Ab-initio Gorkov-Green's function calculations for open shell nuclei

C. Barbieri

Collaborators: V. Somà and T. Duguet
To have “reasonable” ab-initio theories we STILL need:

- Proper realistic forces (**3NF** included)
- Capability of calculating open shells
- Learn to extract information from measured reactions

“reasonable”: not necessary convergent BUT **accurate enough, predictive and useful to an experimental program**
State-of-the-art ab-initio nuclear structure theory

Methods for an ab-initio description of medium-mass nuclei as of 2011

(1) Coupled-cluster [Dean, Papenbrock, Hagen, ...]

(2) In-medium similarity renormalization group [Tsukiyama, Bogner, Schwenk]

(3) Self-consistent Dyson-Green’s function (SCGF) [Barbieri, Dickhoff]

The present status is:

→ Still in need of good nuclear Hamiltonians (3N forces mostly!)

→ Only structure calculations and limited to closed-shells or A±1, A±2
  (BUT calculations are GOOD!!!)

However, Green’s functions can be extended to: Scattering observables
Open shell nuclei
Outline...

• Self-consistent Green's function in closed shells:
  – *Faddeev random-phase approximation (FRPA):* $^4$He benchmark Scattering (N-A)

• Open shells: Gorkov-GF formalism
  – *G-SCGF formalism at 2nd order*
  – *Preliminary results*

• Applications: spectroscopic factors

• Applications: dispersive optical potentials
  – *S. Waldecker, CB, W. Dickhoff, arXiv:1105.4257*
Concepts of Spectral Functions and Many-Body Green’s Functions
One-body Green's function (or propagator) describes the motion of quasi-particles and holes:

\[ g_{\alpha\beta}(E) = \sum_n \frac{\langle \Psi_0^A | c_\alpha | \Psi_n^{A+1} \rangle \langle \Psi_n^{A+1} | c_\beta^\dagger | \Psi_0^A \rangle}{E - (E_n^{A+1} - E_0^A) + i\eta} + \sum_k \frac{\langle \Psi_0^A | c_\beta | \Psi_k^A \rangle \langle \Psi_k^A | c_\alpha | \Psi_0^A \rangle}{E - (E_0^A - E_k^{A-1}) - i\eta} \]

...this contains all the structure information probed by nucleon transfer (spectral function):

[ pics. J. Sadoudi]
Green's functions in many-body theory

One-body Green's function (or propagator) describes the motion of quasi-particles and holes:

\[
g_{\alpha\beta}(E) = \sum_n \frac{\langle \Psi_0^A | c_\alpha | \Psi_{n+1}^A \rangle \langle \Psi_{n+1}^A | c_\beta^\dagger | \Psi_0^A \rangle}{E - (E_{n+1}^A - E_0^A) + i\eta} + \sum_k \frac{\langle \Psi_0^A | c_\beta^\dagger | \Psi_k^{A-1} \rangle \langle \Psi_k^{A-1} | c_\alpha | \Psi_0^A \rangle}{E - (E_0^A - E_k^{A-1}) - i\eta}
\]

...this contains all the structure information probed by nucleon transfer (spectral function):

\[
S(r,\omega) = \frac{\mp 1}{\pi} \text{Im} \ g_{rr}(\omega) = \sum_n |\langle \Psi_n^{A\pm1} | c_r^{(+)} | \Psi_0^A \rangle|^2 \delta(\omega \pm (E_0^A - E_n^{A\pm1}))
\]
Spectral Function of $^{56}$Ni

Faddeev-RPA (FRPA) calculations

$S_{56}^{(n)}(\omega, r)$

$^{56}$Ni

$ f_{7/2}$

$ p_{3/2}$

$ p_{1/2}, f_{5/2}$

$\omega [\text{MeV}]$

$r [\text{fm}]$

$E_F$

scattering

neutron removal

neutron addition


CB, Phys. Rev. Lett. 103, 202502 (2009)]
Dyson equation & self-energy

- Perturbative expansion of one-body propagator

- Irreducible self-energy

- Dyson equation

\[ G_{ab}(\omega) = G_{ab}^{(0)}(\omega) + \sum_{cd} G_{ac}^{(0)}(\omega) \Sigma_{cd}^{*}(\omega) G_{db}(\omega) \]
Solving the Dyson equation

