Recent Advances in NLO QCD for the LHC

Seattle, Institute for Nuclear Theory
Sept 27, 2011
Zvi Bern, UCLA, on behalf of BlackHat

BlackHat Collaboration current members:
ZB, L. Dixon, F. Febres Cordero, G. Diana, S. Hoeche, H. Ita,
D. Kosower, D. Maitre, K. Ozeren
• Recent theoretical progress in performing amplitude and NLO QCD computations.
• Will present $Z, W + 4$ jets as examples.
• Comparison to Tevatron and LHC data.
• Some new theoretical observations for the LHC.
• Specific example of how theory can help experiments.
Example: Susy Search

- Cascade from gluino to neutralino (escapes detector)
- Signal: missing energy + 4 jets
- SM background from Z + 4 jets, $Z \rightarrow$ neutrinos

Previous state of art for Z + 4 jets: ALPGEN, based on LO tree amplitudes $\rightarrow$ normalization still quite uncertain. Issues on shapes of distributions.
Example: Susy Search

We need $pp \rightarrow Z + 4$ jets at NLO

Modern on-shell methods used to solve the problem.
Why we do NLO

CDF collaboration arXiv: 0711.4044

W + 2 jets at the Tevatron

NLO does better, smallest theoretical uncertainty

LO

leading order + parton showering

NLO

QCD

Want similar studies at the LHC also with extra jets.
In 1948 Schwinger computed anomalous magnetic moment of the electron.

60 years later typical example we can calculate via Feynman diagrams:

$$pp \rightarrow W, Z + 2 \text{ jets}$$

For LHC physics we need also four or more final state objects

- Z+3,4 jets not yet done via Feynman diagrams.
- Widespread applications to LHC physics.
## Amusing NLO Wish List

### Run II Monte Carlo Workshop, April 2001

<table>
<thead>
<tr>
<th>Single boson</th>
<th>Diboson</th>
<th>Triboson</th>
<th>Heavy flavour</th>
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<tbody>
<tr>
<td>$W + \leq 5j$</td>
<td>$WW + \leq 5j$</td>
<td>$WWW + \leq 3j$</td>
<td>$t\bar{t} + \leq 3j$</td>
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<td>$W + bb + \leq 3j$</td>
<td>$WW + bb + \leq 3j$</td>
<td>$WWW + bb + \leq 3j$</td>
<td>$t\bar{t} + \gamma + \leq 2j$</td>
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<td>$WW + c\bar{c} + \leq 3j$</td>
<td>$WWW + \gamma\gamma + \leq 3j$</td>
<td>$t\bar{t} + W + \leq 2j$</td>
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<tr>
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<td>$Z\gamma\gamma + \leq 3j$</td>
<td>$t\bar{t} + Z + \leq 2j$</td>
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<tr>
<td>$Z + b\bar{b} + \leq 3j$</td>
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<td>$WZZ + \leq 3j$</td>
<td>$t\bar{t} + H + \leq 2j$</td>
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<td>$ZZZ + \leq 3j$</td>
<td>$t\bar{b} + \leq 2j$</td>
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<td>$\gamma\gamma + b\bar{b} + \leq 3j$</td>
<td>$b\bar{b} + \leq 3j$</td>
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<td>$W\gamma + \leq 3j$</td>
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<td>$Z\gamma + \leq 3j$</td>
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</tbody>
</table>

