{4\pi} \text{Im} \frac{X_q}{\tilde{m}}
\quad w \sim \frac{g_s^3}{(4\pi)^3} \frac{\alpha_s(\tilde{m})}{4\pi} \text{Im} \frac{X_q}{\tilde{m}}
$$

- suppressed by $\alpha_s(\tilde{m})$
- $\sigma$ not studied much. In most models, extra $m_q/M_T$ suppression.

Factors $\delta, \tilde{\delta}, w, \sigma$

- difficult to compare different dim. 6 sources in a way independent of new physics model
- for each source, study relative contributions to different TV observables

T. Ibrahim and P. Nath, ‘08
The $T$-violating Chiral Lagrangian

Expansion in powers of $Q, m_\pi / M_{QCD}$

$$\mathcal{L}_T = \sum_{f, \Delta_\theta} \mathcal{L}^{(\Delta_\theta)}_{T,f} + \sum_{f, \Delta_6} \mathcal{L}^{(\Delta_6)}_{T,f}$$

- $\Delta_{\theta,6}$: count inverse powers of $M_{QCD}$ in coefficients

$$\Delta_\theta = d + 2m + f/2 - 2 \geq 1$$

- $\Delta_6 \geq -1$

$A \leq 1$: perturbative expansion of the amplitudes

$$\mathcal{M} \sim \left( \frac{Q}{M_{QCD}} \right)^\nu$$

$$\nu = 2L + \sum_i \Delta_i, \quad M_{QCD} = 2\pi F_\pi$$

$f = 0, 2$: # of nucleon legs

$d$: # of derivatives or photon fields

$m$: # of quark mass insertions

$A \leq 1$: perturbative expansion of the amplitudes

$\nu = 4$
Chiral Perturbation Theory. $A \geq 2$

- another relevant scale:
  - binding energy $Q^2/m_N$!
- nucleon propagator non static
- enhanced w.r.t chiral power counting

\[
\sim \frac{g_A^2}{F_\pi^2} \quad \text{and} \quad \sim \frac{g_A^2}{F_\pi^2} \frac{m_N Q}{4\pi F_\pi^2}
\]
Chiral Perturbation Theory. $A \geq 2$

- Another relevant scale:
  
  \[ \text{binding energy } \frac{Q^2}{m_N} \]

- Nucleon propagator non static

- Enhanced w.r.t chiral power counting

Weinberg:

- "Irreducible diagram": follow $\chi$PT power counting
  
  Define the potential $V$

\[ \sim \frac{g_A^2}{F_\pi^2} \]

\[ \sim \frac{g_A^2}{F_\pi^2} \frac{m_N Q}{4\pi F_\pi^2} \]

\[ V = \ldots \]
Chiral Perturbation Theory. $A \geq 2$

- another relevant scale:
  binding energy $Q^2/m_N$

- nucleon propagator non static

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Weinberg:

- “irreducible diagram”:
  follow $\chi$PT power counting
  define the potential $V$

- amplitude: iterate $V$
  Lippmann-Schwinger equation!
Chiral Perturbation Theory. $A \geq 2$

- another relevant scale:
  
  binding energy $Q^2/m_N$!

- nucleon propagator non static

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Weinberg:

- “irreducible diagram”:
  follow $\chi$PT power counting
  define the potential $V$

- amplitude: iterate $V$
  Lippmann-Schwinger equation!

- “perturbative pions”

1. LO potential: contact S-wave operator ($C_0$)
2. pion exchange as perturbation: $Q/M_{NN} \ll 1$
3. $\gamma = \sqrt{m_NB}$ only relevant parameter in LO
Chiral Perturbation Theory. $A \geq 2$

- another relevant scale: binding energy $Q^2/m_N$
- nucleon propagator non static
- enhanced w.r.t chiral power counting

Weinberg:

- “irreducible diagram”: follow $\chi$PT power counting define the potential $V$
- amplitude: iterate $V$
  Lippmann-Schwinger equation!

- “perturbative pions”
- “non-perturbative pions”

1. pion exchange leading effect

$Q/M_{NN} \sim 1$
The $T$-violating Chiral Lagrangian: ingredients

- pion-nucleon TV interactions

$$\mathcal{L}_{T,f=2} = - \frac{\bar{g}_0}{F_{\pi}} \bar{N} \pi \cdot \tau N - \frac{\bar{g}_1}{F_{\pi}} \pi_3 \bar{N} N - \frac{\bar{g}_2}{F_{\pi}} \pi_3 \bar{N} \tau_3 N$$

- nucleon-nucleon TV interactions

$$\mathcal{L}_{T,f=4} = \bar{C}_1 \bar{N} N \partial_\mu (\bar{N} S^\mu N) + \bar{C}_2 \bar{N} \tau N \cdot D_\mu (\bar{N} \tau S^\mu N)$$

- nucleon-photon TV interactions

$$\mathcal{L}_{T\gamma,f=2} = - 2 \bar{N} (\bar{d}_0 + \bar{d}_1 \tau_3) S^\mu \nu^\nu N F_{\mu \nu}$$
Discussion

<table>
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<tr>
<th>Source</th>
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<th>photon-nucleon</th>
<th>nucleon-nucleon</th>
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- chiral-breaking sources
  - TV $\pi$-N couplings have lowest chiral index

1. pion loops and short-range EDM operators equally important for nucleon EDM
2. pion-exchange dominate EDMs of light nuclei