- Different approximations to the self-energy (self-consistent approaches)

- **Hartree-Fock**
  - \[
  \text{Diagram of Hartree-Fock}
  \]

- **Second order**
  - \[
  \text{Diagram of Second order}
  \]

- **RPA**
  - \[
  \text{Diagram of RPA}
  \]

- **Ladder (or T-matrix)**
  - \[
  \text{Diagram of Ladder (or T-matrix)}
  \]
Applications to doubly-magic nuclei

- Faddeev-RPA approximation for the self-energy
- collective vibrations
- particle-vibration coupling

[C.B. et al. 2001-2011]

- Successful in medium-mass doubly-magic systems
- Expansion breaks down when pairing instabilities appear

Explicit configuration mixing

Single-reference: Bogoliubov (Gorkov)
**Self-Consistent Green's Function Approach**

**pp/hh-RPA; two-nucleon transfer**

- [CB, Giusti et al., Phys Rev. C70, 014606 (2004)]
- [Middleton, CB, et al., arXiv:0907.1758, EPJA in print]

**Faddeev-RPA**


**ph-RPA; nuclear response function, giant/pygmy resonances, Gamow-Teller**

- [CB, Dickhoff, Phys Rev. C68, 014311 (2003)]
- [CB, Langanke et al., Phys Rev. C77, 024304 (2008)]

**Dyson Eq.**

- optical potential
  - [CB, Jennings, Nucl. Phys A758, 395c (2005)]
  - [Phys Rev. C72, 014613 (2005)]

**single-particle motion**

- [CB, Dickhoff, Phys. Rev. C65, 064313 (2002)]
- [CB, Phys. Rev. Lett. 103, 202502 (2009)]

**Faddeev-RPA** is a *many-body* method:

- Random phase approx. (RPA) for collective vibrations
- Faddeev eqs. for particle-vibration coupling

*Interfaces between structure and reactions...*
Self-consistent FRPA compares well with benchmark calculations on $^4$He

$V_{\text{low-k}}$: $-29.00(2)$  
FRPA/sc0

$V_{\text{low-k}}$: $-29.2 \pm 0.15$  
FRPA/sc

Exact: $-29.19(5)$ (Fadd.-Yak.)  

self-consistency in the mean field only

estimates from different approx. to self-consistency
Comparison to CC benchmark

\[ \Sigma^*(\omega) = R(2p1h) \lor R(2h1p) \]

\[ V_{low-k}, \Lambda = 1.9 \text{ fm}^{-1} \]
\[ N_{max} = 12 \]

\[ E_{gs} (^4\text{He}) \text{ [MeV]} \]

\[ \hbar \omega \text{ [MeV]} \]


Coupled-Cluster benchmark,
**p-^{16}O phase shifts and <^{16}O|^{17}F> overlaps**

- EITHER the phase shifts OR bound states can be made in agreement with the experiment! (with minor phen. corrections)

- BUT the calculation did not reproduce both at the same time

- **AV18 interaction**

- Continuum is treated in full using momentum space

- Non-MF resonances "OK"

---

**FIG. 6.** (Color online) Radial part of the overlap wave functions between \(^{16}O\) and the bound \(d_{3/2}\) and \(s_{1/2}\) states of \(^{17}F\).

Gorkov formalism: open shells

(CB, V. Somà, T. Duguet -- in completion)
**Applications to doubly-magic nuclei**

- Faddeev-RPA approximation for the self-energy
  - collective vibrations
  - particle-vibration coupling

  [C.B. et al. 2001-2011]

- Successful in medium-mass doubly-magic systems
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  Explicit configuration mixing

  Single-reference: Bogoliubov (Gorkov)

---

*Interfaces between structure and reactions...*
Going to open-shells: Gorkov ansatz

\[ \cdots \approx E_0^{N+2} - E_0^N \approx E_0^N - E_0^{N-2} \approx \cdots \approx 2\mu \]

\[ |\Psi_0\rangle = \sum_{N} c_N |\psi_0^N\rangle \]