**Just about every process of interest listed**
The Les Houches Wish List (2005)

<table>
<thead>
<tr>
<th>process wanted at NLO ((V \in {Z, W, \gamma}))</th>
<th>background to</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. (pp \rightarrow VV + \text{jet})</td>
<td>(ttH), new physics</td>
</tr>
<tr>
<td>2. (pp \rightarrow H + 2\text{ jets})</td>
<td>(H) production by</td>
</tr>
<tr>
<td>3. (pp \rightarrow t\bar{t}b\bar{b})</td>
<td>vector boson fusion (VBF)</td>
</tr>
<tr>
<td>4. (pp \rightarrow t\bar{t} + 2\text{ jets})</td>
<td>(ttH)</td>
</tr>
<tr>
<td>5. (pp \rightarrow VVb\bar{b})</td>
<td>(ttH)</td>
</tr>
<tr>
<td>6. (pp \rightarrow VV + 2\text{ jets})</td>
<td>VBF (\rightarrow H \rightarrow VV), (ttH), new physics</td>
</tr>
<tr>
<td>7. (pp \rightarrow V + 3\text{ jets})</td>
<td>VBF (\rightarrow H \rightarrow VV)</td>
</tr>
<tr>
<td>8. (pp \rightarrow VVV)</td>
<td>new physics</td>
</tr>
<tr>
<td></td>
<td>SUSY trilepton</td>
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</tbody>
</table>
The Les Houches Wish List (2010)

<table>
<thead>
<tr>
<th>process wanted at NLO</th>
<th>background to</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $pp \rightarrow VV + \text{jet}$</td>
<td>$t\bar{t}H$, new physics</td>
</tr>
<tr>
<td></td>
<td>Dittmaier, Kallweit, Uwer; Campbell, Ellis, Zanderighi</td>
</tr>
<tr>
<td>2. $pp \rightarrow H + 2 \text{jets}$</td>
<td>$H$ in VBF</td>
</tr>
<tr>
<td></td>
<td>Campbell, Ellis, Zanderighi; Ciccolini, Denner Dittmaier</td>
</tr>
<tr>
<td>3. $pp \rightarrow t\bar{t}b\bar{b}$</td>
<td>$t\bar{t}H$</td>
</tr>
<tr>
<td></td>
<td>Bredenstein, Denner Dittmaier, Pozzorini; Bevilacqua, Czakon, Papadopoulos, Pittau, Worek</td>
</tr>
<tr>
<td>4. $pp \rightarrow t\bar{t} + 2 \text{jets}$</td>
<td>$t\bar{t}H$</td>
</tr>
<tr>
<td></td>
<td>Bevilacqua, Czakon, Papadopoulos, Worek</td>
</tr>
<tr>
<td>5. $pp \rightarrow VVb\bar{b}$</td>
<td>VBF $\rightarrow H \rightarrow VV, t\bar{t}H$, new physics</td>
</tr>
<tr>
<td>6. $pp \rightarrow VV + 2 \text{jets}$</td>
<td></td>
</tr>
<tr>
<td>7. $pp \rightarrow V + 3 \text{jets}$</td>
<td>new physics</td>
</tr>
<tr>
<td>8. $pp \rightarrow VVV$</td>
<td></td>
</tr>
<tr>
<td>9. $pp \rightarrow b\bar{b}b\bar{b}$</td>
<td>Higgs, new physics</td>
</tr>
</tbody>
</table>

2005 list basically done. Amusingly $W,Z + 4 \text{jets}$ was not on this list.

Feynman diagram methods

now joined by

unitarity based methods

Berger, Melia, Melnikov, Rontsch, Zanderighi

VBF: Bozzi, Jäger, Oleari, Zeppenfeld

Mela, Melnikov, Rontsch, Zanderighi

VBF: Bozzi, Jäger, Oleari, Zeppenfeld

Berger, Bern, Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower, Maitre; Ellis, Melnikov, Zanderighi

SUSY trilepton

Lazopoulos, Melnikov, Petriello; Hankele, Zeppenfeld; Binoth, Ossola, Papadopoulos, Pittau

Higgs, new physics

GOLEM
Example of loop difficulty

Consider a tensor integral:

\[
\int \frac{d^{4-2\epsilon} \ell}{(2\pi)^{4-\epsilon}} \frac{\ell^\mu \ell^\nu \ell^\rho \ell^\lambda}{\ell^2 (\ell - k_1)^2 (\ell - k_1 - k_2)^2 (\ell + k_4)^2}
\]

Note: this is trivial on modern computer. Non-trivial for larger numbers of external particles.