...unless selection rules!
### Discussion

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- Chiral-breaking sources
  - TV \(\pi\)-N couplings have lowest chiral index
- Chiral-invariant sources
  - Same chiral index for all interactions

1. Short-range EDM operators dominate nucleon EDM
2. One-body effects & pion-exchange at the same level in light nuclei


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- chiral-breaking sources
  TV $\pi$-N couplings have lowest chiral index
- chiral-invariant sources
  same chiral index for all interactions
- qEDM
  long-distance suppressed by $\alpha_{em}$

1. nucleon and nuclei EDMs dominated by TV currents
Discussion

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$\bar{\theta}$ term
- only isoscalar $\bar{g}_0$ at LO
- isovector $\bar{g}_1$ suppressed by $m^2_\pi / M^2_{QCD}$ important for dEDM!

qCEDM
- $\bar{g}_0$ and $\bar{g}_1$ equally important

TV $\chi$I sources
- $\bar{g}_1$ and $\bar{g}_0$ equally important
- ... but more derivative & short-distance effects equally relevant
Nucleon EDM. Theta Term

\[ J_{\text{ed}}^\mu(q) = 2i (S \cdot qv^\mu - S^\mu v \cdot q) \left( F_0(q^2) + \tau_3 F_1(q^2) \right), \]

\[ F_i(q^2) = d_i - S'_i q^2 + H_i(q^2), \quad q^2 = -q^2. \]

**Leading Order**

- \( F_0 \) purely determined by short-distance physics. No momentum dependence

\[ d_0 = \tilde{d}_0^{(3)}, \quad S'_0 = 0, \]

- \( F_1 \) sensitive to short-distance & charged pions in the loops

\[ \Longrightarrow \bar{g}_0 \text{ only relevant } \pi\text{-N coupling} \]

\[ d_1 = \tilde{d}_1^{(3)} + \frac{eg_A \bar{g}_0}{(2\pi F_\pi)^2} \left[ L - \ln \frac{m_\pi^2}{\mu^2} \right], \quad S'_1 = \frac{eg_A \bar{g}_0}{(2\pi F_\pi)^2} \frac{1}{6m_\pi^2}, \]

Nucleon EDM. Theta Term

Next-to-Leading Order

• first non-analytic contribution & momentum dependence to $F_0(q^2)$

$$d_0 = \tilde{d}^{(3)}_0 + \frac{eg_A \bar{g}_0}{(2\pi F_\pi)^2} \pi \left[ \frac{3m_\pi}{4m_N} - \frac{\delta m_N}{m_\pi} \right] \quad S'_0 = -\frac{eg_A \bar{g}_0}{(2\pi F_\pi)^2} \frac{1}{6m_\pi^2} \pi \frac{\delta m_N}{2m_\pi}$$

• recoil corrections to $F_1$

$$d_1 = \tilde{d}^{(3)}_1 + \frac{eg_A \bar{g}_0}{(2\pi F_\pi)^2} \left[ L - \ln \frac{m_\pi^2}{\mu^2} + \frac{5\pi}{4} \frac{m_\pi}{m_N} - \frac{\delta m_\pi^2}{m_\pi^2} \right],$$

$$S'_1 = \frac{eg_A \bar{g}_0}{(2\pi F_\pi)^2} \frac{1}{6m_\pi^2} \left[ 1 - \frac{5\pi}{4} \frac{m_\pi}{m_N} - \frac{\delta m_\pi^2}{m_\pi^2} \right]$$

• no new $T$-odd LEC
• power counting relations between $\bar{g}_0$, $\bar{d}_{0,1}$ same as for Theta Term,

**LO nucleon EDM identical to Theta Term**

At NLO

• isoscalar

$$d_0 = \bar{d}_0^{(1)} + \frac{e g_A \bar{g}_0}{(2\pi F_\pi)^2} \pi \left[ \frac{3m_\pi}{4m_N} \left( 1 + \frac{\bar{g}_1}{3\bar{g}_0} \right) - \frac{\delta m_N}{m_\pi} \right]$$

$$S'_0 = -\frac{e g_A \bar{g}_0}{(2\pi F_\pi)^2} \frac{1}{6m_\pi^2} \frac{\delta m_N}{2m_\pi}$$

• isovector

$$d_1 = \bar{d}_1^{(1)} + \frac{e g_A \bar{g}_0}{(2\pi F_\pi)^2} \left[ L - \ln \frac{m_\pi^2}{\mu^2} + \frac{5\pi}{4} \frac{m_\pi}{m_N} \left( 1 + \frac{\bar{g}_1}{5\bar{g}_0} \right) - \frac{\bar{g}_0}{m_\pi^2} \right]$$

$$S'_1 = \frac{e g_A \bar{g}_0}{(2\pi F_\pi)^2} \frac{1}{6m_\pi^2} \left[ 1 - \frac{5\pi}{4} \frac{m_\pi}{m_N} - \frac{\delta m_\pi^2}{m_\pi^2} \right]$$

nucleon EDFF cannot distinguish between Theta Term and qCEDM
Nucleon EDM. Theta Term & qCEDM.

