Precision Determination of $\alpha_s(m_Z)$ from Thrust Data

Michael Fickinger
University of Arizona

Phys.Rev. D83 (2011) 074021

Taskforce:  A. Hoang - U. Vienna
            I. Stewart, V. Mateu & R. Abbate - MIT
            M. Fickinger - UA

Frontiers in QCD, INT, 6th October 2011
The Strong Coupling Constant $\alpha_s$

$\alpha_s$ is a key parameter for the analysis of all collider experiments. 

QCD: $\alpha_s(M_Z) = 0.1184(07)$

**Event shape variable:** assigns a number to the shape of an event
**Event shape variable:** assigns a number to the shape of an event

**Thrust:**

\[ T \equiv \max_t \left[ \frac{\sum_j |\vec{p}_j \cdot \hat{t}|}{\sum_j |\vec{p}_j|} \right] \]

use \[ \tau = 1 - T \]
**Event shape variable:** assigns a number to the shape of an event

**Thrust:** \[ T \equiv \max \bigg[ \frac{\sum_j |\vec{p}_j \cdot \hat{t}|}{\sum_j |\vec{p}_j|} \bigg] \]

Use \( \tau = 1 - T \)

Measures 2-jet likeness

- Ideal 2-jet event: \( \tau = 0 \)
- Spherical event: \( \tau = \frac{1}{2} \)
Event shape variable: assigns a number to the shape of an event.

**Thrust:** $T \equiv \max \left[ \frac{\sum_j |\vec{p}_j \cdot \hat{t}|}{\sum_j |\vec{p}_j|} \right]$  
Use $\tau = 1 - T$

Measures 2-jet likeness

ideal 2-jet event

spherical event

$$\frac{1}{\sigma_0} \frac{d\sigma}{d\tau} = \delta(\tau) + \alpha_s c_1 \delta(\tau) + R_1(\tau > 0) + \ldots$$  
Observable is very sensitive to $\alpha_s$!
Outline

• Results
• Theory
• Numerical Analysis
• Moment Fits
• Summary
Results
\[ \alpha_s(m_Z) = 0.1135 \pm (0.0002)_{\text{expt}} \pm (0.0005)_{\text{hadr}} \pm (0.0009)_{\text{pert}} \]

with \[ \frac{\chi^2}{\text{dof}} = 0.91 \]
$$\alpha_s(m_Z) = 0.1135 \pm (0.0002)_{\text{expt}} \pm (0.0005)_{\text{hadr}} \pm (0.0009)_{\text{pert}}$$

with $$\frac{\chi^2}{\text{dof}} = 0.91$$

predict smaller $$\tau$$'s

$$Q = 91.2 \text{ GeV}$$
Fit to Distribution:

$$\alpha_s(m_Z) = 0.1135 \pm 0.0002_{\text{expt}} \pm 0.0005_{\text{hadr}} \pm 0.0009_{\text{pert}}$$

with $\frac{\chi^2}{\text{dof}} = 0.91$

$$\Omega_1(\mu_\Delta, R_\Delta) = 0.323 \pm 0.052$$

487 data points

$\mu_\Delta = R_\Delta = 2\text{GeV}$

Fit to First Moment:

$$\alpha_s(m_Z) = 0.1142 \pm 0.0015_{\text{expt}} \pm 0.0005_{\text{hadr}} \pm 0.0007_{\text{pert}}$$

with $\frac{\chi^2}{\text{dof}} = 1.10$

$$\Omega_1(\mu_\Delta, R_\Delta) = 0.388 \pm 0.061$$

34 data points
\[ \alpha_s(m_Z) = 0.1135 \pm (0.0002)_{\text{expt}} \pm (0.0005)_{\text{hadr}} \pm (0.0009)_{\text{pert}} \]

with \( \frac{\chi^2}{\text{dof}} = 0.91 \)
Theory
- **hard scale** (cm energy)
- **jet scale** (invariant mass of all energetic particles in one hemisphere)
- **soft scale** (uniform soft radiation)
- hard scale (cm energy)
- jet scale (invariant mass of all energetic particles in one hemisphere)
- soft scale (uniform soft radiation)
Soft Collinear Effective Theory (SCET)

Bauer, Fleming, Luke, Pirjol, Rothstein, Stewart

- Describes light-like particles (collinear) interacting with a low energetic background (soft)

- Expansion in

- Power counting:
  - soft: \[ p_\mu = (p_+ , p_- , p_\perp) \propto Q(\lambda^2 , \lambda^2 , \lambda^2) \]
  - collinear: \[ p_\mu = (p_+ , p_- , p_\perp) \propto Q(\lambda^2 , 1 , \lambda) \]
Perturbative Series

\[ \frac{1}{\sigma_0} \frac{d\sigma}{d\tau} = \delta(\tau) + \alpha_s R_1(\tau) + \alpha_s^2 R_2(\tau) + \alpha_s^3 R_3(\tau) + \ldots \]
Perturbative Series

\[ \frac{1}{\sigma_0} \frac{d\sigma}{d\tau} = \delta(\tau) + \alpha_s R_1(\tau) + \alpha_s^2 R_2(\tau) + \alpha_s^3 R_3(\tau) + \ldots \]

