Deformed halo and r-process calculation with covariant density-functional theory

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Outline

- Introduction
- Halos in Density Functional Theory: Skyrme HFB / DD
  RHFB / GF Skyrme HFB
- Deformed halos
- R-process calculation with CDFT Mass table
- Summary & Perspectives
Exotic phenomena in nuclei with extreme N/Z

- Modifications of magic numbers
- New radioactivity
- Halo, skin, …
- …
Halo in spherical nuclei

SPL in mean field model

Contributions from the continuum

- Weakly bound; large spatial extension
- Continuum can not be ignored

连续谱

Stable nucleus

Unstable nucleus
Contribution of continuum in r-HFB

\[ \sum_{\sigma} \int d^3 r' \left( \frac{\hbar}{2M} d^2 \right) U_E(r\sigma) = (\lambda + E) U_E(r\sigma) \]

\[ \sum_{\sigma} \int d^3 r' \left( \frac{\hbar}{2M} d^2 \right) V_E(r\sigma) = (\lambda - E) V_E(r\sigma) \]

When \( r \) goes to infinity, the potentials are zero

\[ \frac{\hbar^2}{2M} \frac{d^2}{dr^2} U_E(r\sigma) = (\lambda + E) U_E(r\sigma) \]

\[ \frac{\hbar^2}{2M} \frac{d^2}{dr^2} V_E(r\sigma) = (\lambda - E) V_E(r\sigma) \]

\( U \) and \( V \) behave when \( r \) goes to infinity

\[ U_E(r\sigma) \sim \begin{cases} \cos(k_U r + \delta) & \text{for } \lambda + E > 0 \\ \exp(-k'_U r) & \text{for } \lambda + E < 0 \end{cases} \]

\[ V_E(r\sigma) \sim \begin{cases} \cos(k_V r + \delta) & \text{for } \lambda - E > 0 \\ \exp(-k'_V r) & \text{for } \lambda - E < 0 \end{cases} \]

Continuum contributes automatically and the density is still localized

Bulgac, 1980 & nucl-th/9907088


Dobaczewski, Flocard & Treiner, NPA422(84)103
Contribution of continuum in r-HFB

\[ \varphi_1(r) \sim U(r), \quad \varphi_2(r) \sim V(r) \]

- \( V(r) \) determines the density
- the density is localized even if \( U(r) \) oscillates at large \( r \)

Positive energy States

Bound States

\[ \text{Dobaczewski, et al., PRC53(96)2809} \]
Quantizing the system;
• Eliminating the mesonic degrees of freedom;
• Factorizing the higher order Greens functions;
• Neglecting retardation effects


RHB Equation

\[
\int \, dr' \begin{pmatrix}
    h(r, r') - \lambda & \Delta(r, r') \\
    \Delta(r, r') & -h(r, r') + \lambda
\end{pmatrix}
\begin{pmatrix}
    \psi_U(r') \\
    \psi_V(r')
\end{pmatrix}
= E \begin{pmatrix}
    \psi_U(r) \\
    \psi_V(r)
\end{pmatrix}
\]
Relativistic continuum Hartree Bogoliubov (RCHB) theory

RHB equations:

\[
\begin{pmatrix}
  \hbar - \lambda & \Delta \\
  -\Delta^* & -h^* + \lambda
\end{pmatrix}
\begin{pmatrix}
  \psi_U \\
  \psi_V
\end{pmatrix}
= E
\begin{pmatrix}
  \psi_U \\
  \psi_V
\end{pmatrix}
\]

\[
h(r) = [\alpha \cdot p + V(r) + \beta(M + S(r))]
\]

\[
\Delta_{kk'}(r, r') = -\int d^3r_1 \int d^3r'_1 \sum_{\bar{k}\bar{k}'} V_{kk',\bar{k}\bar{k}'}(rr'; r_1r'_1) \kappa_{\bar{k}\bar{k}'}(r_1, r'_1)
\]

Pairing tensor

\[
\kappa_{kk'}(r, r') = \langle |a_{k,i}a_{k',i'}| \rangle = \sum_{E_i > 0} \psi_{U,i}^k(r)^* \psi_{V,i'}^{k'}(r')
\]