- EDM depends on $\pi$-N coupling $g_0$, and short-distance LECs $d_{0,1}$
- using non-analytic pieces for estimates

$$|d_n| = |d_0 - d_1| \gtrsim \frac{e g_A g_0}{(2\pi F_\pi)^2} \left[ \ln \frac{m_N^2}{m_{\pi}^2} + \frac{\pi}{2} \frac{m_\pi}{m_N} - \frac{\delta m_\pi^2}{m_{\pi}^2} + \pi \frac{\delta m_N}{m_\pi} \right]$$

$$\simeq (0.130 + 0.008 - 0.002 + 0.002) \frac{g_0}{F_\pi} e \text{ fm}$$

- at NLO, bound on isoscalar EDM

$$|d_0| \gtrsim \frac{e g_A g_0}{(2\pi F_\pi)^2} \pi \left[ \frac{3m_\pi}{4m_N} - \frac{\delta m_N}{m_\pi} \right] \approx (0.012 - 0.002) \frac{g_0}{F_\pi} e \text{ fm}.$$ 

- $S'_{0,1}$ only depends on $g_0$

$$S'_0 = -\frac{e g_A g_0}{12(2\pi F_\pi)^2} \frac{\pi \delta m_N}{m_{\pi}^2} = -0.3 \cdot 10^{-3} \frac{g_0}{F_\pi} e \text{ fm}^3,$$

$$S'_1 = \frac{e g_A g_0}{6(2\pi F_\pi)^2} \frac{1}{m_{\pi}^2} \left[ 1 - \frac{5\pi}{4} \frac{m_\pi}{m_N} - \frac{\delta m_\pi^2}{m_{\pi}^2} \right] \bar{\theta} = 4.7 \cdot 10^{-3} \frac{g_0}{F_\pi} e \text{ fm}^3.$$
Nucleon EDM and EDFF. qEDM & TV $\chi_I$ sources

- EDFF purely short-distance & momentum independent at LO

- isoscalar
\[ F_0(q^2) = d_0 = \bar{d}_0^{(n)}, \quad S_0' = 0 \]

- isovector
\[ F_1(q^2) = d_1 = \bar{d}_1^{(n)}, \quad S_1' = 0. \]
Nucleon EDM and EDFF. qEDM & TV $\chi$I sources

- EDFF purely short-distance & momentum independent at LO
- EDFF acquires momentum dependence at NNLO

- purely short distance for qEDM
- with long distance component for TV $\chi$I sources

- isoscalar

\[ d_0 = \bar{d}_0^{(n)} + \bar{d}_0^{(n+2)}, \quad S_0' = S_0^{(n+2)} \]

- isovector

\[ d_1 = \bar{d}_1^{(n)} + \bar{d}_1^{(n+2)}, \quad S_1' = S_1^{(n+2)} \]
Nucleon EDM and EDFF. Sum up

<table>
<thead>
<tr>
<th>Source</th>
<th>$\theta$</th>
<th>qCEDM</th>
<th>qEDM</th>
<th>TV $\chi I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{QCD} \frac{d_n}{e}$</td>
<td>$O\left(\bar{\theta} \frac{m_\pi^2}{M_{QCD}^2}\right)$</td>
<td>$O\left(\tilde{\delta} \frac{m_\pi^2}{M_T^2}\right)$</td>
<td>$O\left(\delta \frac{m_\pi^2}{M_T^2}\right)$</td>
<td>$O\left(\frac{w}{M_{QCD}^2} \frac{M_{QCD}^2}{M_T^2}\right)$</td>
</tr>
<tr>
<td>$d_p/d_n$</td>
<td>$O\left(1\right)$</td>
<td>$O\left(1\right)$</td>
<td>$O\left(1\right)$</td>
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</tr>
<tr>
<td>$m_\pi^2 S'_1/d_n$</td>
<td>$O\left(1\right)$</td>
<td>$O\left(1\right)$</td>
<td>$O\left(\frac{m_\pi^2}{M_{QCD}^2}\right)$</td>
<td>$O\left(\frac{m_\pi^2}{M_{QCD}^2}\right)$</td>
</tr>
<tr>
<td>$m_\pi^2 S'_0/d_n$</td>
<td>$O\left(\frac{m_\pi}{M_{QCD}}\right)$</td>
<td>$O\left(\frac{m_\pi}{M_{QCD}}\right)$</td>
<td>$O\left(\frac{m_\pi^2}{M_{QCD}^2}\right)$</td>
<td>$O\left(\frac{m_\pi^2}{M_{QCD}^2}\right)$</td>
</tr>
</tbody>
</table>

- measurement of $d_n$ and $d_p$ can be fitted by any source. No signal @ PSI, SNS:

  \[\bar{\theta} \lesssim 10^{-12}, \quad \tilde{\delta}, \delta \lesssim (10^3 \text{ TeV})^{-2}, \quad \frac{w}{M_T^2} \lesssim (5 \cdot 10^3 \text{ TeV})^{-2}\]

- $S'_1$ come at the same order as $d_i$  
- $S'_0$ suppressed by $m_\pi/M_{QCD}$ with respect to $d_i$  
- scale for momentum variation of EDFF set by $m_\pi$  
- $S'_{1,0}$ suppressed by $m_\pi^2/M_{QCD}^2$ with respect to $d_i$
EDMs of Light Nuclei. Power Counting

\[ d_{0,1} \]

\[ \frac{\bar{g}_0}{m_N^2} \frac{Q}{M_{NN}} \]

\[ \frac{\bar{g}_{0,1}}{Q^2} C_{1,2} F_\pi^2 \times \frac{Q}{M_{NN}} \]

\[ \frac{\bar{g}_{0,1}}{m_N^2} \frac{Q^2}{M_{NN}^2} \]
EDMs of Light Nuclei. Power Counting

\[ d_{0,1} = \frac{\bar{g}_0}{m_N^2} \frac{Q}{M_{NN}} \]

\[ \frac{\bar{g}_{0,1}}{Q^2} \bar{C}_{1,2} F_\pi^2 \times \frac{Q}{M_{NN}} \]

\[ \frac{\bar{g}_{0,1}}{m_N^2} \frac{Q^2}{M_{NN}^2} \]