**Problems if:** \( \alpha_s \gtrsim 1 \)

coefficients not of \( O(1) \)
Perturbative Series

\[ \frac{1}{\sigma_0} \frac{d\sigma}{d\tau} = \delta(\tau) + \alpha_s R_1(\tau) + \alpha_s^2 R_2(\tau) + \alpha_s^3 R_3(\tau) + \ldots \]

Problems if: \( \alpha_s \gtrsim 1 \) and coefficients not of \( O(1) \)

\[ R_i(\tau) \propto \frac{\ln^n(\tau)}{\tau} \]
Perturbative Series

\[ \frac{1}{\sigma_0} \frac{d\sigma}{d\tau} = \delta(\tau) + \alpha_s R_1(\tau) + \alpha_s^2 R_2(\tau) + \alpha_s^3 R_3(\tau) + \ldots \]

Problems if: \( \alpha_s \gtrsim 1 \)

coefficients not of \( O(1) \)

SCET gives factorization:

\[ H(\mu_H) \quad \uparrow \quad U_H \]
\[ J(\mu_J) \quad \downarrow \quad U_H, U_J \]

\[ \frac{1}{\sigma_0} \frac{d\hat{\sigma}}{d\tau} \propto H(\mu_H) \otimes J(\mu_J) \otimes S(\mu_s) \otimes S_{\text{mod}}(\Lambda_{QCD}) \]
Perturbative Series

\[ \frac{1}{\sigma_0} \frac{d\sigma}{d\tau} = \delta(\tau) + \alpha_S R_1(\tau) + \alpha_S^2 R_2(\tau) + \alpha_S^3 R_3(\tau) + \ldots \]

Problems if: \( \alpha_S \gtrsim 1 \)
coefficients not of \( O(1) \)

\[ R_i(\tau) \propto \frac{\ln^n(\tau)}{\tau} \]

SCET gives factorization:

\[ \frac{1}{\sigma_0} \frac{d\hat{\sigma}}{d\tau} \propto H(\mu_H) \otimes J(\mu_J) \otimes S(\mu_S) \otimes S_{\text{mod}}(\Lambda_{QCD}) \]

SCET reorganizes the power series in terms of \((\alpha_S^m \ln^n \tau)\)
Factorization Theorem for Thrust

\[
\frac{d\sigma}{d\tau} = \int dk \left( \frac{d\hat{\sigma}_s}{d\tau} + \frac{d\hat{\sigma}_{ns}}{d\tau} + \Delta \frac{d\hat{\sigma}_b}{d\tau} \right) \left( \tau - \frac{k}{Q} \right) S^\text{mod}_\tau (k - 2\Delta) + O \left( \sigma_0 \frac{\alpha_s \Lambda_{QCD}}{Q} \right)
\]

Method to simultaneously describe all three regions and multiple Q:
profile functions

Singular partonic:
- NNLO matching, including full 3-loop hard function
- N^3LL resummation of large logs

Nonsingular partonic:
- fixed order thrust distribution (subleading orders in SCET)

Nonperturbative soft function:
- nonperturbative effects treated within field theory
  Operator Product Expansion in tail \( \rightarrow \Omega_1 \)

Interface between power and perturbative corrections:
- renormalon subtraction

b-mass effects (~2% effect)
QED effects (~2% effect)
axial anomaly at \( O(\alpha_s^2) \) (~1% effect)
Factorization Formula for all Thrust

peak: sum large logs, nonperturbative soft fct.

\[ Q \gg \frac{Q \sqrt{\Lambda_{QCD}}}{Q} \gg \Lambda_{QCD} \]

tail: sum large logs, series of nonperturbative power corrections

\[ Q \gg Q\sqrt{\tau} \gg Q\tau \gg \Lambda_{QCD} \]

far tail: fixed order perturbation theory, power corrections

\[ Q \gg \Lambda_{QCD} \]

Profile Functions:

Scales must equal in the far tail: turns of resummation
Far Tail

\[ \frac{1}{\sigma} \frac{d\sigma}{d\tau} \]

**N^3 LL' results**

- **full** \((\alpha_s = 0.1135, \Omega_1 = 0.324 \text{ GeV})\)
- +theory scan error
- **no \(\Omega_1\)** \((\alpha_s = 0.1135)\)
- **full, BS profile** \((\alpha_s = 0.1001, \Omega_1 = 0.371 \text{ GeV})\)
- **no \(\Omega_1\), BS profile** \((\alpha_s = 0.1172)\)

\(Q = m_Z\)

ALEPH

\[ \tau \]

\[ 0.32 \quad 0.34 \quad 0.36 \quad 0.38 \quad 0.40 \quad 0.42 \]
Factorization Theorem for Thrust

\[ \frac{d\sigma}{d\tau} = \int dk \left( \frac{d\hat{\sigma}_s}{d\tau} + d\hat{\sigma}_{ns} + \Delta d\hat{\sigma}_b \right) \left( \tau - \frac{k}{Q} \right) S_{\tau}^{\text{mod}} \left( k - 2\Delta \right) + O\left( \sigma_0 \frac{\alpha_s\Lambda_{QCD}}{Q} \right) \]

Method to simultaneously describe all three regions and multiple Q:
profile functions

**Singular partonic:**
- NNLO matching, including full 3-loop hard function
- \( N^3\text{LL} \) resummation of large logs