Evaluate this integral via Passarino-Veltman reduction. Result is …
Result of performing the integration

Calculations explode for larger numbers of particles or loops. Clearly, there should be a better way!
Why are Feynman diagrams clumsy for high-loop or multiplicity processes?

• Vertices and propagators involve gauge-dependent off-shell states. Origin of the complexity.

\[ p^2 \neq 0 \]

• To get at root cause of the trouble we must rewrite perturbative quantum field theory.

\[ p^2 \neq m^2 \]

• All steps should be in terms of gauge invariant on-shell states. \[ p^2 = m^2 \] On shell formalism.

• Radical rewrite of gauge theory needed.
On-shell Methods

Key idea: Rewrite quantum field theory so only gauge invariant on-shell quantities appear in intermediate steps.

Loops amplitudes constructed from tree amplitudes

Generalized unitarity as a practical tool

Unitarity method
Bern, Dixon, Dunbar and Kosower (BDDK)

Rules for assembling $n$-point amplitudes from tree amplitudes

many new advances
Bern, Dixon and Kosower
Britto, Cachazo and Feng,
Ossola, Papadopoulos, Pittau;
Giele, Melniokov, Kunszt, Forde; Badger

Use of complex momenta
Britto, Cachazo and Feng,
On-Shell Recursion

A very general machinery for constructing tree-level scattering amplitudes are on-shell recursion relations.

Building blocks are on-shell amplitudes
General replacement for tree-level Feynman diagrams

Contrast with Feynman diagram which are based on off-shell unphysical states with \( p^2 \neq m^2 \)

Proof relies on so little. Power comes from generality

• Cauchy’s theorem
• Basic field theory factorization properties
• Applies as well to massive theories.
• Applies as well to gravity theories.
On-Shell Recursion for Tree Amplitudes

Britto, Cachazo, Feng and Witten

Consider amplitude under complex shifts of the momenta

\[ p_1^\mu(z) = p_1^\mu - zq^\mu \quad p_n^\mu(z) = p_n^\mu + zq^\mu \quad q^2 = 0, \quad p \cdot q = 0 \]

\[ (p_i^\mu(z))^2 = 0 \quad \text{complex momenta} \]

If \( A(z) \rightarrow 0, \quad z \rightarrow \infty \)

\[
\oint_{C_\infty} \frac{A(z)}{z} \, dz = 0 \quad \Rightarrow \quad A(z = 0) = - \sum_{\alpha} \text{Res}_\alpha \frac{A(z)}{z}
\]

\[
A(z) = \sum_{\alpha} \frac{c_\alpha}{z - z_\alpha}
\]

\( A(z) \) is amplitude with shifted momenta

\[
\text{on-shell amplitude}
\]

Sum over residues
gives the on-shell recursion relation

Poles in \( z \) come from kinematic poles in amplitude.
Recent Applications of Unitarity Method

On-shell methods applied in a variety of problems:

- **$N = 4$ super-Yang-Mills ansatz for planar 4,5 point amplitudes to all loop orders.** Non-trivial place to study AdS/CFT duality.
  
  Anastasiou, ZB, Dixon, Kosower;
  ZB, Dixon, Smirnov; Alday and Maldacena
  Drummond, Henn, Korchemsky, Sokatchev
  Brandhuber, Heslop, Travaglini; Arkani-Hamed, Cachazo, etc.

- **Applications to gravity.**

  Direct challenge to accepted wisdom on impossibility of constructing point-like UV finite theories of quantum gravity.
  
  ZB, Bjerrum-Bohr and Dunbar;
  Bjerrum-Bohr, Dunbar, Ita, Perkins, Risager;
  ZB, Dixon and Roiban;
  ZB, Carrasco, Dixon, Johanson, Kosower, Roiban; etc.

