Exploring the isovector equation of state at high densities with HIC

Vaia D. Prassa

Aristotle University of Thessaloniki
Department of Physics
Introduction

- Nuclear Equation of state
- Symmetry Energy
- Phase diagram
- Spacetime evolution

Quantum Hadrodynamics (QHD)

- Meson exchange model
- Asymmetric Nuclear Equation of State

Transport theory

- Vlasov term
- Collision term

Particle Production

- Cross sections
- Kaon-nucleon potential

Summary & Outlook

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Exploring the isovector equation of state at high densities with HIC
First questions to be answered:

Why is the Symmetry Energy so important?

- **At low densities**: Nuclear structure (neutron skins, pigmy resonances), Nuclear Reactions (neutron distillation in fragmentation, charge equilibration), and Astrophysics, (neutron star formation, and crust),

- **At high densities**: Relativistic Heavy ion collisions (isospin flows, particle production), Compact stars (neutron star structure), and for fundamental properties of strong interacting systems (transition to new phases of the matter).

Why HIC?

- Probing the in-medium nuclear interaction in regions away from saturation.
- Reaction observables sensitive to the symmetry term of the nuclear equation of state.
- High density symmetry term probed from isospin effects on heavy ion reactions at relativistic energies.

In this talk:

- HIC at intermediate energies 400AMeV-2AGeV.
- Investigation of particle ratios: Probes for the symmetry energy density dependence. In-medium effects in inelastic cross sections & Kaon potential choices.
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Equation of State (EOS)

The nuclear matter thermodynamical properties are described by an equation of state.

The nuclear EOS gives the binding energy $E/A$ or the pressure $P$ of the system per nucleon, as a function of the baryon density or the temperature.

$$E(\rho, T) = E_{th}(\rho, T) + E_c(\rho, T = 0) + E_0$$

$E_{th}(\rho, T)$: thermic energy, consists of a kinetic and a dynamic term.

$E_c(\rho, T = 0)$: compression energy at $T = 0$.

$E_0$: binding energy at $T = 0$ and $\rho_0$.

Incompressibility: $K = 9\rho_0^2 \left( \frac{\partial^2 E}{\partial \rho_B^2} \right)_{\rho_B = \rho_0}$
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At $\rho_{sat}$: well defined by studies on stable nuclei.

High $\rho_B$: is predicted by the theoretical models and is adjusted to fit the HIC data.

Constraints from HIC: **Multiplicities and Flows of n and p**.
Symmetry Energy

The EOS depends on the asymmetry parameter $\alpha = \frac{N-Z}{N+Z}$:

$$E(\rho_B, \alpha) \equiv \frac{\epsilon(\rho_B, \alpha)}{\rho_B} = E(\rho_B, 0) + E_{\text{sym}}(\rho_B)\alpha^2$$

Thus, it gives a definition of the symmetry energy:

$$E_{\text{sym}} \equiv \frac{1}{2} \left. \frac{\partial^2 E(\rho_B, \alpha)}{\partial \alpha^2} \right|_{\alpha=0} = \frac{1}{2} \rho_B \left. \frac{\partial^2 \epsilon}{\partial \rho_B^2} \right|_{\rho_B^3=0}$$

$$K_{\text{sym}} = 9\rho_0^2 \left. \frac{\partial^2 E_{\text{sym}}}{\partial \rho_B^2} \right|_{\rho_B=\rho_0}$$

The symmetry energy describes the difference between the binding energy of the symmetric matter and that of the pure neutron matter.
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Different behavior even at saturation point.

**High $\rho_B$, discrepancies between the models increases.**

**Constraints from HIC:**

- **Multiplicities and Differential flows of n and p.**
- **Pion flows** and **Isospin ratios** $\pi^-/\pi^+, K^0/K^+$. 

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Exploring the isovector equation of state at high densities with HIC
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Liquid phase: $T \approx 10\text{MeV}$.
Gas phase: RHIC
$T \approx 10 - 100\,\text{MeV}$.

Liquid phase:
$T \approx 10\,\text{MeV}$.
- **Quark-gluon plasma**: Ultra RHIC  
  \[ T > 100 \text{MeV} \]

- **Gas phase**: RHIC  
  \[ T \approx 10 - 100 \text{MeV} \]

- **Liquid phase**:  
  \[ T \approx 10 \text{MeV} \]
Initial stage. Nuclei at ground state. \( P = 0, \ T = 0, \ \rho = \rho_0.\)
Compression. Sequential N-N binary collisions. Incoming matter of the target & the projectile is mixed & compressed forming a short-lived stage of nuclear matter of high $\rho_B$ that depends on the EOS.