Moch, Vermaseren, Vogt
Becher, Neubert
Schwartz; Fleming et al.
Becher, Schwartz; Hoang, Kluth
Dasgupta, Salam

**Nonsingular partonic:**
- fixed order thrust distribution (subleading orders in SCET)

Becher, Schwartz

**Nonperturbative soft function:**
- nonperturbative effects treated within field theory
- Operator Product Expansion in tail \( \rightarrow \Omega_1 \)

**Interface between power and perturbative corrections:**
- renormalon subtraction

**b-mass effects** (~2% effect)

**QED effects** (~2% effect)

axial anomaly at \( O(\alpha_s^2) \) (~1% effect)
Summing large Logarithms

\[ \frac{1}{\sigma} \frac{d\sigma}{d\tau} \]

\( Q = m_Z \)

**Fixed Order**

- \( \mathcal{O}(\alpha_s^3) \)
- \( \mathcal{O}(\alpha_s^2) \)
- \( \mathcal{O}(\alpha_s) \)

**Sum Logs, no \( S_{\text{mod}} \)**

- \( N^3\text{LL}' \)
- \( N^3\text{LL} \)
- \( \text{NNLL}' \)
- \( \text{NNLL} \)
- \( \text{NLL}' \)

Setting \( \mu_H, \mu_J, \mu_S \) to their natural scales

\[ \rightarrow \text{much better convergence!} \]
Factorization Theorem for Thrust

\[ \frac{d\sigma}{d\tau} = \int dk \left( \frac{d\hat{\sigma}_s}{d\tau} + \left( \frac{d\hat{\sigma}_{ns}}{d\tau} + \frac{\Delta d\hat{\sigma}_b}{d\tau} \right) \right) \left( \tau - \frac{k}{Q} \right) S_{\tau}^{\text{mod}}(k - 2\Delta) + O\left( \frac{\alpha_s \Lambda_{QCD}}{Q} \right) \]

Method to simultaneously describe all three regions and multiple Q:
- profile functions

Singular partonic:
- NNLO matching, including full 3-loop hard function
- \(N^3LL\) resummation of large logs

Nonsingular partonic: \(Ellis\ et\ al.,\ Catani\ et\ al.,\ Gehrmann\ et\ al.,\ Weinzierl\)
- fixed order thrust distribution (subleading orders in SCET)

Nonperturbative soft function:
- nonperturbative effects treated within field theory
- Operator Product Expansion in tail \(\rightarrow \Omega_1\)

Interface between power and perturbative corrections:
- renormalon subtraction

b-mass effects (~2% effect)
QED effects (~2% effect)
axial anomaly at \(O(\alpha_s^2)\) (~1% effect)
Factorization Theorem for Thrust

\[
\frac{d\sigma}{d\tau} = \int dk \left( \frac{d\hat{\sigma}_s}{d\tau} + \frac{d\hat{\sigma}_{ns}}{d\tau} + \frac{\Delta d\hat{\sigma}_b}{d\tau} \right) \left( \tau - \frac{k}{Q} \right) S_{\tau}^{\text{mod}}(k - 2\bar{\Delta}) + O\left(\sigma_0 \frac{\alpha_s \Lambda_{QCD}}{Q}\right)
\]

Method to simultaneously describe all three regions and multiple $Q$:
profile functions

Singular partonic:
\- NNLO matching, including full 3-loop hard function
\- N$^3$LL resummation of large logs

Nonsingular partonic:
\- fixed order thrust distribution (subleading orders in SCET)

**Nonperturbative soft function:**
\- nonperturabtive effects treated within field theory
\- Operator Product Expansion in tail $\rightarrow \Omega_1$

Interface between power and perturbative corrections:
renormalon subtraction

b-mass effects ($\sim 2\%$ effect)
QED effects ($\sim 2\%$ effect)
axial anomaly at $O(\alpha_s^2)$ ($\sim 1\%$ effect)

Hoang, Stewart
Ligeti, Stewart, Tackmann
Lee, Sterman
Korchemsky, Sterman
Nonperturbative Corrections

Soft function from SCET factorization:

\[ S_\tau(k, \mu) = \frac{1}{N_c} \langle 0 | Tr \tilde{Y}_n Y_n \delta(k - i \partial_\tau) Y_n^+ \tilde{Y}_n^+ | 0 \rangle \]

\[ i \partial_\tau \equiv \theta(\bar{n} \cdot \partial - in \cdot \partial) in \cdot \partial + \theta(in \cdot \partial - \bar{n} \cdot \partial) \bar{n} \cdot \partial \]

Factorization in perturbative and non-perturbative part:

\[ S_\tau(k, \mu) = \int dk' S_\tau^{part}(k - k', \mu) S_\tau^{mod}(k') \]

OPE:

\[ S_\tau(k, \mu) = S_\tau^{part}(k, \mu) - 2 \Omega_1 \frac{d S_\tau^{part}}{dk}(k, \mu) + \ldots \]

Hoang, Stewart Ligeti, Stewart, Tackmann

complete basis of functions
Are non-perturbative effects negligible?