- **NLO computations for LHC physics.**

  Anastasiou, Badger, Bedford, Berger, ZB, Bernicot, Brandhuber, Britto, Buchbinder, Cachazo, Del Duca, Dixon, Dunbar, Ellis, Feng, Febres Cordero, Forde, Giele, Glover, Guillet, Ita, Kilgore, Kosower, Kunszt; Lazopolous, Mastroia; Maitre, Melnikov, Spence, Travaglini; Ossola, Papadopoulos, Pittau, Risager, Yang; Zanderighi, etc.
Any one-loop amplitude can be expressed in terms of basis of scalar integrals:

\[ A_n = \sum_i d_i I_4^i + \sum_i c_i I_3^i + \sum_i b_i I_2^i + \sum_i a_i I_1^i + \text{rational} \]

- **Known basis of scalar integrals.** 't Hooft, Veltman; van Oldenborgh, Vermaseren; Beenakker, Denner; Denner, Nierste, Scharf; ZB, Dixon, Kosower; etc

- **Problem of computing one-loop amplitudes is “just” to compute rational coefficients of integrals.**

On-shell formalism reduces the problem to tree-like calculations.
Some One-loop On-Shell Developments

• Generalized unitarity – used to produce $pp \rightarrow W, Z + 2$ partons
  Used in MCFM


• Realization of the remarkable power of complex momenta in generalized cuts. Inspiration from Witten’s twistor string paper.
  Britto, Cachazo, Feng (2004); Britto et al series of papers.

• $D$ dimensional unitarity to capture rational pieces of loops.
  ZB, Morgan (1995); ZB, Dixon, Dunbar, Kosower (1996), ZB, Dixon, Kosower (2000);
  Anastasiou, Britto, Feng, Kunszt, Mastroia (2006); Giele, Kunszt, Melnikov (2008); Badger (2009)

• On-shell recursion for loops (based on BCFW)
  Berger, ZB, Dixon, Forde, Kosower; + Febres Cordero, Ita, Maitre

• Efficient on-shell reduction of integrals, in a way designed for numerical integration (OPP).
  Ossola, Papadopoulos, Pittau (OPP) (2006); Giele, Kunszt, Melnikov (2008);

• Efficient on-shell integration using analytic properties consistent with numerical approaches.
  Forde (2007); Berger et al [BlackHat]
Quadruple Cut Freezes Box Integral

Britto, Cachazo, Feng

Solve on-shell conditions $l_i^2 = m_i^2$ and plug 2 solutions into product of tree amplitudes. Gives coefficient.

Momента complex

Box integral coefficient

$$d_i = \frac{1}{2} \sum_{\sigma = \pm} A_{\text{tree}}^{(1)} A_{\text{tree}}^{(2)} A_{\text{tree}}^{(3)} A_{\text{tree}}^{(4)} | l_i = l_i^{(\sigma)}$$

If all particles massless and $K_1$ also massless, very simple solution:

Berger, ZB, Dixon, Febres Cordero, Forde, Ita, Kosower, Maitre; Risager

Simplicity helps with numerical stability

Very neat!

$$\begin{align*}
(l_1^{(\pm)})^\mu &= \frac{\langle 1^\mp | K_2 K_3 K_4 \gamma^\mu | 1^\pm \rangle}{2 \langle 1^\mp | K_2 K_4 | 1^\pm \rangle}, \\
(l_2^{(\pm)})^\mu &= -\frac{\langle 1^\mp | \gamma^\mu K_2 K_3 K_4 | 1^\pm \rangle}{2 \langle 1^\mp | K_2 K_4 | 1^\pm \rangle}, \\
(l_3^{(\pm)})^\mu &= \frac{\langle 1^\mp | K_2 \gamma^\mu K_3 K_4 | 1^\pm \rangle}{2 \langle 1^\mp | K_2 K_4 | 1^\pm \rangle}, \\
(l_4^{(\pm)})^\mu &= -\frac{\langle 1^\mp | K_2 K_3 \gamma^\mu K_4 | 1^\pm \rangle}{2 \langle 1^\mp | K_2 K_4 | 1^\pm \rangle}
\end{align*}$$
Subtracting box contributions from triple cut cleans complex plane. Triangle coefficients extracted from discrete Fourier transform.