- Theta & qCEDM: pion-exchange dominates
- qEDM: contribs. from neutron and proton EDMs dominate
- \( \chi I \): one-body, pion-exchange & short range equally important.
EDMs of Light Nuclei. Power Counting

\[ d_{0,1} \]

\[ \frac{\bar{g}_0}{m_N^2} \frac{Q}{M_{NN}} \]

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- Theta & qCEDM: pion-exchange dominates
- qEDM: contribs. from neutron and proton EDMs dominate
- \( \chi \): one-body, pion-exchange & short range equally important.

selection rules!
especially for Theta Term
Deuteron EDM and MQM

Spin 1, Isospin 0 particle

\[ H_T = -2d_d D^\dagger S \cdot E D - \frac{\mathcal{M}_d}{2} D_j^\dagger D_i \nabla^{(iB^j)} \]

\( d_d \): deuteron EDM \hspace{1cm} \mathcal{M}_d \): deuteron magnetic quadrupole moment (MQM).

\[ \text{dEDM} \]

- isoscalar \((\bar{g}_0, \bar{C}_{1,2})\) TV corrections to wavefunction vanish at LO.

\[ \text{dMQM} \]

- both isoscalar & isovector corrections contribute
Deuteron EDM and MQM

Spin 1, Isospin 0 particle

\[ H_T = -2d_d D^\dagger S \cdot E D - \frac{M_d}{2} D_j^\dagger D_i \nabla^{(i} B^{j)} \]

\( d_d \): deuteron EDM \( M_d \): deuteron magnetic quadrupole moment (MQM).

- isoscalar (\( \bar{g}_0, \bar{C}_{1,2} \)) TV corrections to wavefunction vanish at LO.

- both isoscalar & isovector corrections contribute
Deuteron EDM

One-body

- only sensitive to isoscalar nucleon EDM

\[ F_D(q^2) = 2d_0 \frac{4\gamma}{|q|} \arctan \left( \frac{|q|}{4\gamma} \right) = 2d_0 \left( 1 - \frac{1}{3} \left( \frac{|q|}{4\gamma} \right)^2 + \ldots \right) \]

- sensitive to isobreaking $\bar{g}_1$

\[ F_D(q^2) = -\frac{2}{3} e \frac{g_A \bar{g}_1}{m_\pi^2} m_N m_\pi \frac{1 + \xi}{4\pi F_\pi^2} \left( 1 - 0.45 \left( \frac{|q|}{4\gamma} \right)^2 + \ldots \right), \quad \xi = \frac{\gamma}{m_\pi} \]

TV corrections to wavefunction

- relative size different for different sources!
Deuteron EDM. qCEDM

qCEDM: chiral breaking & isospin breaking

\[ d_d = 2d_0 - \frac{2}{3}e \frac{g_A \bar{g}_1}{m_\pi^2} \frac{m_N m_\pi}{4\pi F_\pi^2} \frac{1 + \xi}{(1 + 2\xi)^2} = d_n + d_p - 0.23 \frac{\bar{g}_1}{F_\pi} e \text{ fm} \]

\[ \mathcal{O} \left( \frac{\tilde{\delta}}{M_T^2} \frac{m_\pi^2}{M_{QCD}} \right) \]

\[ \mathcal{O} \left( \frac{\tilde{\delta}}{M_T^2} \frac{M_{QCD} m_\pi}{M_{NN}} \right) \]

deuteron EDM enhanced w.r.t. nucleon!

- \( \bar{g}_1 \) leading interaction
- \( d_0 \) suppressed by two powers of \( M_{QCD} \)

\[ \frac{d_d}{d_n + d_p} \lesssim 10 \frac{\bar{g}_1}{g_0} \]

using non-analytic piece of \( d_0 \)
Deuteron EDM. Theta Term & TV $\chi$I Sources

**Theta term:** chiral breaking & isospin symmetric

**TV $\chi$I Sources:** chiral invariant

\[ \bar{g}_1 \text{ suppressed!} \]

\[ D_0 \text{ enhanced!} \]

\[ d_d = 2d_0 - \frac{2}{3} e \frac{g_A \bar{g}_1}{m_N m_\pi} \frac{m_\pi}{4\pi F_\pi^2} \frac{1 + \xi}{(1 + 2\xi)^2} = d_n + d_p - 0.23 \frac{\bar{g}_1}{F_\pi} e \text{ fm} \]

\[ \mathcal{O} \left( \bar{\theta} \frac{m_\pi}{M_{QCD}^3} \right) \]

\[ \mathcal{O} \left( \bar{\theta} \varepsilon \frac{m_\pi^2}{M_{QCD}^3} \frac{m_\pi}{M_{NN}} \right) \]

- $\bar{g}_1$ & $d_0$ appear at the same level in the Lagrangian
- dEDM well approximated by $d_n + d_p$
Deuteron EDM. Theta Term & TV $\chi$I Sources