Leading effect: shift in the thrust distribution \(\tau \rightarrow \tau - 2 \Lambda/Q\)

\[
\frac{1}{\sigma} \frac{d\sigma}{d\tau} = h(\tau - 2 \frac{\Lambda}{Q})
\]

Manohar, Wise; Webber; Dokshitzer, Weber; Akhoury, Zakharov; Nason, Seymour; Korchemsky, Sterman; Movilla Fernandez, Bethke, Biebel, Kluth
Are non-perturbative effects negligible?

Leading effect: shift in the thrust distribution \( \tau \rightarrow \tau - 2 \frac{\Lambda}{Q} \)

\[
\frac{1}{\sigma} \frac{d\sigma}{d\tau} = h(\tau - 2 \frac{\Lambda}{Q}) \approx h(\tau) - 2 \frac{\Lambda}{Q} h'(\tau) \quad \Lambda/Q \ll 1
\]
Are non-perturbative effects negligible?

Leading effect: shift in the thrust distribution \( \tau \rightarrow \tau - 2 \frac{\Lambda}{Q} \)

\[
\frac{1}{\sigma} \frac{d\sigma}{d\tau} = h(\tau - 2 \frac{\Lambda}{Q}) \approx h(\tau) - 2 \frac{\Lambda}{Q} h'(\tau) \quad \Lambda / Q \ll 1
\]

\( h \) proportional to \( \alpha_s \) =>

\[
\frac{\delta \alpha_s}{\alpha_s} \approx \frac{h(\tau - 2 \frac{\Lambda}{Q}) - h(\tau)}{h(\tau)} \approx -2 \frac{\Lambda}{Q} \frac{h'(\tau)}{h(\tau)}
\]
Are non-perturbative effects negligible?

Leading effect: shift in the thrust distribution \( \tau \rightarrow \tau - 2 \frac{\Lambda}{Q} \)

\[
\frac{1}{\sigma} \frac{d\sigma}{d\tau} = h(\tau - 2 \frac{\Lambda}{Q}) \approx h(\tau) - 2 \frac{\Lambda}{Q} h'(\tau) \quad \Lambda / Q \ll 1
\]

\( h \) proportional to \( \alpha_s \) \=>

\[
\frac{\delta \alpha_s}{\alpha_s} \approx \frac{h(\tau - 2 \frac{\Lambda}{Q}) - h(\tau)}{\left[ h(\tau) \right]} \approx -2 \frac{\Lambda}{Q} \frac{h'(\tau)}{h(\tau)}
\]
Are non-perturbative effects negligible?

Leading effect: shift in the thrust distribution \( \tau \rightarrow \tau - 2 \Lambda / Q \)

\[
\frac{1}{\sigma} \frac{d\sigma}{d\tau} = h(\tau - 2 \frac{\Lambda}{Q}) \approx h(\tau) - 2 \frac{\Lambda}{Q} h'(\tau) \quad \text{if} \quad \Lambda / Q \ll 1
\]

\( h \) proportional to \( \alpha_s \) \( \Rightarrow \frac{\delta \alpha_s}{\alpha_s} \approx \frac{h(\tau - 2 \frac{\Lambda}{Q}) - h(\tau)}{h(\tau)} \approx -2 \frac{\Lambda}{Q} \frac{h'(\tau)}{h(\tau)} \)

\( \Lambda \approx 0.3 \text{ GeV} \) and \( h'/h \approx -14 \pm 4 \) in tail region (see figure)

\( \Rightarrow \frac{\delta \alpha_s}{\alpha_s} \approx -(9 \pm 3)\% \)
Are non-perturbative effects negligible?

Leading effect: shift in the thrust distribution \( \tau \to \tau - 2 \Lambda / Q \)

\[
\frac{1}{\sigma} \frac{d\sigma}{d\tau} = h(\tau - 2 \frac{\Lambda}{Q}) \approx h(\tau) - 2 \frac{\Lambda}{Q} h'(\tau) \quad \Lambda / Q \ll 1
\]

\( h \) proportional to \( \alpha_s \) \( \Rightarrow \frac{\delta \alpha_s}{\alpha_s} \approx \frac{h(\tau - 2 \frac{\Lambda}{Q}) - h(\tau)}{h(\tau)} \approx -2 \frac{\Lambda}{Q} \frac{h'(\tau)}{h(\tau)} \)

\( \Lambda \approx 0.3 \text{ GeV} \) and

\( h'/h \approx -14\pm4 \) in tail region (see figure)

\( \Rightarrow \delta \alpha_s/\alpha_s \approx -(9\pm3)\% \)

\( \Rightarrow \text{NOT negligible for 2\% analysis} \)
Non-perturbative effects

Use tail fit results to predict peak
Non-perturbative effects

Use tail fit results to predict peak

$\Rightarrow$ much better prediction for peak
Non-perturbative effects

Use tail fit results to predict peak

⇒ much better prediction for peak

<table>
<thead>
<tr>
<th>$\alpha_s(m_Z)$±(pert. error)</th>
<th>$\chi^2/(\text{dof})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N^3\text{LL}'$ with $\Omega_1^{\text{Rgap}}$</td>
<td>0.1135 ± 0.0009</td>
</tr>
<tr>
<td>$N^3\text{LL}'$ with $\Omega_1^{\text{MS}}$</td>
<td>0.1146 ± 0.0021</td>
</tr>
<tr>
<td>$N^3\text{LL}'$ without $S_{\tau}^{\text{mod}}$</td>
<td>0.1241 ± 0.0034</td>
</tr>
<tr>
<td>$O(\alpha_s^3)$ fixed-order without $S_{\tau}^{\text{mod}}$</td>
<td>0.1295 ± 0.0046</td>
</tr>
</tbody>
</table>