- Mixes various particle numbers

- Introduce a “grand-canonical” potential \( \Omega = H - \mu N \)

\[ |\Psi_0\rangle \text{ minimizes } \Omega_0 = \langle \Psi_0 | \Omega | \Psi_0 \rangle \]

under the constraint \( N = \langle \Psi_0 | N | \Psi_0 \rangle \)

\[ \Omega_0 = \sum_{N'} |c_{N'}|^2 \Omega_0^{N'} \approx E_0^N - \mu N \]
Gorkov Green's functions and equations

Set of 4 Green's functions

\[
i G_{ab}^{11} (t, t') \equiv \langle \Psi_0 | T \left\{ a_a(t) a_b^\dagger (t') \right\} | \Psi_0 \rangle \equiv
\]

\[
i G_{ab}^{21} (t, t') \equiv \langle \Psi_0 | T \left\{ a_a^\dagger (t) a_b (t') \right\} | \Psi_0 \rangle \equiv
\]

\[
i G_{ab}^{12} (t, t') \equiv \langle \Psi_0 | T \left\{ a_a(t) \bar{a}_b (t') \right\} | \Psi_0 \rangle \equiv
\]

\[
i G_{ab}^{22} (t, t') \equiv \langle \Psi_0 | T \left\{ a_a^\dagger (t) \bar{a}_b (t') \right\} | \Psi_0 \rangle \equiv
\]

\[
G_{ab}(\omega) = G_{ab}^{(0)}(\omega) + \sum_{cd} G_{ac}^{(0)}(\omega) \Sigma_{cd}(\omega) G_{db}(\omega)
\]

Gorkov equations

\[
\Sigma_{ab}(\omega) = \begin{pmatrix}
\Sigma_{ab}^{11}(\omega) & \Sigma_{ab}^{12}(\omega) \\
\Sigma_{ab}^{21}(\omega) & \Sigma_{ab}^{22}(\omega)
\end{pmatrix}
\]

\[
\Sigma_{ab}^*(\omega) \equiv \Sigma_{ab}(\omega) - U_{ab}
\]
1st & 2nd order diagrams and eigenvalue problem

1st order ➞ energy-independent self-energy

\[ \Sigma_{ab}^{11} = \begin{array}{c}
\begin{tikzpicture}
  \node (a) at (0,0) {$a$};
  \node (b) at (2,0) {$b$};
  \node (c) at (1,1) {$c$};
  \node (d) at (1,-1) {$d$};
  \draw (a) -- (b);
  \draw (c) edge[->] (d);
  \draw (d) edge[->] (a);
  \node at (1,0) {$\downarrow \omega'$};
\end{tikzpicture}
\end{array} \]

2nd order ➞ energy-dependent self-energy

\[ \Sigma_{ab}^{12} = \begin{array}{c}
\begin{tikzpicture}
  \node (a) at (0,0) {$a$};
  \node (b) at (2,0) {$b$};
  \node (c) at (1,1) {$c$};
  \node (d) at (1,-1) {$d$};
  \draw (a) -- (b);
  \draw (c) edge[->] (d);
  \draw (d) edge[<-] (a);
  \node at (1,0) {$\downarrow \omega'$};
\end{tikzpicture}
\end{array} \]

Gorkov equations ➞ eigenvalue problem

\[
\sum_b \left( t_{ab} - \mu_{ab} + \Sigma_{ab}^{11}(\omega) \right) - t_{ab} + \mu_{ab} + \Sigma_{ab}^{22}(\omega) \right|_{\omega_k} \begin{pmatrix} U_k^b \\ V_k^b \end{pmatrix} = \omega_k \begin{pmatrix} U_k^a \\ V_k^a \end{pmatrix}
\]

\[ U_k^a \equiv \langle \Psi_k | a^\dagger \Psi_0 \rangle \]

\[ V_k^a \equiv \langle \Psi_k | a \Psi_0 \rangle \]
Gorkov equations

\[ \sum_b \left( t_{ab} - \mu_{ab} + \Sigma_{ab}^{11}(\omega) \frac{\Sigma_{ab}^{21}(\omega)}{-t_{ab} + \mu_{ab} + \Sigma_{ab}^{22}(\omega)} \right) \left| \omega_k \right\rangle \begin{pmatrix} U_b^k \\ V_b^k \end{pmatrix} = \omega_k \begin{pmatrix} U_a^k \\ V_a^k \end{pmatrix} \]

\[
\begin{pmatrix}
T - \mu + \Lambda & \tilde{h} \\
\tilde{h}^\dagger & -T + \mu - \Lambda \\
C^\dagger & -D \\
-C & D
\end{pmatrix}
\begin{pmatrix}
U^k \\
V^k \\
W_k \\
Z_k
\end{pmatrix} = \omega_k \begin{pmatrix}
U^k \\
V^k \\
W_k \\
Z_k
\end{pmatrix}
\]