$E = 1\text{AGeV}, \ T \approx 30 - 60\text{MeV}, \ \rho_B \approx 2 - 3\rho_0$.

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**Expansion.** Expansion of the density energy. $T, \rho \downarrow$. Hot matter interacts with the cold matter of the spectator. $E_{\text{beam}} > 10 A GeV \Rightarrow QGP_{\text{limit}}$, $P$ lower than $P_{\text{hadronic phase}}$.

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**Initial stage.** Nuclei at ground state. $P = 0$, $T = 0$, $\rho = \rho_0$. 

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**Nuclear Equation of state**

**Symmetry Energy**

**Phase diagram**

**Spacetime evolution**
Freeze-out. Low $\rho_B \Rightarrow$ no further interaction.

Expansion. Expansion of the density energy. $T, \rho \downarrow$. Hot matter interacts with the cold matter of the spectator. $E_{\text{beam}} > 10\text{AGeV} \Rightarrow \text{QGP limit}$, $P$ lower than $P_{\text{hadronic phase}}$.

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Exploring the isovector equation of state at high densities with HIC
The NN Interaction is described by the exchange of **mesons**.

**SCALAR** (attraction): $\sigma, \delta$ (isospin dependence).

**VECTOR** (repulsion): $\omega, \rho$ (isospin dependence).
The NN Interaction is described by the exchange of mesons. 
**SCALAR** (attraction): $\sigma$, $\delta$ (isospin dependence). 
**VECTOR** (repulsion): $\omega$, $\rho$ (isospin dependence).

### Lagrangian density

\[
\mathcal{L} = \bar{\psi} (i \gamma^\mu \partial_\mu - m) \psi + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - U(\sigma) + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4} \Omega_{\mu \nu} \Omega^{\mu \nu} \\
+ \frac{1}{2} m_\rho^2 \vec{\rho}_\mu \cdot \vec{\rho}^\mu - \frac{1}{4} \vec{R}_{\mu \nu} \vec{R}^{\mu \nu} + \frac{1}{2} \left( \partial_\mu \vec{\delta} \cdot \partial^\mu \vec{\delta} - m_\delta^2 \vec{\delta}^2 \right) \\
+ \bar{\psi} \left( -g_\omega \omega_\mu - g_\rho \gamma^\tau \cdot \vec{\rho}_\mu + g_\sigma \sigma + g_\delta \vec{\tau} \cdot \vec{\delta} \right) \psi
\]

Lagrangian density of: the **free nucleons**, the $\sigma$ **meson** with $U(\sigma) = \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{3} g_2 \sigma^3 + \frac{1}{4} g_3 \sigma^4$, the $\omega$ **meson** with the field tensor $\Omega_{\mu \nu} \equiv \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$, the $\rho$ **meson** with the field tensor $\vec{R}_{\mu \nu} \equiv \partial_\mu \vec{\rho}_\nu - \partial_\nu \vec{\rho}_\mu$, the $\delta$ **meson** and of their interaction.
The nuclear EOS for asymmetric nuclear matter in the QHD picture:

$$\mathcal{E} = \sum_{i=n,p} 2 \int \frac{d^3 k}{(2\pi)^3} E_i^*(k) + U(\Phi) + \frac{1}{2} f_V \rho_B^2 + \frac{1}{2} f_\rho \rho B_3 + \frac{1}{2} f_\delta \rho S_3$$

The nuclear Symmetry energy in the QHD picture:

$$E_{sym} = \frac{1}{6} \frac{k_F^2}{E_F^*} + \frac{1}{2} \left[ f_\rho - f_\delta \left( \frac{m^*}{E_F^*} \right)^2 \right] \rho_B$$

![Graphs showing the nuclear symmetry energy and implications for high densities with HIC.](image)
Relativistic transport equation

Relativistic Boltzmann-Uehling-Uhlenbeck (RBUU)

\[
\begin{align*}
[p^*_{\mu i} & \partial^\mu_{\chi} + (g_\omega p^*_{\nu i} F^\mu_{\nu i} + m^*_i (\partial^\mu_{\chi} m^*_i)) \partial^\mu_{p^*_i}] f_i(x, p^*) = \\
g \frac{1}{(2\pi)^3} \int \frac{d^3 p^*_2}{p^*_2} \frac{d^3 p^*_3}{p^*_3} \frac{d^3 p^*_4}{p^*_4} W(p^*, p^*_2, p^*_3, p^*_4) \\
\{ f_3 f_4 [1 - f][1 - f_2] - ff_2 [1 - f_3][1 - f_4] \}
\end{align*}
\]

- **Vlasov term.** Temporal evolution of the system, which is described by the phase-space distribution function \( f(x, p^*) \), under the influence of a mean field \( (m^*_i, p^*_i) \).