$\alpha_S$ from full analysis is approximately 9% smaller than $\alpha_S$ without model, as predicted.
Factorization Theorem for Thrust

\[ \frac{d\sigma}{d\tau} = \int dk \left( \frac{d\hat{\sigma}_s}{d\tau} + \frac{d\hat{\sigma}_{ns}}{d\tau} + \frac{\Delta d\hat{\sigma}_b}{d\tau} \right) (\tau - \frac{k}{Q}) S_\tau^{\text{mod}} (k - 2\Delta) + O(\sigma_0 \frac{\alpha_s \Lambda_{QCD}}{Q}) \]

Method to simultaneously describe all three regions and multiple $Q$:
profile functions

**Singular partonic:**
- NNLO matching, including full 3-loop hard function
- $N^3LL$ resummation of large logs

**Nonsingular partonic:**
- fixed order thrust distribution (subleading orders in SCET)

**Nonperturbative soft function:**
- nonperturbative effects treated within field theory
- Operator Product Expansion in tail $\rightarrow \Omega_1$

**Interface between power and perturbative corrections:**
- renormalon subtraction

$b$-mass effects ($\sim 2\%$ effect)

QED effects ($\sim 2\%$ effect)

axial anomaly at $O(\alpha_s^2)$ ($\sim 1\%$ effect)

Hoang, Stewart
Hoang, Jain, Scimemi, Stewart
**Renormalon Subtraction**

\( \overline{\text{MS}} \) perturbative series includes fluctuations with arbitrarily small momenta \( \rightarrow \) large unphysical corrections

Both \( S_{\tau}^{\text{part}} \) and \( \Omega_1 \) suffer from renormalon

Introduce gap parameter \( \Delta \):

\[
S_{\tau}^{\text{mod}}(k) \rightarrow S_{\tau}^{\text{mod}}(k - 2\Delta)
\]

and \( \Delta = \bar{\Lambda}(R, \mu_S) + \delta(R, \mu_S) \)

\( \rightarrow \) renormalon free soft function:

\[
S_{\tau}(k, \mu) = \int dk' \left( e^{-2\delta \frac{\partial}{\partial k} \delta} S_{\tau}^{\text{part}}(k - k', \mu) \right) S_{\tau}^{\text{mod}}(k' - 2\Delta)
\]

Hoang, Stewart
renormalon substracted

renormalon \textbf{NOT} substracted

\[ \bar{\Omega}_1 \text{ (GeV)} \]
\[ \alpha_s (m_Z) \]

\[ \frac{\chi^2}{\text{dof}} \]

\[ \alpha_s (m_Z) \]

\textit{MS} perturbative series includes fluctuations with arbitrarily small momenta → large unphysical corrections
Factorization Theorem for Thrust

\[
\frac{d\sigma}{d\tau} = \int dk \left( \frac{d\hat{\sigma}_s}{d\tau} + \frac{d\hat{\sigma}_{ns}}{d\tau} + \frac{\Delta d\hat{\sigma}_b}{d\tau} \right) \left( \tau - \frac{k}{Q} \right) S_{\tau}^{\text{mod}} (k - 2\Delta) + O(\sigma_0 \frac{\alpha_s \Lambda_{QCD}}{Q})
\]

Method to simultaneously describe all three regions and multiple Q:
- profile functions

Singular partonic:
- NNLO matching, including full 3-loop hard function
- N^3LL resummation of large logs

Nonsingular partonic:
- fixed order thrust distribution (subleading orders in SCET)

Nonperturbative soft function:
- nonperturbative effects treated within field theory
  Operator Product Expansion in tail $\rightarrow \Omega_1$

Interface between power and perturbative corrections:
- renormalon subtraction

**b-mass effects** (~2% effect)

QED effects (~2% effect)

axial anomaly at $O(\alpha_s^2)$ (~1% effect)
Factorization Theorem for Thrust

\[ \frac{d\sigma}{d\tau} = \int dk \left( \frac{d\hat{\sigma}_s}{d\tau} + \frac{d\hat{\sigma}_{ns}}{d\tau} + \frac{\Delta d\hat{\sigma}_b}{d\tau} \right) \left( \tau - \frac{k}{Q} \right) S_{\tau}^{\text{mod}}(k - 2\bar{\Delta}) + O\left(\sigma_0 \frac{\alpha_s \Lambda_{QCD}}{Q}\right) \]

Method to simultaneously describe all three regions and multiple Q:
profile functions

Singular partonic:
NNLO matching, including full 3-loop hard function
N^3LL resummation of large logs

Nonsingular partonic:
fixed order thrust distribution (subleading orders in SCET)

Nonperturbative soft function:
nonperturbative effects treated within field theory
Operator Product Expansion in tail \(\rightarrow\) \(\Omega_1\)

Interface between power and perturbative corrections:
renormalon subtraction

b-mass effects (~2% effect)