Energy independent eigenvalue problem

with the normalization condition

\[ \sum_a \left[ |U_a^k|^2 + |V_a^k|^2 \right] + \sum_{k_1 k_2 k_3} \left[ |W_{k_1 k_2 k_3}^k|^2 + |Z_{k_1 k_2 k_3}^k|^2 \right] = 1 \]
Green's functions: important features

- Self-consistent approach
- Direct connection to observables
- Improvability (diagrammatic expansion)
- Control over many-body requirements (conserving approximations)
- Possible connection to nuclear reactions (dispersive optical models)

* Drawbacks
- Technically and computationally involved
Preliminary Gorgov results
Results

Calculations of $^{40-48}$Ca isotopes

- Spherical HO basis (no-core): 7 shells, $\hbar \omega = 22$ MeV (very preliminary!)
- $V_{\text{low-k}}$ from Ch-EFT $N^3LO$ potential with cutoff $\Lambda = 2.1 \& 2.5$ fm$^{-1}$
- NN interaction only

CEA-CCRT massively-parallel high-performance cluster

- ~ 40,000 cores, ~ 300 Tflops total
- Parallelized code

Essential for converged self-consistent second-order calculations

[Entem and Machleidt 2003]
Systematic along isotopic/isotonic chains has become available

- Overbinding with $A$: traces need for (at least) NNN forces
- Correlation energy close to CCSD and FRPA (thorough comparison planned)
- Effect of self-consistency significant; i.e. less bound than MBPT2
Spectral function

Dyson 1st order (HF)

Gorkov 1st order (HFB)

Fragmentation

Static pairing

Dynamical fluctuations

Dyson 2nd order

Gorkov 2nd order

Interfaces between structure and reactions...

INT, Aug. 19th, 2011
ESPE collect fragmentation of “single-particle” strengths from both $N \pm 1$

$$\epsilon_{ab}^{cent} \equiv h_{ab}^{cent} \delta_{ab} = t_{aa} + \sum_{cd} \tilde{V}_{acda}^{NN} \rho_{dc}^{[1]} + \sum_{cdef} \tilde{V}_{acdaef}^{NNN} \rho_{efcd}^{[2]} \equiv \sum_{k} S_{k}^{+} E_{k}^{+} + \sum_{k} S_{k}^{-} E_{k}^{-}$$


Separation energies and transfer strengths

ESPE not to be confused with quasiparticle peak

 Particularly true for low-lying state in open-shell due to pairing
ESPE collect fragmentation of “single-particle” strengths from both $N\pm1$

$$\epsilon^\text{cent}_a \equiv h^\text{cent}_{ab} \delta_{ab} = t_{aa} + \sum_{cd} \bar{V}_{acad}^{[1]} \rho_{dc}^{[1]} + \sum_{cdef} \bar{V}_{acdaef}^{[2]} \rho_{efcd}^{[2]} \equiv \sum_k S_k^+ E_k^+ + \sum_k S_k^- E_k^-$$


Quasiparticle peaks

Centroids

$\rightarrow$ ESPE not to be confused with quasiparticle peak

$\rightarrow$ Particularly true for low-lying state in open-shell due to pairing
Natural single-particle occupation

- Natural orbit $a$: $\rho_{ab}^{[1]} = n_a^{\text{nat}} \delta_{ab}$

- Associated energy: $\epsilon_a^{\text{nat}} = h_{aa}^{\text{cent}}$

- Dynamical correlations similar for doubly-magic and semi-magic

- Static pairing essential to open-shells
Pairing gaps

• Three-point mass differences

\[ \Delta_n^{(3)}(N) = \frac{(-1)^N}{2} \frac{\partial \mu_n}{\partial N} + \Delta_n \]

→ Systematic underestimation of experimental gaps

→ Missing NNN in \( \Sigma^{11} \) changes picture qualitatively
Summary

- Self-Consistent Green's Functions (SCGF), is a microscopic ab-initio method applicable to medium mass nuclei.