**Theta term:** chiral breaking & isospin symmetric

**TV $\chi$I Sources:** chiral invariant

\[ \bar{g}_1 \text{ suppressed!} \]

\[ D_0 \text{ enhanced!} \]

\[ d_d = 2d_0 - \frac{2}{3} e \frac{g_A \bar{g}_1}{m^2} \frac{m_N m_\pi}{4 \pi F^2_\pi} \frac{1 + \xi}{(1 + 2\xi)^2} = d_n + d_p - 0.23 \frac{\bar{g}_1}{F_\pi} e \text{ fm} \]

\approx 0.02 \frac{\bar{g}_0}{F_\pi} e \text{ fm}

\approx 0.23 \times 0.01 \frac{\bar{g}_0}{F_\pi} e \text{ fm}

- $\bar{g}_1 \& d_0$ appear at the same level in the Lagrangian
- dEDM well approximated by $d_n + d_p$
qEDM: $\pi - N$ coupling suppressed by $\alpha_{\text{em}}$

\[
d_d = 2d_0 - \frac{2}{3} e \frac{g_A g_1}{m^2_\pi} \frac{m_N m_\pi}{4\pi F^2_\pi} \frac{1 + \xi}{(1 + 2\xi)^2} = d_n + d_p - 0.23 \frac{g_1}{F_\pi} \text{ e fm} \]

\[
\mathcal{O} \left( \frac{\delta}{M^2_\pi \frac{m^2_\pi}{M^2_T \cdot M_{\text{QCD}}}} \right)
\]

- dEDM well approximated by $d_n + d_p$
Deuteron EDM. Non perturbative results

“Hybrid approach”

- realistic potentials for TC interactions (AV18, Reid93, Nijmegen II)
- EFT potential for TV interactions

$$d_d = d_n + d_p - 0.19 \frac{g_1}{F_\pi} e \text{ fm},$$

for AV18, different potentials agree at $\sim 5\%$

- in good agreement with perturbative calculation!

1. $g_1$ contrib. agrees at $\sim 20\%$
2. for theta, formally LO pion-exchange terms are small
Deuteron EDM. Non perturbative results

“Hybrid approach”

- realistic potentials for TC interactions (AV18, Reid93, Nijmegen II)
  - ok...
  - if observable not too sensitive to short distance details
- EFT potential for TV interactions

\[
d_d(\bar{\theta}) = d_n + d_p + \left[ -0.19 \frac{g_1}{F_\pi} + \left( 0.2 - 0.7 \cdot 10^2 \beta_1 \right) \cdot 10^{-3} \frac{g_0}{F_\pi} \right] e \text{ fm},
\]

for AV18,
- different potentials agree at \( \sim 5\% \)

- in good agreement with perturbative calculation!
  - \( \bar{g}_1 \) contrib. agrees at \( \sim 20\% \)
  - for theta, formally LO pion-exchange terms are small
Deuteron EDM. Summary

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<tr>
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<tr>
<td>$M_{QCD} d_d/e$</td>
<td>$\mathcal{O}\left(\frac{\bar{\theta} m^2_\pi}{M^2_{QCD}}\right)$</td>
<td>$\mathcal{O}\left(\frac{\bar{\delta} m^2_\pi M^2_{QCD}}{M_{NN} M^2_T}\right)$</td>
<td>$\mathcal{O}\left(\frac{\bar{\delta} m^2_\pi}{M^2_T}\right)$</td>
<td>$\mathcal{O}\left(\frac{M^2_{QCD}}{M^2_T}\right)$</td>
</tr>
<tr>
<td>$d_d/d_n$</td>
<td>$\mathcal{O}(1)$</td>
<td>$\mathcal{O}\left(\frac{M^2_{QCD}}{m^2_\pi M_{NN}}\right)$</td>
<td>$\mathcal{O}(1)$</td>
<td>$\mathcal{O}(1)$</td>
</tr>
</tbody>
</table>

- deuteron EDM signal can be fitted by any source
- deuteron EDM well approximated by $d_n + d_p$ for $\bar{\theta}$, qEDM and TV $\chi_I$ sources
- only for qCEDM, $d_d \gg d_n + d_p$

**qCEDM**
- deuteron EDM experiment more sensitive than neutron & proton EDM

$$d_d \lesssim 10^{-16} \ e\text{ fm} \implies \frac{\bar{\delta}}{M^2_T} \lesssim (3 \cdot 10^4 \text{ TeV})^{-2}$$

- nucleon and deuteron EDM *qualitatively* pinpoint qCEDM.
Deuteron MQM. Chiral Breaking Sources

Corrections to wavefunction

\[ m_d \mathcal{M}_d = 2e \frac{g_A \bar{g}_0}{m_\pi^2} \frac{m_N m_\pi}{2\pi F_\pi^2} \left[ (1 + \kappa_0) + \frac{\bar{g}_1}{3\bar{g}_0} (1 + \kappa_1) \right] \frac{1 + \xi}{(1 + 2\xi)^2}, \]

qCEDM

- \( \bar{g}_0 \) and \( \bar{g}_1 \) equally important
- dEDM and dMQM comparable

\[ \left| \frac{m_d \mathcal{M}_d}{2d_d} \right| = (1 + \kappa_1) + \frac{3\bar{g}_0}{\bar{g}_1} (1 + \kappa_0) \]

ratio independent of deuteron details!
Deuteron MQM. Chiral Breaking Sources

Corrections to wavefunction

\[ m_d \mathcal{M}_d = 2e \frac{g_A \bar{g}_0}{m^2_\pi} \frac{m_N m_\pi}{2\pi F^2_\pi} (1 + \kappa_0) \frac{1 + \xi}{(1 + 2\xi)^2}, \]