Bubble and tadpole coefficient can also be solved along these lines.
Rational Terms

Two basic approaches:

1) $D$-dimensional unitarity in the cuts
   — gets rational terms which would be dropped if
     $D = 4$ momenta used.
     van Neerven; ZB and Morgan; Anastasiou, Britto, Feng, Kunst, Mastrolia; Giele, Kunszt, Melnikov; Badger, Berger et al (BlackHat)

2) On-shell recursion
   — based on BCFW tree level recursion
     Berger, ZB, Dixon, Forde, Kosower

BlackHat uses both approaches.
Stability and Scaling with Number of Legs

Extremely mild scaling with number of legs

Berger, ZB, Dixon, Febres Cordero, Forde, Ita, Kosower, Maitre

2.33 GHz Xeon

Relative precision = $\log_{10}\left(\frac{|A^{\text{num}} - A^{\text{ref}}|}{|A^{\text{ref}}|}\right)$

Amusing count for 8 gluons

+ 3,017,489 Feynman diagrams
The NLO revolution

G. Salam, ICHEP 2010

2009: NLO $W+3j$ [Rocket: Ellis, Melnikov & Zanderighi]
2009: NLO $W+3j$ [BlackHat: Berger et al]
2009: NLO $t\bar{t}b\bar{b}$ [Bredenstein et al]
2009: NLO $t\bar{t}b\bar{b}$ [HELAC-NLO: Bevilacqua et al]
2009: NLO $q\bar{q} \rightarrow b\bar{b}b\bar{b}$ [Golem: Binoth et al]
2010: NLO $t\bar{t}jj$ [HELAC-NLO: Bevilacqua et al]
2010: NLO $Z+3j$ [BlackHat: Berger et al]

[unitarity]
[unitarity]
[traditional]
[unitarity]
[traditional]
The NLO revolution

BlackHat: C++ implementation of on-shell methods for one-loop amplitudes

Berger, ZB, Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower, Maitre

BlackHat is a C++ package for numerically computing one-loop matrix elements with 6 or more external particles.
- Input is on-shell tree-level amplitudes.
- Output is numerical on-shell one-loop amplitudes.

On-shell methods used to achieve the speed and stability required for LHC phenomenology at NLO.

Other (semi) on-shell packages under construction

— Helac-NLO: Bevilacqua, Czakon, Ossola, Papadopoulos, Pittau, Worek
— Rocket: Ellis, Giele, Kunszt, Melnikov, Zanderighi
— SAMURAI: Mastrolia, Ossola, Reiter, Tramontano
— MadLoop: Hirchi, Maltoni, Frixione, Frederix, Garzelli, Pittau
Berger, ZB, Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower, Maitre
New Members (not shown): Hoeche, Diana and Ozeren
Some differences in BlackHat

BlackHat has some differences with other programs

• We use helicity for tree amplitude input.
• We use BCFW recursion to generate tree amplitudes, many cases compact analytic expressions.
• We use primitive amplitudes. These are color-stripped building blocks.
• We use Sherpa to deal with real emission and phase-space integration.
• Our stability safety system recomputes only small pieces of the amplitude if an instability is detected.

Dixon, Henn, Plefka, & Schuster
ZB, Dixon and Kosower
BlackHat + Sherpa


\[ \sigma^{NLO}_n = \int_n \sigma_{tree}^n + \int_n (\sigma_{virt}^n + \Sigma_{sub}^n) + \int_{n+1} (\sigma_{real}^{n+1} - \sigma_{sub}^{n+1}) \]

Sherpa integrates phase space.
Uses Catani-Seymour dipole formalism for IR singularities, automated in Amegic package.