Baryon density

\[
\rho(r, r') = \sum_{k, E_i > 0} \psi_{V,i}^k(r)^* \psi_{V,i}^{k'}(r')
\]

Pairing force

\[
V(r_1, r_2) = V_0 \delta(r_1 - r_2) \frac{1}{4} [1 - \sigma_1 \sigma_2] \left(1 - \frac{\rho(r)}{\rho_0}\right)
\]
To describe bound states, continuum and the coupling between them, RHB equation must be solved in suitable methods

Radial RHB equation in spherical case


**Relativistic Continuum Hartree-Bogoliubov (RCHB) theory**

for spherical nuclei

\[
\phi_U^i = \frac{1}{r} \left( iG_U^{i\kappa}(r)Y_{jm}^l(\theta, \phi) \right) \chi_t(t) \quad \psi_V^i = \frac{1}{r} \left( iG_V^{i\kappa}(r)Y_{jm}^l(\theta, \phi) \right) \chi_t(t)
\]

\[
\begin{align*}
\frac{dG_U(r)}{dr} + \frac{\kappa}{r} G_U(r) - (E + \lambda - V(r) + S(r)) F_U(r) + r^2 \Delta(r) F_V(r) &= 0, \\
\frac{dF_U(r)}{dr} - \frac{\kappa}{r} F_U(r) + (E + \lambda - V(r) - S(r)) G_U(r) + r^2 \Delta(r) G_V(r) &= 0, \\
\frac{dG_V(r)}{dr} + \frac{\kappa}{r} G_V(r) + (E - \lambda + V(r) - S(r)) F_V(r) + r^2 \Delta(r) F_U(r) &= 0, \\
\frac{dF_V(r)}{dr} - \frac{\kappa}{r} F_V(r) - (E - \lambda + V(r) + S(r)) G_V(r) + r^2 \Delta(r) G_U(r) &= 0,
\end{align*}
\]
Some comments

- **Nucleus has finite volume:** the asymptotic RCHB equations for \( r \to \infty \):

\[
\begin{align*}
\frac{d^2 G_U(r)}{dr^2} &= -(E + \lambda)(2M + E + \lambda)G_U(r) \\
\frac{d^2 G_V(r)}{dr^2} &= -(\lambda - E)(2M + \lambda - E)G_V(r)
\end{align*}
\]

similar for \( F_U(r) \)

similar for \( F_V(r) \)

- **Asymptotic solutions:**

\[
\begin{align*}
F_U(r), G_U(r) &\sim \begin{cases} 
\cos(k_U r), & \lambda + E > 0 \\
\exp(-k_U' r), & \lambda + E < 0
\end{cases} \\
F_V(r), G_V(r) &\sim \begin{cases} 
\cos(k_V r), & \lambda - E > 0 \\
\exp(-k_V' r), & \lambda - E < 0
\end{cases}
\end{align*}
\]

if \( E < -\lambda \), U components is localized, discrete

if \( E > -\lambda \), U components is non-localized, continuum
RCHB theory

Densities

\[
\begin{align*}
4\pi r^2 \rho_s(r) &= \sum_i (|G^i_V(r)|^2 - |F^i_V(r)|^2), \\
4\pi r^2 \rho_v(r) &= \sum_i (|G^i_V(r)|^2 + |F^i_V(r)|^2), \\
4\pi r^2 \rho_3(r) &= \sum_i \tau_3(|G^i_V(r)|^2 + |F^i_V(r)|^2), \\
4\pi r^2 \rho_c(r) &= \sum_i \frac{1}{2}(1 - \tau_3)(|G^i_V(r)|^2 + |F^i_V(r)|^2),
\end{align*}
\]

Total binding energy

\[
E = E_{\text{nucleon}} + E_\sigma + E_\omega + E_\rho + E_c + E_{\text{CM}},
\]

\[
E_{\text{nucleon}} = \sum_i \int dr (\lambda - E^i) \left[|G^i_V(r)|^2 + |F^i_V(r)|^2\right] - 2E_{\text{pair}},
\]

\[
E_{\text{pair}} = -\frac{1}{2} \text{Tr} \Delta \kappa
\]
$^{11}\text{Li}$: self-consistent RCHB description