QED effects (~2% effect)

axial anomaly at O(\(\alpha_s^2\)) (~1% effect)
Factorization Theorem for Thrust

\[ \frac{d\sigma}{d\tau} = \int dk \left( \frac{d\hat{\sigma}_s}{d\tau} + \frac{d\hat{\sigma}_{ns}}{d\tau} + \frac{\Delta d\hat{\sigma}_b}{d\tau} \right) \left( \tau - \frac{k}{Q} \right) S_{\tau}^{\text{mod}} (k - 2\bar{\Delta}) + O \left( \sigma_0 \frac{\alpha_s \Lambda_{QCD}}{Q} \right) \]

Method to simultaneously describe all three regions and multiple Q:
profile functions

Singular partonic:
- NNLO matching, including full 3-loop hard function
- N^3LL resummation of large logs

Nonsingular partonic:
- fixed order thrust distribution (subleading orders in SCET)

Nonperturbative soft function:
- nonperturbative effects treated within field theory
  - Operator Product Expansion in tail \( \rightarrow \Omega_1 \)

Interface between power and perturbative corrections:
- renormalon subtraction

b-mass effects (\( \sim 2\% \) effect)
QED effects (\( \sim 2\% \) effect)
al axial anomaly at \( O(\alpha_s^2) \) (\( \sim 1\% \) effect)

Kniehl, Kuhn
Hagiwara, Kuruma, Yamada
\(\alpha_s(m_Z)\) from global thrust fits

\[\pm \rightarrow \text{perturbative error}\]

- O(\(\alpha_s^3\))
  \[0.1300 \pm 0.0047\]
- Multijet boundary
  \[0.1245 \pm 0.0034\]
- Power Corrections
  \[0.1152 \pm 0.0021\]
- + N^3LL summation
- + R-scheme
  \[0.1140 \pm 0.0009\]
- + b-mass & QED
  \[0.1135 \pm 0.0009\]

\[
\alpha_s(m_Z) = 0.1300 \pm 0.0047
\]
Comparison with similar analysis

<table>
<thead>
<tr>
<th></th>
<th>sum logs</th>
<th>power corrections</th>
<th>data</th>
<th>(\alpha_s(M_Z))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dissertori et al.</td>
<td>no</td>
<td>Monte Carlo (MC)</td>
<td>ALEPH</td>
<td>0.1240(34)</td>
</tr>
<tr>
<td>Dissertori et al.</td>
<td>NLL</td>
<td>Monte Carlo</td>
<td>ALEPH</td>
<td>0.1224(39)</td>
</tr>
<tr>
<td>Becher, Schwartz</td>
<td>N3LL</td>
<td>uncertainty from MC</td>
<td>ALEPH, OPAL</td>
<td>0.1172(21)</td>
</tr>
<tr>
<td>Davison, Webber</td>
<td>NLL</td>
<td>effective coupling model</td>
<td>Most of data</td>
<td>0.1164(28)</td>
</tr>
<tr>
<td>Bethke et al.</td>
<td>NLL</td>
<td>Monte Carlo</td>
<td>JADE</td>
<td>0.1172(51)</td>
</tr>
</tbody>
</table>

Becher, Schwartz:
- no nonperturbative soft function
- different profile function
- different way of calculating binned cross section
Numerical Analysis
Two parameter fit in tail region (factorization formula valid for all tau)

$\Omega_1$ is the 1st nonperturbative power correction

$Q = 91.2 \text{ GeV}$

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Values of $Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALEPH</td>
<td>{91.2, 133.0, 161.0, 172.0, 183.0, 189.0, 200.0, 206.0}</td>
</tr>
<tr>
<td>DELPHI</td>
<td>{45.0, 66.0, 76.0, 89.5, 91.2, 93.0, 133.0, 161.0, 172.0, 183.0, 189.0, 192.0, 196.0, 200.0, 202.0, 205.0, 207.0}</td>
</tr>
<tr>
<td>OPAL</td>
<td>{91.0, 133.0, 161.0, 172.0, 177.0, 183.0, 189.0, 197.0}</td>
</tr>
<tr>
<td>L3</td>
<td>{41.4, 55.3, 65.4, 75.7, 82.3, 85.1, 91.2, 130.1, 136.1, 161.3, 172.3, 182.8, 188.6, 194.4, 200.0, 206.2}</td>
</tr>
<tr>
<td>SLD</td>
<td>{91.2}</td>
</tr>
<tr>
<td>TASSO</td>
<td>{(14.0), (22.0), 35.0, 44.0}</td>
</tr>
<tr>
<td>JADE</td>
<td>{35.0, 44.0}</td>
</tr>
<tr>
<td>AMY</td>
<td>{55.2}</td>
</tr>
</tbody>
</table>
Order Counting

\[
\ln \left( \frac{d\sigma}{dy} \right) \sim (\alpha_s \ln)^k \ln + (\alpha_s \ln)^k + \alpha_s (\alpha_s \ln)^k + \alpha_s^2 (\alpha_s \ln)^k + \ldots
\]