- **Collision term.** Transition rate \( W \), is expressed by the differential cross section \( \frac{d\sigma}{d\Omega(s,\Theta)} \),

\[
W(p^*, p^*_2, p^*_3, p^*_4) = (p^* + p^*_2)^2 \frac{d\sigma}{d\Omega} \delta^4(p^* + p^*_2 - p^*_3 - p^*_4)
\]

where \( \Theta \) is the scattering angle in the cms frame and \( s \) the square of the total energy, \( s = (p^* + p^*_2)^2 \).
Test particle method

Representation of the phase-space distribution function by a number of test particles.

**Gaussian test particles**

\[ g(p^* - p_i^*(\tau)) = \alpha_p e^{(p^* - p_i^*(\tau))^2/\sigma_p^2} \delta[p^*_\mu p^{*\mu}_i(\tau) - m_i^{*2}] \]

**Distribution function**

\[ f(x, p^*) = \frac{1}{N(\pi \sigma_p)} \sum_{i=1}^{A \cdot N} \int_{-\infty}^{+\infty} d\tau \ e^{R_{i\mu}(x)R^\mu_i(x)/\sigma^2} e^{(p^* - p_i^*(\tau))^2/\sigma_p^2} \]
\[ \times \delta[(x_\mu - x_{i\mu}(\tau))u^\mu_i(\tau)] \delta[p^*_\mu p^{*\mu}_i(\tau) - m_i^{*2}] \]

**Test particles equations of motions**

\[ \frac{d}{d\tau} x^\mu_i = \frac{p^*_i(\tau)}{m^{*}_i(x_i)} ; \]
\[ \frac{d}{d\tau} p^{*\mu}_i = \frac{p^{*\nu}_i(\tau)}{m^{*}_i(x_i)} F^\mu\nu_i(x_i(\tau)) + \partial^\mu m^{*}_i(x_i) \]
Collision term

\[ I_c = \frac{g}{(2\pi)^3} \int \frac{d^3 p_2^*}{p_2^* 0} \frac{d^3 p_3^*}{p_3^* 0} \frac{d^3 p_4^*}{p_4^* 0} W(p^*, p_2^*, p_3^*, p_4^*) \]
\[ \{ f(x, p_1^*) f(x, p_4^*) [1 - f(x, p_3^*)] [1 - f(x, p_2^*)] - f(x, p^*) f(x, p_2^*) [1 - f(x, p_3^*)] [1 - f(x, p_4^*)] \} \]

- Factors \((1 - f_i), (f_i = f(x, p_i^*))\), Pauli principle.
- Transition rate:
  \[ W = (2\pi)^4 \delta^4 (k + k_2 - k_3 - k_4) (m^*)^4 |T|^2. \]
- Two particles collide if:
  \[ d < d_0 = \sqrt{\frac{\sigma_{\text{tot}}}{\pi}}. \]

Elastic channels

1. \(NN \iff NN\)
2. \(N\Delta \iff N\Delta\)
3. \(\Delta\Delta \iff \Delta\Delta\)

Inelastic channels

<table>
<thead>
<tr>
<th>Ingoing Channel</th>
<th>Outgoing Channel</th>
<th>Isospin coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>nn</td>
<td>(p\Delta^-)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>(n\Delta^0)</td>
<td>2/3</td>
</tr>
<tr>
<td>np</td>
<td>(p\Delta^0)</td>
<td>1/3</td>
</tr>
<tr>
<td></td>
<td>(n\Delta^+)</td>
<td>2/3</td>
</tr>
<tr>
<td>pp</td>
<td>(p\Delta^+)</td>
<td>1/3</td>
</tr>
<tr>
<td></td>
<td>(n\Delta^{++})</td>
<td>1</td>
</tr>
</tbody>
</table>

Decay width:

\[ \Gamma(q) = \tilde{\Gamma} \frac{q^3 R^2}{1 + q^2 R^2} z(q), \quad z(q) = \frac{q_r^2 + \delta^2}{q^2 + \delta^2} \]

Resonance decay probability \(P\),

\[ P = 1 - \exp \left[ -\frac{\Gamma(M)\Delta t}{\gamma \hbar c} \right] \]
Elastic Baryon-Baryon collisions

In-medium effects: Dirac-Brueckner.

Suppression of cross sections at $E_{\text{beam}} < 300\text{AMeV}$ and high $\rho_B$.

At high $E_{\text{lab}}$, the $\sigma_{\text{eff}}$ approaches asymptotically $\sigma_{\text{free}}$.