Contribution of continuum

Important roles of low-$l$ orbitals close to the threshold

Meng & Ring, PRL77,3963 (96)
Giant halo: predictions of RCHB

Halos consisting of up to 6 neutrons

Important roles of low-\(l\) orbitals close to the threshold

Meng & Ring, PRL80,460 (1998)
Prediction of giant halo


Zhang, Meng, Zhou & Zeng, CPL19,312 (2002)


Giant halos in lighter isotopes
Neutron halos in hyper Ca nuclei

Lu & Meng, CPL 19, 1775(2002)

Hyperon halos in $^{13}\text{C}_\Lambda$

Exotic Phenomena
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Giant halos from non-rela. HFB
Different predictions for drip line

Terasaki, Zhang, Zhou, & Meng,
PRC74 (2006) 054318
Role of pion and Exchange term DDRHFB

RHFB equation

\[
\int dr' \begin{pmatrix}
\frac{h(r, r') - \lambda}{\Delta(r, r')} & \frac{\Delta(r, r')}{\Delta(r, r') - h(r, r') + \lambda}
\end{pmatrix}
\begin{pmatrix}
\psi_U(r') \\
\psi_V(r')
\end{pmatrix}
= E
\begin{pmatrix}
\psi_U(r) \\
\psi_V(r)
\end{pmatrix}
\]

Single particle Hamiltonian:

\[ h = h^{\text{kin}} + h^D + h^E \]

Kinetic energy:
\[ h^{\text{kin}}(r, r') = [\alpha \cdot p + \beta M] \delta(r, r'), \]

Local potentials:
\[ h^D(r, r') = [\Sigma_T(r) \gamma_5 + \Sigma_0(r) + \beta \Sigma_S(r)] \delta(r, r'), \]

Non-local Potentials:
\[ h^E(r, r') = \begin{pmatrix}
Y_G(r, r') & Y_F(r, r') \\
X_G(r, r') & X_F(r, r')
\end{pmatrix} \]

Pairing Force: Gogny D1S

\[ V(r, r') = \sum_{i=1,2} e^{((r-r')/\mu_i)^2} \left( W_i + B_i P^\sigma - H_i P^T - M_i P^\sigma P^T \right) \]


To Solve the integro-differential RHFB equation
Halo and giant halo in DDRHFB

2. Long, Ring, Meng, Giai, and Bertulani, Physical Review C 81, 031302(R) (2010)
Halo structures in Cerium isotopes

Halos: $^{186, 188, 190}\text{Ce}$; Giant halos: $^{192, 194, 196, 198}\text{Ce}$
Continuum Skyrme HFB with Green’s function method

Density in discretized and continuum HFB approach

\[ \rho(r\sigma, r'\sigma') \equiv \langle \Phi_0 \mid \psi^\dagger(r'\sigma') \psi(r\sigma) \mid \Phi_0 \rangle \]

\[ \tilde{\rho}(r\sigma, r'\sigma') \equiv \langle \Phi_0 \mid \psi(r'\sigma') \psi(r\sigma) \mid \Phi_0 \rangle \]

\[ \rho(r\sigma, r'\sigma') = \sum_{0 < E_i < |\lambda|} \varphi_2(E_i, r\sigma) \varphi_2^*(E_i, r'\sigma') + \int_{|\lambda|} dE \ \varphi_2(E, r\sigma) \varphi_2^*(E, r'\sigma') \]

\[ \tilde{\rho}(r\sigma, r'\sigma') = - \sum_{0 < E_i < |\lambda|} \varphi_2(E_i, r\sigma) \varphi_1^*(E_i, r'\sigma') - \int_{|\lambda|} dE \ \varphi_2(E, r\sigma) \varphi_1^*(E, r'\sigma') \]