<table>
<thead>
<tr>
<th></th>
<th>cusp</th>
<th>non-cusp</th>
<th>matching</th>
<th>alphas</th>
</tr>
</thead>
<tbody>
<tr>
<td>LL</td>
<td>1</td>
<td>-</td>
<td>tree</td>
<td>1</td>
</tr>
<tr>
<td>NLL</td>
<td>2</td>
<td>1</td>
<td>tree</td>
<td>2</td>
</tr>
<tr>
<td>NNLL</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>N^3LL</td>
<td>4^{pade}</td>
<td>3</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>LL'</td>
<td>1</td>
<td>-</td>
<td>tree</td>
<td>1</td>
</tr>
<tr>
<td>NLL'</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>NNLL'</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>N^3LL'</td>
<td>4^{pade}</td>
<td>3</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

Primed counting is better if fixed order results are important.
Experimental and Hadronic Uncertainty

with theory error

$\Delta \chi^2 = 1$

$\sqrt{\sigma^2_{\exp} + \sigma^2_{\Omega_1}}$
Perturbative Uncertainty

12 theory parameters:

- 6 parameters for the variation of the renormalization scales
- 3 parameters related to the statistical uncertainties of numerical fixed order calculations
- 3 parameters for Padé approximants of unknown constants

1. Flat random scan over theory parameters
2. Fitting for each parameter set
3. Range of best fits \(\rightarrow\) perturbative uncertainty
\[ Q = 91.2 \text{ GeV} \]

\[ \alpha_s(m_Z) = 0.1135 \pm (0.0002)_{\text{expt}} \pm (0.0005)_{\text{hadr}} \pm (0.0009)_{\text{pert}} \]

with \[ \frac{\chi^2}{\text{dof}} = 0.91 \]
same analysis for other event shape variables: Heavy Jet Mass

AHMS+Schwartz

fits to moment data

AFHMS
Moment Fits
\(n^{th}\) moment: 
\[
M_n = \int_0^{\tau_{\text{max}}} d\tau \tau^n \int_0^{Q\tau} dp \frac{d\hat{\sigma}}{d\tau} (\tau - \frac{p}{Q}) S_{\tau}^{\text{mod}} (p) + O\left(\frac{\alpha_s \Lambda_{\text{QCD}}}{Q}\right)
\]

purely perturbative

\[
M_n = \sum_{k=0}^{n} \binom{n}{k} \hat{M}_k \left(\frac{2}{Q}\right)^{n-k} \Omega_{n-k} + O\left(\frac{\alpha_s \Lambda_{\text{QCD}}}{Q}\right)
\]

where \(\Omega_i = \int dp \left(\frac{p}{2}\right)^i S_{\tau}^{\text{mod}} (p)\) is defined as for the distribution

Higher sensitivity for higher moments with primed moments:

\[
M'_n = \hat{M}'_n + \left(\frac{2}{Q}\right)^n \Omega'_n + O\left(\frac{\alpha_s \Lambda_{\text{QCD}}}{Q}\right)
\]

\[
\Omega'_1 = \Omega_1
\]
\[
\Omega'_2 = \Omega_2 - \Omega_1^2
\]
\[
\Omega'_3 = \Omega_3 - 3\Omega_2 \Omega_1 + 2\Omega_1^3
\]

appear to also be \(1/Q^n\)!!!!
Global fits to $M_1$ data
Global fits to $M_1$ data

theory errors only
experimental errors are dominant

$N^3LL'$
$NNLL'$
$NLL'$
Fits for \( (2/Q)^i \Omega_i \) at \( \alpha_s(m_Z)=0.114 \), MS scheme:
cancellation for primed moments → no new information from higher moment data

\( \Omega_1 = 0.37 \pm 0.02 \) GeV

\( \Omega_2' = 0.54 \pm 0.28 \) GeV

\( \Omega_3' = 4.5 \pm 3.7 \) GeV

\( \Omega_4' = 38 \pm 42 \) GeV

\( \Omega_5' = 200 \pm 550 \) GeV
Comparison to Gehrmann

\[ M_{n}^{\text{Gehrmann}} = \sum_{k=0}^{n} \binom{n}{k} \hat{M}_{k} (2P)^{n-k} \]

\[ M_{n} = \sum_{k=0}^{n} \binom{n}{k} \hat{M}_{k} \left( \frac{2}{Q} \right)^{n-k} \Omega_{n-k} \]

\[ \rightarrow \text{fits to the higher moments are all sensitive to the same } \hat{M}_{1} \]

with dispersive model for power corrections:

\[ P = \frac{4C_{F}}{\pi^{2}} \cdot M \cdot \left\{ \alpha_{0} - \left( \alpha_{S}(\mu_{R}) + \frac{\beta_{0}}{\pi} \alpha_{S}^{2}(\mu_{R}) \left( \ln \frac{\mu_{R}}{\mu_{I}} + 1 + \frac{K}{2\beta_{0}} \right) + O(\alpha_{S}^{3}) \right) \right\} \cdot \frac{\mu_{I}}{Q} \]
Summary

- $\alpha_s = 0.1135 \pm (0.0002)_{\text{expt}} \pm (0.0005)_{\text{hadr}} \pm (0.0009)_{\text{pert}}$

- SCET can **improve convergence** and therefore precision
- SCET allows to include **non-perturbative effects** in a systematic manner
- Profile functions allow to combine several kinematic regions
Backup Slides
Why do we need a global fit?