Theta Term

- only \( \bar{g}_0 \) contributes
- dMQM bigger than dEDM

\[ \left| \frac{m_d \mathcal{M}_d}{d_d} \right| = \frac{2}{3} (1 + \kappa_0) \frac{1 + \xi}{(1 + 2\xi)^2} \left( \frac{m_N}{m_\pi} \right)^2 \lesssim 12 \]

using non-analytic piece of \( d_0 \).
Deuteron MQM. qEDM & TV $\chi$I Sources

Corrections to wavefunction + TV currents

- new two-body low-energy constants

  loss of predictive power!

- for both sources $m_d \mathcal{M}_d \lesssim d_d$

  no useful new info from observation of dMQM
EDM of $^3\text{He}$ and $^3\text{H}$

- AV18, EFT potentials for TC interactions
- code of I. Stetcu et al., '08
- agreement at the level of 15% for one-body & long-range contribs.
- no agreement for short range contribution ($\bar{C}_{1,2}$)

\[
d_{3\text{He}} = 0.88\, d_n - 0.047\, d_p - \left( 0.15\, \frac{\bar{g}_0}{F_\pi} + 0.28\, \frac{\bar{g}_1}{F_\pi} + 0.01\, F_\pi^3\, \bar{C}_1 - 0.02\, F_\pi^3\, \bar{C}_2 \right) e\, \text{fm}
\]

and

\[
d_{3\text{H}} = -0.050\, d_n + 0.90\, d_p + \left( 0.15\, \frac{\bar{g}_0}{F_\pi} - 0.28\, \frac{\bar{g}_1}{F_\pi} + 0.01\, F_\pi^3\, \bar{C}_1 - 0.02\, F_\pi^3\, \bar{C}_2 \right) e\, \text{fm},
\]

for AV18

- for EFT, $\bar{C}_{1,2}$ contribs. five time bigger
- need fully consistent calculation for $\chi I$ sources...

... maybe...
EDM of $^3$He and $^3$H. qCEDM

$$d_{3\text{He}}(\text{qCEDM}) = 0.83 \, d_0 - 0.93 \, d_1 - \left( 0.15 \, \frac{\bar{g}_0}{F_\pi} + 0.28 \, \frac{\bar{g}_1}{F_\pi} \right) e\,\text{fm},$$

$$d_{3\text{H}}(\text{qCEDM}) = 0.85 \, d_0 + 0.95 \, d_1 + \left( 0.15 \, \frac{\bar{g}_0}{F_\pi} - 0.28 \, \frac{\bar{g}_1}{F_\pi} \right) e\,\text{fm}.$$

$$\sim 0.1 \frac{\bar{g}_0}{F_\pi} e\,\text{fm}$$

- one-body pieces more important than expected by naive power counting
- qualitatively: $^3$He and $^3$H EDMs significantly different from neutron and proton EDMs
- quantitatively: if nucleon & deuteron observed
  1. $d_{3\text{He}} + d_{3\text{H}}$ testable prediction of the theory
     $$d_{3\text{He}} + d_{3\text{H}} = 1.68d_0 + 0.02d_1 - 0.56 \frac{\bar{g}_1}{F_\pi} e\,\text{fm}$$
  2. use $d_{3\text{He}} - d_{3\text{H}}$ to extract $\bar{g}_0$ & predict other TV observable
deuteron MQM, proton Schiff moment
EDM of $^3$He and $^3$H. Theta term

\[
d_{3\text{He}}(\bar{\theta}) = 0.83 \, d_0 - 0.93 \, d_1 - 0.15 \, \frac{g_0}{F_\pi} \, e \, \text{fm},
\]

\[
d_{3\text{H}}(\bar{\theta}) = 0.85 \, d_0 + 0.95 \, d_1 + 0.15 \, \frac{g_0}{F_\pi} \, e \, \text{fm}.
\]

- one-body piece more important than naive power counting
- $d_{3\text{He}} + d_{3\text{H}}$ well approximated by $d_n + d_p$
- $d_{3\text{He}} - d_{3\text{H}}$ significantly different from $2d_1$

\[
d_{3\text{He}} - d_{3\text{H}} = -0.02d_0 - 1.88d_1 - 0.30 \, \frac{g_0}{F_\pi} \, e \, \text{fm}
\]

- one more observable for quantitative prediction

  deuteron MQM, proton Schiff moment
EDM of $^3$He and $^3$H. qEDM & $\chi_I$ sources

\[
\begin{align*}
  d_{^3\text{He}}(\text{qEDM}) &= 0.83 d_0 - 0.93 d_1 , \\
  d_{^3\text{H}}(\text{qEDM}) &= 0.85 d_0 + 0.95 d_1 .
\end{align*}
\]