Discretized HFB

Continuum HFB

\[ \rho(r\sigma, r'\sigma') = \sum_{0 < E_i < E_{cut}} \varphi_2(E_i, r\sigma) \varphi_2^*(E_i, r'\sigma') \]

\[ \tilde{\rho}(r\sigma, r'\sigma') = - \sum_{0 < E_i < E_{cut}} \varphi_2(E_i, r\sigma) \varphi_1^*(E_i, r'\sigma') \]

\[ \rho(r\sigma, r'\sigma') = \frac{1}{2\pi i} \oint_{C_{E<0}} dE \ \varphi_2(E, r\sigma) \varphi_2^*(E, r'\sigma') \]

\[ \tilde{\rho}(r\sigma, r'\sigma') = - \frac{1}{2\pi i} \oint_{C_{E<0}} dE \ \varphi_2(E, r\sigma) \varphi_1^*(E, r'\sigma') \]

PRC83, 054301(2011)
Density obtained from discretized and continuum HFB approach

independent on the box size for continuum HFB
Continuum Skyrme HFB with Green’s function method

- $n(E)$: occupation number density by continuum HFB cal.
- $v^2$: occupation probability by discretized HFB cal.

New information: width of q.p. resonance for continuum HFB
Continuum Skyrme HFB with Green’s function method

- Quasiparticle resonance

![Graphs showing quasiparticle resonance](image)
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Deformed RHB in a Woods-Saxon basis

Axially deformed nuclei

\[ \beta_{km}^+ = \sum_{(i\kappa)} u_{k,(i\kappa)}^{(m)} a_{i\kappa m}^+ + v_{k,(i\bar{\kappa})}^{(m)} \tilde{a}_{i\kappa m} \]

\[ \left( U_k^{(m)}(r\sigma p) \right) = \sum_{i\kappa} \left( u_{k,(i\kappa)}^{(m)} \varphi_{i\kappa m}(r\sigma p) \right) \]

\[ \left( V_k^{(m)}(r\sigma p) \right) = \sum_{i\kappa} \left( v_{k,(i\bar{\kappa})}^{(m)} \tilde{\varphi}_{i\kappa m}(r\sigma p) \right) \]

\[ \varphi_{i\kappa m}(r\sigma p) = \frac{1}{r} \left( i G_{i\kappa}(r) Y_{\kappa m}(\Omega \sigma) \right) \]

\[ \varphi_{i\kappa m}(r\sigma p) = \frac{1}{r} \left( - F_{i\kappa}(r) Y_{\kappa m}(\Omega \sigma) \right) \]

\[ \sum_{\sigma p} \int d^3r \left( h(r\sigma p; r' \sigma' p') - \lambda \Delta(r\sigma p; r' \sigma' p') - h(r\sigma p; r' \sigma' p') + \lambda \right) \left( U_E(r' \sigma' p') \right) \left( V_E(r' \sigma' p') \right) = E \left( U_E(r\sigma p) \right) \left( V_E(r\sigma p) \right) \]

\[ \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} U \\ V \end{pmatrix} = E \begin{pmatrix} U \\ V \end{pmatrix} \]

\[ U = \begin{pmatrix} u_{k,(i\kappa)}^{(m)} \end{pmatrix} \quad V \equiv \begin{pmatrix} v_{k,(i\bar{\kappa})}^{(m)} \end{pmatrix} \]
**DRHB matrix elements**

\[
A_{(i\kappa),(i'\kappa')} = \left( h_{(i\kappa),(i'\kappa')}^{(m)} \right) - \lambda I \\
B_{(i\kappa),(i'\kappa')} = \left( \Delta_{(i\kappa),(i'\kappa')}^{(m)} \right)
\]

\[
C_{(i\bar{\kappa}),(i'\bar{\kappa}')} = \left( - \Delta_{(i\bar{\kappa}),(i'\bar{\kappa}')}^{(m)} \right) = \Delta_{(i\kappa),(i'\bar{\kappa}')}^{(m)} \\
D_{(i\bar{\kappa}),(i'\bar{\kappa}')} = \left( - h_{(i\bar{\kappa}),(i'\bar{\kappa}')}^{(m)} \right) + \lambda I
\]

\[
V(r) = \sum_{\lambda, \mu} V_{\lambda} (\hat{r}) Y_{\lambda} (\Omega) \\
S(r) = \sum_{\lambda, \mu} S_{\lambda} (\hat{r}) Y_{\lambda} (\Omega)
\]