- The greatest advantage is the link to experimental information (→ spectroscopy)

- The bigger challenges are:
  - Approach open-shells
  - Consistent description of structure and reactions

- SCGF are the optimal choice
  - extension to Gorkov-formalism
    → Open-shell nuclei
    → Reactions at driplines
    → structure of next generation EDF

- Three nucleon forces (3NF) are a MUST for accurate predictions of exotic isotopes.

Thank you for your attention!!!
Collaborators

V. Somà, T. Duguet

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A. Rios

A. Polls

D. Van Neck, M. Degroote

T. Otsuka

M. Hjorth-Jensen

C. Giusti, F.D. Pacati
Quasiparticle states and spectroscopic factors
N3LO needs a monopole correction to fix the p-h gap:

\[
\begin{align*}
\Delta V^T_{fr} &\rightarrow \Delta V^T_{fr} - (-1)^T \kappa_M, \\
\Delta V^T_{ff} &\rightarrow \Delta V^T_{ff} - 1.5(1-T)\kappa_M,
\end{align*}
\]

\[r \equiv p_{3/2}, p_{1/2}, f_{5/2}, f_{7/2}\]

Experimental Eph is found for \(k_M = 0.57\).
Particle-vibration coupling dominates the quenching of spectroscopic factors

Relative strength among fragments requires shell-model approach


<table>
<thead>
<tr>
<th>10 osc. shells</th>
<th>Exp. [30]</th>
<th>1p0f space</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FRPA</td>
<td>full</td>
</tr>
<tr>
<td></td>
<td>(SRC)</td>
<td>full</td>
</tr>
<tr>
<td>57Ni:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ν1p_{1/2}</td>
<td>0.96</td>
<td>0.63</td>
</tr>
<tr>
<td>ν0f_{5/2}</td>
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<td>0.59</td>
</tr>
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</tr>
<tr>
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<td>0.72</td>
</tr>
<tr>
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<tr>
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<td>0.95</td>
<td>0.72</td>
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<tr>
<td>57Cu:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>π1p_{1/2}</td>
<td>0.96</td>
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</tr>
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<td>0.96</td>
<td>0.60</td>
</tr>
<tr>
<td>π1p_{3/2}</td>
<td>0.96</td>
<td>0.67</td>
</tr>
<tr>
<td>55Co:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>π0f_{7/2}</td>
<td>0.95</td>
<td>0.73</td>
</tr>
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</table>

[CB, Phys. Rev. Lett. 103, 202502 (2009)]
Overall quenching of spectroscopic factors is driven by:

**SRC** $\rightarrow$ ~10%

part-vibr. coupling $\rightarrow$ dominant

"shell-model" $\rightarrow$ in open shell

\[ S_{56}\text{Ni}(r,\omega)\text{[fm}^{-3}\text{MeV]} \]

\[ Z_\alpha = \int d^3r |\psi_{\alpha}^{\text{overlap}}(r)|^2 = \frac{1}{1 - \left| \frac{\partial \Sigma_{\alpha\Omega}(\omega)}{\partial \omega} \right|_{\omega=\varepsilon_\alpha}} \]


---

**with analogous conclusions for \(^{48}\text{Ca}\)**

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</tr>
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Optical Potentials Based on the Nuclear Self-energy

(CB, Jennings, and Waldecker, CB, Dickhoff)

• Proton-$^{16}$O scattering
  [CB, B. Jennings, Phys. Rev. C72, 014613 (2005)]

• Optical model for the $^{40}$Ca chain
Self-Consistent Green's Function Approach

**Faddeev-RPA**

$g^{II}(\omega)$

$\Pi^{(ph)}(\omega)$

pp/hh-RPA; two-nucleon transfer

ph-RPA; nuclear response function, giant/pygmy resonances, Gamow-Teller

optical potential

S(r,\omega)

Dyson Eq.

single-particle motion

CB, Jennings, Nucl. Phys A758, 395c (2005)