Gleisberg and Krauss
New W,Z + 3,4-Jet Predictions for LHC

BlackHat Collaboration
First NLO calculations of $W, Z + 4$ jets

Berger, ZB, Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower, Maitre (BlackHat collaboration)

$W + 4$ jets $H_T$ distribution

BlackHat + Sherpa

NLO QCD provides the best available theoretical predictions.

- On-shell methods really work!
- 2 legs beyond Feynman diagrams for this type of process.

Uses leading color approx good to ~ 3 percent
First Useful NLO $W+3$ Jets Prediction

Berger, ZB, Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower, Maitre (BlackHat collaboration)


- Excellent agreement between NLO theory and experiment.
- Best available predictions
- Beyond what has been done via Feynman diagrams.

Subsequent results from Ellis, Melnikov and Zanderighi

Methods validated on Tevatron data. Apply to LHC.
Comparison to LHC Data

- Fresh from ATLAS at the EPS conference.
- $3^{\text{rd}}$ jet $p_T$ in $W+$jets [ATLAS-CONF-2011-060].
- Small scale variation at NLO, good agreement with data.
- Much more to come including four jets!

Ntuples give experiments the ability to use BlackHat results without needing to master the program.
Shape Changes in $W+4$ jets

Some distributions can have sizable shape changes between LO and NLO
Renormalization Scale Dependence

Renormalization and factorization scale dependence gets stronger as number of legs increases, but NLO tames it.
Big improvement in scale stability
Numerical reliability
Fourth jet $p_T$ has little LO to NLO change in shape...
...but for leading three jet $p_T$s, shape changes

Ita, ZB, Febres Cordero, Dixon, Kosower, Maitre (2011)
Importance of Sensible Scale Choices

For Tevatron $\mu = E_T^W$ is a common renormalization scale choice. For LHC this is a poor choice. Does not set the correct scale for the jets.

- LO/NLO ratio goes haywire.
- NLO scale dependence is large at high ET.
- NLO cross-section becomes negative!

Energy of $W$ boson does not represent typical jet energy.
Better Scale Choices

What is happening? Consider two configurations

- If (a) dominates, \( \mu = E_T^W \equiv \sqrt{M_W^2 + p_T^2(W)} \) is a fine choice.
- But if (b) dominates then \( E_T^W \) is too low a scale.
- Looking at large \( E_T \) of 2nd jets forces (b) to dominate.
- The total (partonic) transverse energy is a better variable; gets large properly for both (a) and (b).
- Other reasonable scales are possible.

\[ \hat{H}_T = \sum_p E_T^p + E_T^e + E_T^{\nu} \]

Bauer and Lange; Melnikov and Zanderighi
Importance of Sensible Scale Choices

2nd jet $E_T$ in $W^- + 3$ jet production

$W^- + 3$ jets + X

$\sqrt{s} = 14$ TeV

$\mu = \sqrt{\hat{H}_T}$

- LO/NLO ratio sensible.
- NLO scale dependence very good.
- NLO cross sections positive.

Scale choice $\mu = E_T^{W}$ can cause trouble
**New $W$ Polarization Effect**

- **Both** $W^-$ and $W^+$ dominantly left-handed at high $p_{T,W}$
- Stable under QCD-corrections.
- Similar for $W$+1,2,3 jets.
- Not to be confused with longitudinal polarization effect.
Polarization Effects of $W$’s

\[
\frac{1}{\sigma} \frac{d\sigma}{d \cos \theta^*} = \frac{3}{8} (1 \mp \cos \theta^*)^2 f_L + \frac{3}{8} (1 \pm \cos \theta^*)^2 f_R + \frac{3}{4} \sin^2 \theta^* f_0
\]

Effect is non-trivial, depending on an unobvious property of the matrix elements.
Up to 80 percent left-handed polarization.