\[
h_{(i\kappa),(i'\kappa')}^{(m)} = \sum_{\lambda} \int \! dr \{ G_{i\kappa} (r) G_{i'\kappa'} (r) \left[ V_{\lambda} (r) + S_{\lambda} (r) \right] + F_{i\kappa} (r) F_{i'\kappa'} (r) \left[ V_{\lambda} (r) - S_{\lambda} (r) \right] \} A(\lambda, \kappa, \kappa', m)
\]

\[
\Delta(r, \sigma_1 \sigma_2) = \sum_{\lambda} \sum_S Y_{\lambda} (\Omega) \chi_S (\sigma_1 \sigma_2) \Delta_{\lambda \mu M}^{S_{p_1 p_2}} (r) \\
\lambda, \text{ even or odd} \\
\mu = 0, \pm 1
\]

\[
\Delta_{(i_1\kappa_1),(i_2\bar{\kappa}_2)}^{(m)} = \frac{1}{2} \sum_{\lambda, \mu} \sum_{SM_S} \delta_{MS_S,-\mu} \sum_{p_1 p_2} \eta_{SM_S}^{\lambda \mu \alpha_1 \alpha_2} \int \! dr R_{i_1\kappa_1}^{p_1} (r) R_{i_2\bar{\kappa}_2}^{p_2} (r) \Delta_{\lambda \mu ; p_1 p_2}^{SM_S} (r)
\]
Pairing interaction

- Phenomenological pairing interaction with parameters: $V_0$, $\rho_0$, $\gamma$, and the smooth cut off parameters $E_{\text{cut}}$ and $\Gamma$

$$V^\text{pair} = \frac{1}{4} V_0 \delta(r_1 - r_2) \left(1 - \frac{\rho(r_1)}{\rho_0}\right)^\gamma$$

$$s(E_k) = \frac{1}{2} \left(1 - \frac{E_k - E_{k,\text{cut}}^{q,p}}{\sqrt{(E_k - E_{k,\text{cut}}^{q,p})^2 + (\Gamma_{\text{cut}}^{q,p})^2}}\right)$$

Finite range?  Volume or surface?  Microscopic?

PHYSICAL REVIEW C57 (1988) 1229
20\textsuperscript{Mg}: spherical from DRHBWS calculation

\[ R_{\text{max}} = 20 \text{ fm}, \quad \Delta r = 0.1 \text{ fm} \]

**Zero pairing energy for the neutron**

<table>
<thead>
<tr>
<th>Model</th>
<th>Pairing force</th>
<th>Parameters</th>
<th>$E_{\text{pair}}^p$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SRHBHO</td>
<td>Gogny</td>
<td>D1S</td>
<td>$-9.2382$</td>
</tr>
<tr>
<td>RCHB</td>
<td>Surface $\delta$</td>
<td>\begin{align*} V_0 &amp;= 374 \text{ MeV fm}^3 \ \rho_0 &amp;= 0.152 \text{ fm}^3 \ E_{\text{cut}}^{\text{q.p.}} &amp;= 60 \text{ MeV} \end{align*}</td>
<td>$-9.2387$</td>
</tr>
<tr>
<td>DRHBWS</td>
<td>Surface $\delta$</td>
<td>\begin{align*} V_0 &amp;= 380 \text{ MeV fm}^3 \ \rho_0 &amp;= 0.152 \text{ fm}^3 \ E_{\text{cut}}^{\text{q.p.}} &amp;= 60 \text{ MeV} \ \Gamma &amp;= 5.65 \text{ MeV} \end{align*}</td>
<td>$-9.2383$</td>
</tr>
</tbody>
</table>
### RMF in a Woods-Saxon basis: progress

<table>
<thead>
<tr>
<th>Shapes</th>
<th>Model</th>
<th>Schrödinger W-S basis</th>
<th>Dirac W-S basis</th>
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<td>Spherical</td>
<td>Rela. Hartree</td>
<td>SRH  SWS</td>
<td>SRH  DWS</td>
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<td></td>
<td></td>
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<td>Zhou, Meng &amp; Ring, PRC68,034323(03); PRL91, 262501 (03)</td>
</tr>
<tr>
<td>Axially deformed</td>
<td>Rela. Hartree + BCS</td>
<td>DRH  DWS</td>
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<td></td>
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</tr>
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<td>Rela. Hartree-Bogoliubov</td>
<td>DRHB  DWS</td>
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<td></td>
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<td>Zhou, Meng, Ring, ISPUN 2007</td>
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<tr>
<td>Triaxially deformed</td>
<td>Rela. Hartree-Bogoliubov</td>
<td>TRHB  DWS</td>
<td></td>
</tr>
</tbody>
</table>