in the tail $\alpha_s(m_Z)$ and $\Omega_1$ are degenerated for a single $Q$

degeneracy is lifted by simultaneously fitting multiple $Q$
QED and b-mass effects

b-mass:
horizontal shift of thrust distribution towards larger $\tau$
$\rightarrow$ lower $\Omega_1$

QED:
effective increase of coupling strength
$\rightarrow$ lower $\alpha_s$
Cut on Dataset

$\Omega_2$ effects increase $\tau_{\text{min}}$ decrease of $\Delta\tau$ → increase of statistical uncertainty

$2\Omega_1$ (GeV)

$\alpha_s(m_Z)$

missing $\alpha_s\Lambda_{QCD}/Q$ effects become important
## Theory Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Default Value</th>
<th>Range of Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_0$</td>
<td>2 GeV</td>
<td>1.5 to 2.5 GeV</td>
</tr>
<tr>
<td>$n_1$</td>
<td>5</td>
<td>2 to 8</td>
</tr>
<tr>
<td>$t_2$</td>
<td>0.25</td>
<td>0.20 to 0.30</td>
</tr>
<tr>
<td>$e_J$</td>
<td>0</td>
<td>-1,0,1</td>
</tr>
<tr>
<td>$e_H$</td>
<td>1</td>
<td>0.5 to 2.0</td>
</tr>
<tr>
<td>$n_s$</td>
<td>0</td>
<td>-1,0,1</td>
</tr>
<tr>
<td>$s_2$</td>
<td>-39.1</td>
<td>-36.6 to -41.6</td>
</tr>
<tr>
<td>$\Gamma_3^{\text{cusp}}$</td>
<td>1553.06</td>
<td>-1553.06 to +4569.18</td>
</tr>
<tr>
<td>$j_3$</td>
<td>0</td>
<td>-3000 to +3000</td>
</tr>
<tr>
<td>$s_3$</td>
<td>0</td>
<td>-500 to +500</td>
</tr>
<tr>
<td>$\epsilon_2$</td>
<td>0</td>
<td>-1,0,1</td>
</tr>
<tr>
<td>$\epsilon_3$</td>
<td>0</td>
<td>-1,0,1</td>
</tr>
</tbody>
</table>

- **profile function** (variation of renormalization scales)
- **Padé approximants**
- **non-singular stat. uncertainty**
Include b-mass effects in Factorization Thm: \( \sim 2\% \) effect

\[
\frac{\Delta d\hat{\sigma}^b}{d\tau} = \frac{d\hat{\sigma}^{NNLL}_{\text{massive}}}{d\tau} - \frac{d\hat{\sigma}^{NNLL}_{\text{massless}}}{d\tau}
\]

- at this order it effects only the jet function and \( \tau \) limits
- use SCET massive fact. thm
- charm quarks are much smaller effect

Include QED effects in Factorization Thm: \( \sim 2\% \) effect

- count \( \alpha \sim \alpha_s^2 \), include only final state radiation
- include \( \mathcal{O}(\alpha_s^2 \alpha) \) corrections to QCD \( \beta \)-function
- include one-loop QED corrections to \( H_Q, J_\tau, S_\tau \)

Include axial anomaly contribution \( \sim 1\% \) effect

- affects \( H^{ua}_Q, H^{da}_Q \) at \( \mathcal{O}(\alpha_s^2) \)
  \[
  H^a_Q = H^v_Q + H^{\text{singlet}}
  \]
  \[
  f^{da}(\tau, 1) = f^v(\tau, 1) + \frac{\alpha_s^2}{4\pi^2} f_{\text{singlet}}(\tau, \frac{Q^2}{4 m_t^2})
  \]
- due to large top-bottom mass splitting

Fleming, Hoang, Mantry, Stewart

Kniehl, Kuhn
Hagiwara, Kuruma, Yamada
Calculating binned cross-sections

At this level of precision, it is important to calculate theory results for the entire bin (even though they are quite fine).

How should we calculate the bins?

Difference of cumulants
\[ \sum(\tau_2, \mu_i(\tau_2)) - \sum(\tau_1, \mu_i(\tau_1)) \]
[all classic resummation analyses]
[Becher & Schwartz] [Chien & Schwartz]

Integrating the differential distribution
\[ \int_{\tau_1}^{\tau_2} d\tau' \frac{d\sigma}{d\tau'}(\tau', \mu_i(\tau')) \]
[AFHMS] [AHMS+Schwartz]

The two procedures look the same, but actually they are not.
\[ \Sigma(\tau_2, \mu_i(\tau_2)) - \Sigma(\tau_1, \mu_i(\tau_1)) = \]
\[ \int_{\tau_1}^{\tau_2} d\tau' \frac{d\sigma}{d\tau'}(\tau', \mu_i(\tau_2)) + \Sigma(\tau_1, \mu_i(\tau_2)) - \Sigma(\tau_1, \mu_i(\tau_1)) \]
\[ \approx \int_{\tau_1}^{\tau_2} d\tau' \frac{d\sigma}{d\tau'}(\tau', \mu_i(\tau')) + (\tau_2 - \tau_1) \frac{d\mu_i(\tau_1)}{d\tau_1} \int_{0}^{\tau_1} d\tau' \frac{d\sigma}{d\tau'}(\tau', \mu_i(\tau_1)) \]

Enhanced uncertainty from the peak!