100% left handed
mostly right handed but 1/4 the weight.

left-handed gluon

right-handed gluon
Polarization Effects of $W$’s

The shapes are due to a preference for both $W$ bosons to be left handed at high transverse energies.

$W$ polarization can be used to separate out $W$’s from top (or perhaps new physics)! Under study by CMS.
**Measurement by CMS**

<table>
<thead>
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<th></th>
<th>CMS</th>
<th>NLO</th>
<th>ME+PS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W^+ (f_L - f_R)$</td>
<td>$0.300 \pm 0.031 \pm 0.034$</td>
<td>$0.308$</td>
<td>$0.283$</td>
</tr>
<tr>
<td>$W^- (f_L - f_R)$</td>
<td>$0.226 \pm 0.031 \pm 0.050$</td>
<td>$0.248$</td>
<td>$0.222$</td>
</tr>
<tr>
<td>$W^+ f_0$</td>
<td>$0.192 \pm 0.075 \pm 0.089$</td>
<td>$0.200$</td>
<td>$0.187$</td>
</tr>
<tr>
<td>$W^- f_0$</td>
<td>$0.162 \pm 0.078 \pm 0.136$</td>
<td>$0.193$</td>
<td>$0.179$</td>
</tr>
</tbody>
</table>

Recent CMS measurement confirms predictions!

$W$ polarization may be usable to separate out prompt $W$’s from ones from top (or perhaps new physics)
Jet production ratios in $Z + n$ jets

Ellis, Kleiss, Stirling; Berends, Giele, Kuijf, Klaiss, Stirling; Berends, Giele, Kuijf, Tausk

Also called ‘Berends’ or ‘staircase’ ratio.

<table>
<thead>
<tr>
<th>jet ratio</th>
<th>CDF</th>
<th>LO</th>
<th>NLO</th>
</tr>
</thead>
<tbody>
<tr>
<td>2/1</td>
<td>0.099 ± 0.012</td>
<td>0.093+0.015−0.012</td>
<td>0.093+0.004−0.006</td>
</tr>
<tr>
<td>3/2</td>
<td>0.086 ± 0.021</td>
<td>0.057+0.008−0.006</td>
<td>0.065+0.008−0.007</td>
</tr>
<tr>
<td>4/3</td>
<td>—</td>
<td>0.040+0.005−0.004</td>
<td>—</td>
</tr>
</tbody>
</table>

- Ratios should mitigate dependence on e.g.: jet energy scales, pdfs, nonperturbative effects, etc.
- Strong dependence on kinematics and cuts.
- Note: Lore that $n/(n+1)$ jet ratio independent of $n$ is too simplistic, depends strongly on cuts. Berger et al (BlackHat)

Z+1, 2, 3 jets with CDF setup

Differential ratios in $p_{T,Z}$
Data Driven Background Estimation

CMS uses photons to estimate $Z$ background to susy searches.

$\sigma(pp \rightarrow Z(\rightarrow \nu \bar{\nu}) + \text{jets}) = \sigma(pp \rightarrow \gamma + \text{jets}) \times R_{Z/\gamma}$

irreducible background

measure this

theory input

Has better statistics than $Z \rightarrow \mu \bar{\mu}$

Our task was to theoretically understand conversion and give theoretical uncertainty to CMS.

See also recent paper from Stirling et al.
CMS Setup

Set 1: \( H_T^{\text{jet}} > 300 \text{ GeV}, |\text{MET}| > 250 \text{ GeV} \)

Set 2: \( H_T^{\text{jet}} > 500 \text{ GeV}, |\text{MET}| > 150 \text{ GeV} \)

Set 3: \( H_T^{\text{jet}} > 300 \text{ GeV}, |\text{MET}| > 150 \text{ GeV} \)

\[
H_T = \sum_j E_T^j \\
\text{MET} = -\sum_j p_j
\]

\( \Delta(\phi)(\text{MET, jet}) > 0.5 \) to suppress QCD multijet background

Used Frixione photon isolation
\[
\sum_i E_{iT} \Theta(\delta - R_{i\gamma}) \leq \mathcal{H}(\delta) \quad \delta < \delta_0
\]