**Woods-Saxon basis might be a reconciler between the HO basis and r space**
Origin of the symmetry - Anti-nucleons

Zhou, Meng & Ring, PRL92(03)262501
**44Mg from DRHBWS**

- **Prolate deformation**
- **Large spatial extension in neutron density distribution**

Zhou, Meng, Ring, Zhao
PRC82(10)011301R
The 3rd & 4th states contribute to tail part of neutron density distribution

Main component: $2p_{3/2}$

$R_{\text{core}} = 3.72$ fm, $R_{\text{halo}} = 5.86$ fm
Prolate core, but slightly oblate halo with sizable hexadecapole component!
Decoupling of deformation betw. core & halo
Density of core & halo

- Prolate core, but slightly oblate halo with sizable hexadecapole component
- Decoupling of deformation between core & halo
Introduction

Halos in Density Functional Theory: Skyrme HFB / DD
  RHFB / GF Skyrme HFB

Deformed halos

R-process calculation with CDFT Mass table

Summary & Perspectives
71 nuclides covered
27 nuclides were measured
8 measured for the first time
8 unresolved ground state and isomeric states
1 isomeric state of $^{133}$Sb ($E^*=4564(170)$ keV)

Mass accuracy: $1.0 \cdot 10^{-6}$ (~120 keV)
Resolving power: 200,000
First RMF mass table in 2005: Theo. vs. Exp.

rms deviation of nuclear masses: 2.1 MeV

Binding energy differences: $B_{\text{theo.}} - B_{\text{exp.}}$

RMF+BCS

Proton Number $Z$

Neutron Number $N$
Nuclear single neutron separation energy: Theo. vs. Exp.

rms deviation of $S_n$: 0.654 MeV
Assume:

- \((n,\gamma) \leftrightarrow (\gamma, n)\) equilibrium within isotopic chain, and
- elemental distribution of neighboring \(Z\)-chain is determined by the \(\beta\)-decays
- neglect the effect of fission
- constant \(T_\beta\), multi \(r\)-process components with \(n_n = 10^{20-27}\).

The nucleus with maximum abundance in each isotopic chain has smaller neutron capture rate and must wait for the longer time to continue via \(\beta\)-decay.
Astrophysical conditions:

$T_9 = 1.5,$

16-component fit with $n_n = 10^{20} - 3 \times 10^{27} \text{ cm}^{-3},$ which fulfill the following equations:

$$\omega(n_n) = n_n^a, \tau(n_n) = bn_n^c$$

where $\omega$ and $\tau$ are respectively the weight and neutron irradiation time, and $a, b, c$ are the alterable parameters, which will be determined by least-square fit to the solar r-process abundance.

Good approximation for astrophysical environment studies!
nuclear inputs: $S_n$ (RMF), $T_{1/2}$ (β-decay), $P_{1n}$, $P_{2n}$, $P_{3n}$ (FRDM),
astrophysical parameters: $T_9 = 1.5$, $n_n = 10^{20-28}$, $\omega$, $\tau$ (least-square fit),
Nuclear Mass Model dependence
The age of the universe is one of the most important physical quantities in cosmology. The metal-poor star is formed at the early stage of the universe, so its age provides constraint to the age of the universe. The age of metal-poor star:

\[
\frac{Th}{U}_{\text{present}} = \frac{Th}{U}_{\text{initial}} e^{-(\lambda_{Th}-\lambda_{U})t}
\]

- Present abundances: astronomical observations.

- Initial abundances: r-process calculations (Th, U are r-only nuclei).