- no deviation from $d_n, d_p$

\[
\begin{align*}
  d_{^3\text{He}}(\chi_I) &= 0.83 d_0 - 0.93 d_1 - \left(0.15 \frac{\bar{g}_0}{F_\pi} + 0.28 \frac{\bar{g}_1}{F_\pi} + 0.01 F_\pi^3 \bar{C}_1 - 0.02 F_\pi^3 \bar{C}_2\right) e \text{ fm} , \\
  d_{^3\text{H}}(\chi_I) &= 0.85 d_0 + 0.95 d_1 + \left(0.15 \frac{\bar{g}_0}{F_\pi} - 0.28 \frac{\bar{g}_1}{F_\pi} + 0.01 F_\pi^3 \bar{C}_1 - 0.02 F_\pi^3 \bar{C}_2\right) e \text{ fm} .
\end{align*}
\]

\[\sim 1.9 \frac{\bar{g}_0}{F_\pi} \text{ e fm} \quad \text{naive dim. analysis}\]

- formally, all of the same size
- numerically, one-body contribution dominates

hard to differentiate between qEDM and $\chi_I$ sources!
Summary & Conclusion

EFT approach
1. consistent framework to treat one, two and three nucleon TV observables
2. qualitative relations between one, two and three nucleon observables, specific to TV source
3. particularly promising for qCEDM and Theta Term

identify/exclude them in next generation of experiments?

4. not much hope to distinguish between qEDM and $\chi$I sources

other observables? TV observables w/o photons?

To-do list
1. beyond NDA
2. improve calculation
3. other observables, deuteron MQM, proton Schiff moment

- compute LECs on the lattice
- NLO with perturbative pions
- fully consistent non ptb. calculation
- study atomic EDMs?
Backup Slides
Electromagnetic and $T$-violating operators

- chiral properties of $(P_3 + P_4) \otimes (I + T_{34})$
- lowest chiral order $\Delta = 3$
- $P_3 + P_4$

$$\mathcal{L}^{(3)}_{\mathcal{K}, f=2, \text{em}} = c_{1, \text{em}}^{(3)} \frac{1}{D} \left[ \frac{2\pi_3}{F_\pi} + \rho \left( 1 - \frac{\pi^2}{F^2_\pi} \right) \right] \bar{N} (S^\mu \nu^\nu - S^\nu \nu^\mu) N eF_{\mu \nu}$$

- $(P_3 + P_4) \otimes T_{34}$

$$\mathcal{L}^{(3)}_{\mathcal{K}, f=2, \text{em}} = c_{3, \text{em}}^{(3)} \bar{N} \left[ - \frac{2}{F_\pi D} \pi \cdot t - \rho \left( t_3 - \frac{2\pi_3}{F^2_\pi D} \pi \cdot t \right) \right] (S^\mu \nu^\nu - S^\nu \nu^\mu) N eF_{\mu \nu} + \text{tensor}$$

- isoscalar and isovector EDM related to pion photo-production.
Electromagnetic and $T$-violating operators

At the same order $S_4 \otimes (1 + T_{34})$

- $S_4$
  \[ \mathcal{L}_{\mathcal{K}, f=2, \text{em}}^{(3)} = c_{6, \text{em}}^{(3)} \left( -\frac{2}{F \pi D} \right) \bar{N} \pi \cdot t \left( S^\mu S^\nu - S^\nu S^\mu \right) N e F_{\mu\nu} \]

- $S_4 \otimes T_{34}$
  \[ \mathcal{L}_{\mathcal{K}, f=2, \text{em}}^{(3)} = c_{8, \text{em}}^{(3)} \frac{2 \pi^3}{F \pi D} \bar{N} \left( S^\mu S^\nu - S^\nu S^\mu \right) N e F_{\mu\nu} + \text{tensor} \]

- same chiral properties as partners of $\mathcal{T}$ operator
- pion-photoproduction constrains only $c_{1, \text{em}}^{(3)} + c_{6, \text{em}}^{(3)}$ and $c_{3, \text{em}}^{(3)} + c_{8, \text{em}}^{(3)}$
- but $\mathcal{T}$ only depends on $c_{1, \text{em}}^{(3)}$ and $c_{3, \text{em}}^{(3)}$

no $T$-conserving observable constrains short distance contrib. to nucleon EDM

- true only in $SU(2) \times SU(2)$
- larger symmetry of $SU(3) \times SU(3)$ leaves question open
Deuteron EDM and MQM. KSW Power Counting

\( T \)-even sector

\[
\mathcal{L}_{f=4} = -C_0^3 S_1^1 (N^t P^i N)^\dagger N^t P^i N + \frac{C_2^3 S_1}{8} \left[ (N^t P_i N)^\dagger N^t D^2_\text{L} P_i N + \text{h.c.} \right] + \ldots, \quad P^i = \frac{1}{\sqrt{8}} \sigma_2 \sigma_i \tau_2
\]

- enhance \( C_0 \) to account for unnatural large scattering lengths. In PDS scheme

\[
C_0^3 S_1 = \mathcal{O} \left( \frac{4\pi}{m_N \mu} \right), \quad \mu \sim Q
\]

- iterate \( C_0 \) at all orders

\[
C_0 \quad C_0 \frac{m_N Q}{4\pi} C_0 \quad C_0 \left( \frac{m_N Q}{4\pi} C_0 \right)^2
\]
Deuteron EDM and MQM. KSW Power Counting

*T*-even sector

\[ L_{f=4} = -C_0^3 S_1 (N^i P^i N) \dagger N^i P^i N + \frac{C_2^3 S_1}{8} \left[ (N^i P_i N)^\dagger N^i D^2_\pi P_i N + \text{h.c.} \right] + \ldots, \quad P^i = \frac{1}{\sqrt{8}} \sigma_2 \sigma_i \tau_2 \]