\( \epsilon = 0.025, \delta_0 = 0.3 \) and \( n = 2 \)

\[
\mathcal{H}(\delta) = E_T^\gamma \epsilon \left( \frac{1 - \cos \delta}{1 - \cos \delta_0} \right)^n
\]

Technical Aside: Experiments use cone photon isolation. Confirmed via JetPhox (Binoth et al) and Vogelsang’s code, that difference very small with this setup.
Differences between ME+PS and NLO small in the ratio.

## Data Driven Background Estimation

Set 1

<table>
<thead>
<tr>
<th>process</th>
<th>LO</th>
<th>ME+PS</th>
<th>NLO</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z + 2j$</td>
<td>$0.521(0.001)^{+0.180}_{-0.125}$</td>
<td>$0.416(0.004)$</td>
<td>$0.560(0.002)^{+0.012}_{-0.042}$</td>
</tr>
<tr>
<td>$\gamma + 2j$</td>
<td>$2.087(0.005)^{+0.716}_{-0.494}$</td>
<td>$1.943(0.027)$</td>
<td>$2.448(0.008)^{+0.142}_{-0.225}$</td>
</tr>
<tr>
<td>$Z/\gamma$ ratio</td>
<td>$0.250$</td>
<td>$0.214$</td>
<td>$0.229$</td>
</tr>
</tbody>
</table>
Different theoretical predictions track each other. This conversion directly used by CMS in their estimate of theory uncertainty.
Longer Term Prospects

• More automation needed to allow any process. BlackHat is investing into this, as are other groups.

• Upcoming Gold Standard: NLO + parton showering (+ non-perturbative)

Multiple groups working on this: MC@NLO, POWHEG, SHERPA, VINCHIA, GenEvA

WW+ dijets is current state-of-the art example but expect larger numbers of jets in the coming years. NLO programs can provide the needed virtual and real emission contributions.

See Zuberi’s talk

Frixione and Webber; Alioli, Nason, Oleari, Re; Hoche, Krauss, Shonherr, Siegert; Giele, Kosower, and Skands; Bauer, Tackman, Thaler et al, Melia, Nason, Rontch, Zanderighi, etc.
Summary

• The on-shell formulation of quantum field theory leads to powerful new ways to compute important quantities of experimental interest at the LHC.

• Huge advance in NLO QCD. For multijet processes these are currently the best available theoretical predictions.

• Many new processes $t\bar{t}b\bar{b}$, $t\bar{t}jj$, $W,Z + 3$ jets, $W,Z + 4$ jets, etc.

• Discovery of new SM $W$ polarization effect. Separate out $W$’s from top decay or perhaps new physics. Under study by CMS.

• Theory can help with data driven determinations of backgrounds, by providing conversions and uncertainties.

• In the near future expect many more processes brought under NLO QCD control.
Extra Transparancies
• **W,Z+4 jets done.**

• **Processes with heavy quarks:**  $ttbb, tt+2$-jets recently completed by various groups. In the future years you can expect to see NLO $tt + 3,4$ jets.

• **ntuples will allow experiments to make their own NLO studies.**

• **Public release of code.** (Non-trivial task for state of the art NLO)
What’s New in BLACKHAT

• Automation of subprocess assembly
  – Primitive → partial (color-ordered) & partial → complete amplitude
  – Crucial to obtaining recent physics results

• BlackHat-supplied trees
  – Compact analytic expressions
  – First use of $N = 4$ derived expressions (Dixon, Henn, Plefka, & Schuster)
  – Important to obtaining recent new physics results

• Six-quark processes

• Improved assessment of numerical stability of rational terms
  – Less recomputation
  – Sometimes it’s the Born that needs higher precision!

• ROOT $n$-tuples actively generated & used by experimenters
  – Efficient computation of scale bands & PDF uncertainties