- The classical r-process model is usually employed in r-process calculations.
Ages of metal-poor stars

Age (HE 1523-0901) = 11.8 ± 3.7 Gyr
Age (CS 31082-001) = 13.5 ± 2.9 Gyr

Z. Niu et al., PRC 80 065806 (2009)
Calibrating the cosmic clock

Influence of nuclear physics inputs and astrophysical conditions on the Th-U chronometer
Zhongming Niu (牛中明), Baohua Sun (孙保华), and Jie Meng (孟杰)
Phys. Rev. C 80, 065806 (Published December 22, 2009)

Knowing when nucleosynthesis—the formation of new nuclei from existing nuclei—occurred in astrophysical sites can be crucial to our understanding of cosmology. One method to pin the process down in time is to compare the current abundance ratio of thorium to uranium (both of which have lifetimes of the order of the age of the universe) with calculations of this ratio at the time at which the nucleosynthesis that formed those elements took place. The assumption is that the nucleosynthesis itself happens over a time scale that is short compared to the time since it occurred.

In a paper published in Physical Review C, Zhongming Niu of Peking University and Baohua Sun and Jie Meng of Beihang University, both in China, present a study of the uncertainties in the calculation of the initial ratio of thorium to uranium. In particular, they focus on the importance of the models used to determine the nuclear masses and the nucleosynthesis processes themselves. Utilizing the abundances of uranium and thorium in the sun to restrict the models, Niu et al. are able to minimize the impact of the uncertainties. They find that the error due to the nuclear input alone is about 1.6–2.2 billion years (for reference, the age of the universe is about 14-16 billion years). This estimate is lower, but not significantly so, than the observational uncertainties. In addition, they determine when nucleosynthesis occurred for three stars. New observations for other elements in stars and improved mass models could make a major impact on the thorium-uranium chronometer, and in general, studies of this kind help us learn what parts of the universe were undergoing the extreme conditions needed for nucleosynthesis to occur; and when. — William Gibbons

"Phys. Rev. C (2009) 80, 065806" was reported as an important progress of nuclear physics and cosmology on "physics.aps.org" webpage
Future: β-decay half-lives from CDFT

- RHFB+QRPA: well reproduces the experimental data
- RHB+QRPA: underestimates the half-lives of $^{108,110}$Zr, $^{112,114}$Mo, and $^{116,118}$Ru.
- FRDM+QRPA: systematically overestimates the nuclear half-lives.
Observed neutron-richest e-e nuclei with $26 \leq Z \leq 100$

rms mass deviations:

$$\sigma_{\text{rms}} = \sqrt{\frac{\sum_{i=1}^{N} (M_i^{\text{cal}} - M_i^{\text{exp}})^2}{N}}$$

- PC-PK1 improves the description remarkably.
- Similar accuracy for the others

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Mass model: HFB-17

\[ F = \sum_{A=90}^{209} \left| \log(Y_{A,\text{baseline}}) - \log(Y_A) \right| \]

β-decay properties: FRDM+QRPA

S. Goriely, N. Chamel, and J. M. Pearson, PRL 102, 152503.

P. Moller, B. Pfeiffer, and K.-L. Kratz, PRC 67, 055802.

- \( S_n \) of Ag, Cd, Sn have great influence on the r-process abundances
- \(^{131}\text{Ag}, ^{133,134}\text{Cd}, ^{141,142}\text{Sn}: F>4 \) using both HFB-17 and FRDM
Summary and Perspectives

- **Recent development**
  - RH or RHB in WS basis
  - Time-odd (constrained) Triaxial RMF
  - Resonance in RMF+ACCC
  - DDRHF + RPA
  - DDRHFB+QRPA
  - 3D AMP+GCM
  - Tilted Cranking RMF
  - Bohr H based on CDFT

- **Recent application**
  - (Deformed) Exotic phenomena: halos
  - CDFT mass formula and r-process calculation
  - Symmetry: Spin and pseudo-spin symmetry
  - Low-lying nuclear spectra
  - Magnetic rotation & chirality
  - Quantum shape transitions
  - Spin-Dipole Resonances
  - CKM matrix | $V_{ud}$ | . . .

- **Open issues**
  - (Q)RPA for excited states
  - 3D Cranking RMF for chirality
  - Projected RMF: H-K isomers | shape transition | NP pairing
  - Deformed RHF & RHFB & (Q)RPA
Thank You!