Cross sections
Kaon-nucleon potential

Inelastic Baryon-Baryon collisions

In-medium effects: DBHF.

\[ \sigma_{\text{eff}}^{\text{inel}} = f(\rho)\sigma_{\text{free}}^{\text{inel}}(E_{\text{lab}}) \]

\[ f(\rho) = 1 + a_0(\rho_B/\rho_0) + a_1(\rho_B/\rho_0)^2 + a_2(\rho_B/\rho_0)^3. \]

Inverse channel:

\[ \sigma_{N\Delta\rightarrow NN} = \frac{(2S_N+1)(2S_N+1)}{(2S_N+1)(2S_\Delta+1)} q_i^2 q_f^2 \sigma_{NN\rightarrow N\Delta}. \]
Cross sections
Kaon-nucleon potential

<table>
<thead>
<tr>
<th>State</th>
<th>$I_3$</th>
<th>Decay channel</th>
<th>Weights</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta^-$</td>
<td>$-3/2$</td>
<td>$n\pi^-$</td>
<td>1</td>
</tr>
<tr>
<td>$\Delta^0$</td>
<td>$-1/2$</td>
<td>$p\pi^-$</td>
<td>1/3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$n\pi^0$</td>
<td>2/3</td>
</tr>
<tr>
<td>$\Delta^+$</td>
<td>$+1/2$</td>
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</tr>
<tr>
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**Centrality dependence**

\( Au + Au \) collision at 1AGeV.
\( A_{part} \): number of participants in a collision.

**Overestimation of data.**
**Introduction**

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**Centrality dependence**

$Au + Au$ collision at 1AGeV.

$A_{part}$: number of participants in a collision.

**Overestimation of data.**

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**Transverse momentum spectra**

$Au + Au$ collision at 1AGeV, at mid-rapidity ($-0.2 < y^0 < 0.2$).

**Very good agreement with data.**
Pion rapidity distribution

$Au + Au$ collision at $E_{\text{beam}} = 1 \text{ AGeV}$, with $p_t > 0.1 \text{ GeV}/c$.

Mid rapidity region: good agreement.

Spectator region: overestimation.
Kaon production

1. $\pi B \rightarrow YK$
2. $BB \rightarrow BYK$

$Au + Au$ central collision at 1AGeV.
Kaon production

1. $\pi B \rightarrow YK$
2. $BB \rightarrow BYK$

$Au + Au$ central collision at 1AGeV.

Rapidity distribution of $K^+$

$Ni + Ni$ collision at 1.93AGeV.

$\sigma_{\text{eff}}$: reduction of $K^+ \Rightarrow$ towards a better agreement with data.
Yield ratios: **Determination** of the $E_{sym}$ behavior.

$\pi^-/\pi^+$: partially affected from the in-medium cross sections.

$K^0/K^+$: appears **robust** against the in-medium cross sections.

**Kaons equation of motion**

\[
[(\partial^\mu + iV_\mu)^2 + m_K^*]^2 \phi_K(x) = 0
\]

**Chiral perturbation theory potential**

\[
V_\mu = \frac{3}{8f_\pi^2} j_\mu
\]

\[
m_K^* = \sqrt{m_K^2 - \frac{\Sigma_{KN}}{f_\pi^2} \rho_s + V_\mu V^\mu}
\]

**One boson exchange model potential**

\[
V^\mu = \frac{1}{3} f_\omega^* j_\mu
\]

\[
m_K^* = \sqrt{m_K^2 + \frac{m_K}{3} g_{\sigma N} \rho_s}
\]

**Kaon in-medium energy**

\[
E_K(k) = k_0 = \sqrt{k^2 + m_K^* + V_0}
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**Kaons equation of motion**

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**Chiral perturbation theory potential**

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\]

\[
m_K^* = \sqrt{m_K^2 - \frac{\Sigma_{KN}}{f_\pi^2} \rho_s + V_\mu V_\mu \mp \frac{C}{f_\pi^2} \rho_s}
\]

**One boson exchange model potential**

\[
V_\mu = \frac{1}{3} \left( f_{\omega} j^\mu \pm f_{\rho} j_3^\mu \right)
\]

\[
m_K^* = \sqrt{m_K^2 + \frac{m_K}{3} g_{\sigma NN}}
\]

upper sign \( K^+ \)

---

**Kaon in-medium energy**

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**Kaons equation of motion**
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\left[ (\partial^\mu + iV_\mu)^2 + m_K^* \right] \phi_K(x) = 0
\]