- enhance \( C_0 \) to account for unnaturally large scattering lengths. In PDS scheme

\[ C_0^3 S_1 = \mathcal{O} \left( \frac{4\pi}{m_N \mu} \right), \quad \mu \sim Q \]

- iterate \( C_0 \) at all orders

- operators which connect \( S \)-waves get enhanced \( C_2^3 S_1 = \mathcal{O} \left( \frac{4\pi}{m_N \Lambda_{NN}} \frac{1}{\mu^2} \right) \)

\[
\begin{align*}
C_0 \frac{Q}{\Lambda_{NN}} & \quad C_0 \frac{Q}{\Lambda_{NN}} \frac{m_N Q}{4\pi} C_0 & \quad C_0 \frac{Q}{\Lambda_{NN}} \left( \frac{m_N Q}{4\pi} C_0 \right)^2
\end{align*}
\]
Deuteron EDM and MQM. KSW Power Counting

- treat pion exchange as a perturbation

\[ C_0 \frac{g_\Lambda^2 m_N Q}{4\pi F_\pi^2} \quad C_0 \frac{g_\Lambda^2 m_N Q}{4\pi F_\pi^2} \frac{m_N Q}{C_0} \]

- identify \( \Lambda_{NN} = 4\pi F_\pi^2 / m_N \approx 300 \text{ MeV} \).

Perturbative pion approach:

- expansion in \( Q / \Lambda_{NN} \), with \( Q \in \{ |q|, m_\pi, \gamma = \sqrt{m_N B} \} \)

- competing with the \( m_\pi / M_{QCD} \) of ChPT Lagrangian

  - successful for deuteron properties at low energies
    Kaplan, Savage and Wise, Phys. Rev. C 59, 617 (1999);

  - problems in \( ^3S_1 \) scattering lengths,
    ptb. series does not converge for \( Q \sim m_\pi \)
    Fleming, Mehen, and Stewart, Nucl. Phys. A 677, 313 (2000);
Deuteron EDM and MQM. KSW Power Counting

- treat pion exchange as a perturbation

\[
\frac{g_A^2}{F_\pi^2} \quad \frac{g_A^2 m_N Q}{4\pi F_\pi^2}
\]

- identify \( \Lambda_{NN} = 4\pi F_\pi^2 / m_N \sim 300 \text{ MeV} \).

Perturbative pion approach:

- expansion in \( Q / \Lambda_{NN} \), with \( Q \in \{|q|, m_\pi, \gamma = \sqrt{m_N B}\} \)
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Deuteron EDM and MQM. KSW Power Counting

$T$-odd sector

a. four-nucleon $T$-odd operators

$$\mathcal{L}_{T,f=4} = C_{1,T} \bar{N} S \cdot (\mathcal{D} + \mathcal{D}^\dagger) N \bar{N} N + C_{2,T} \bar{N} \tau S \cdot (\mathcal{D} + \mathcal{D}^\dagger) N \cdot \bar{N} \tau N.$$ 

• in the PDS scheme

1. Theta

\[ C_{i,T} \frac{4\pi}{\mu m_N} \bar{\theta} \frac{m^2}{M_{QCD}^2 \Lambda_{NN}^2} \]

2. qCEDM

\[ \frac{4\pi}{\mu m_N} \bar{\delta} \frac{m^2}{M_{QCD}^2} \]

3. qEDM

\[ 0 \]

4. gCEDM

\[ \frac{4\pi}{\mu m_N} \frac{w}{M_T^2} \Lambda_{NN} \]

b. four-nucleon $T$-odd currents

$$\mathcal{L}_{T,em,f=4} = C_{1,T,em} \bar{N} (S^\mu \nu^\nu - \nu^\nu \nu^\mu) N \bar{N} N F_{\mu \nu}.$$ 

• in the PDS scheme

1. Theta

\[ C_{i,T,em} \frac{4\pi}{\mu^2 m_N} \bar{\theta} \frac{m^2}{M_{QCD}^2 \Lambda_{NN}^2} \]

2. qCEDM

\[ \frac{4\pi}{\mu^2 m_N} \bar{\delta} \frac{m^2}{M_{QCD}^2} \]

3. qEDM

\[ \frac{4\pi}{\mu^2 m_N} \delta \frac{m^2}{M_{QCD}^2} \]

4. gCEDM

\[ \frac{4\pi}{\mu^2 m_N} \frac{w}{M_T^2} \Lambda_{NN} \]
Deuteron EDM. Formalism

- crossed blob: insertion of interpolating field $\mathcal{D}^i(x) = N(x) P^3_i S^1 P^S_1 N(x)$
- two-point and three-point Green’s functions expressed in terms of irreducible function $\text{irreducible}$
  - do not contain $C^3_0 S^1$
- by LSZ formula
  $$\langle p' j_{\text{em}} | J_{\text{em}}^\mu | p i \rangle = i \left[ \frac{\Gamma_{ij}^{\mu} (\bar{E}, \bar{E}', \mathbf{q})}{d\Sigma(\bar{E})/dE} \right]_{\bar{E}, \bar{E}' = -B}$$
- two-point function
  $$\frac{d\Sigma_{(1)}}{d\bar{E}} \bigg|_{\bar{E} = -B} = -i \frac{m_N^2}{8\pi \gamma}$$