**Chiral perturbation theory potential**
\[
V_\mu = \frac{3}{8f_\pi^2} j^\mu \pm \frac{1}{8f_\pi^2} j^\mu_3
\]
\[
m_K^* = \sqrt{m_K^2 - \frac{\Sigma_{KN}}{f_\pi^2} \rho_s + V_\mu V_\mu \mp \frac{C}{f_\pi^2} \rho_3}
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**One boson exchange model potential**
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V_\mu = \frac{1}{3} \left( f_\omega^* j_\mu^\pm f_\rho j_3^\mu \right)
\]
\[
m_K^* = \sqrt{m_K^2 + \frac{m_K}{3} \left( g_\sigma N \sigma \mp f_\delta \rho S_3 \right)}
\]

upper sign \(K^+\)

**Kaon in-medium energy**
\[
E_K(k) = k_0 = \sqrt{k^2 + m_K^* + V_0}
\]
Rapidity distribution: Dependence on the $V_K$

Central $Au + Au@1AGeV$.

Reduction in the whole rapidity region.

OBE: less stopping.

Combination of $V_K$ and $\sigma_{\text{eff}}$: further reduction.
Rapidity distribution: Dependence on the $V_K$

Central $Au + Au@1AGeV$.
Reduction in the whole rapidity region.
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Combination of $V_K$ and $\sigma_{eff}$: further reduction.

Rapidity distribution: Isospin dependent-$V_K$

Central $Au + Au@1AGeV$.
Rapidity distributions not affected.
Main contribution: mid-rapidity region.
Combination $V_K$ and $\sigma_{eff}$: further reduction.
Rapidity distributions of $K^+$

$Ni + Ni@1.93AGeV \ b < 4fm$.

ChPT: $\sigma_{\text{free}}$ good agreement with exp. data.
$\sigma_{\text{eff}}$: underestimation of the exp. data.

OBE: $\sigma_{\text{free}}$ good agreement with exp. data.
$\sigma_{\text{eff}}$: on the exp. data.
**Introduction**

Quantum Hadrodynamics (QHD)
Transport theory
Particle Production
Summary & Outlook

**Cross sections**

Kaon-nucleon potential

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**Centrality dependence $K^+$**

$Au + Au@1AGeV$ collision.

Underestimation of the experimental data.

**OBE**: closer to the exp.data.


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**Rapidity distributions of $K^+$**

$Ni + Ni@1.93AGeV$ $b < 4fm$.

**ChPT**: $\sigma_{\text{free}}$ good agreement with exp. data.

$\sigma_{\text{eff}}$: underestimation of the exp. data.

**OBE**: $\sigma_{\text{free}}$ good agreement with exp. data.

$\sigma_{\text{eff}}$: on the exp. data.
Temporal evolution of $K^0/K^+$

Central Au + Au@1AGeV.

OBE: reduction $K^0/K^+$. Favors $K^+$ production.

ChPT: raise. Favors $K^0$ production.

IOBE: raise $K^0/K^+$.

IChPT: sharp drop.
**Temporal evolution of $K^0/K^+$**

Central $Au + Au@1AGeV$.

- **OBE**: reduction $K^0/K^+$. Favors $K^+$ production.
- **ChPT**: raise. Favors $K^0$ production.
- **IOBE**: raise $K^0/K^+$.
- **IChPT**: sharp drop.

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**$K^0/K^+$ dependence on the EOS**

Central $Au + Au@1AGeV$.

- **IChPT**: reduction $\approx 20\%$.
- **IOBE**: $NL\rho \approx 3\%$, while $NL\rho\delta \approx 5\%$. 

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**Cross sections**

**Kaon-nucleon potential**
Summary

- **Effective N-N cross sections:**
  - Dirac Brueckner Hartree Fock.
  - Pions: reduction of production. At mid-rapidity good agreement with the data.
  - $\pi^-/\pi^+$: depends on the effective inelastic cross sections.
  - Kaons: more affected ($\approx 30\%$).
  - $K^0/K^+$ almost unchanged (large mean free path).

- **Kaon-nucleon potential**
  1. Chiral Perturbation Theory, ChPT.
  2. One-Boson-Exchange, OBE.
     - Reduction of kaon production.
     - Good agreement with the data (particularly with OBE).
     - ChPT: $K^0/K^+$ depends on the parametrization of the EOS.
     - OBE: $K^0/K^+$ more robust against the EOS parametrization.

Outlook

- Inclusion of momentum dependence.
- Improvement of NN-interactions.
THANK YOU FOR YOUR ATTENTION

Collaborations:

M. Di Toro, M. Colonna
*LNS, Catania*

H. H. Wolter
*LMU, Muenchen*

Theo Gaitanos
*U. Giessen*

G. A. Lalazissis
*U. Thessaloniki*