Interfaces between structure and reactions...
The irreducible self-energy is a nucleon-nucleus optical potential [see e.g. Mahaux and Sartor, Adv. Nucl. Phys. 20, (1991)]

\[
\Sigma^*(\mathbf{r}, \mathbf{r}'; \varepsilon) = \Sigma^{HF}_{\alpha\beta} - \frac{1}{\pi} \int_{E'}^{\infty} dE' \frac{Im \Sigma^*(\mathbf{r}, \mathbf{r}'; E')}{\varepsilon - E' + i\eta} \\
+ \frac{1}{\pi} \int_{-\infty}^{E'} dE' \frac{Im \Sigma^*(\mathbf{r}, \mathbf{r}'; E')}{\varepsilon - E' - i\eta}
\]

This provides consistent overlaps and scattering wave functions
FRPA self-energy is calculated in h.o. basis and re-expressed in k-space:

\[ \Sigma^*(\omega) = \text{Diagram}\]

**Correlated HF potential (cHF)**

\[
\Sigma_{lj}^{MF,Fadd}(k, k') = \sum_{n_\alpha,n_\beta \in \mathcal{P}} \phi_\alpha(k) \Sigma_{lj;n_\alpha,n_\beta}^{MF,Fadd} \phi_\beta^*(k')
\]

**Non mean-field, part.-vib. coupling, etc...**

\[
\Sigma_{lj}^{(2p1h),Fadd}(k, k') = \sum_{n_\alpha,n_\beta \in \mathcal{P}} \phi_\alpha(k) \left[ \sum_{n+} \frac{(m_\alpha^n)^* m_\beta^n}{\omega - \epsilon_{ij}^{n+} + i\eta} \right] \phi_\beta^*(k')
\]

\[
\Sigma_{lj}^{(2h1p),Fadd}(k, k') = \sum_{n_\alpha,n_\beta \in \mathcal{P}} \phi_\alpha(k) \left[ \sum_{k-} \frac{(m_\alpha^k)^* m_\beta^k}{\omega - \epsilon_{ij}^{k-} - i\eta} \right] \phi_\beta^*(k')
\]

Then one can solve for the scattering and spectroscopic factors

\[
\frac{k^2}{2\mu} \psi(k) + \int_0^\infty dk' k'^2 \left\{ \Sigma^*_{lj}(k, k'; E_{\text{c.m.}}) + V_{\text{Coul.}}(k, k') \right\} \psi(k') = E_{\text{c.m.}} \psi(k)
\]

\[
Z_{lj}^n = \int_0^\infty dk \ k^2 |\psi^n(k)|^2 = \left[ 1 - \langle \tilde{\psi}^n \mid \frac{d \Sigma^*_{lj}}{d\omega} \mid \tilde{\psi}^n \rangle \bigg|_{\omega=E_{\text{c.m.}}^n} \right]^{-1}
\]
Comparison of Calculated Microscopic and DOM Optical Potentials

Comparison to DOM potential, tone through volume integrals:

\[
J^\ell_W(E) = 4\pi \int dr r^2 \int dr' r'^2 \text{Im} \Sigma_0^\ell(r, r'; E)
\]

\[
J^\ell_V(E) = 4\pi \int dr r^2 \int dr' r'^2 \text{Re} \Sigma^\ell_0(r, r'; E)
\]

For the local DOM \( U(r, r') = U(r)\delta(r-r') \):

\[
J^\ell_U = 4\pi \int dr r^2 \int dr' r'^2 U^\ell(r, r') = 4\pi \int U(r) r^2 dr = \int U(r) dr,
\]

for any \( \ell \)

N-A^4Ca scattering calculated with the chiral NN N3LO and AV18 interactions
**Imaginary self-energy/optical pot. for $^{40}\text{Ca}$**

![Graphs showing imaginary volume integrals $J_W$ for $^{40}\text{Ca}$ self-energy for neutrons with $\ell = 0 - 5$.]

$J_W$ gives the overall inelastic absorption.

**FIG. 6.** Imaginary volume integral $J_W^l$ of $^{40}\text{Ca}$ self-energy for neutrons with $\ell = 0 - 5$.

**Dispersive real parts \( J_v \)**

**FRPA**

\[
\begin{array}{c}
\text{FRPA} \\
\begin{array}{c}
\ell = 0 \\
\ell = 1 \\
\ell = 2 \\
\ell = 3 \\
\ell = 4 \\
\ell = 5 \\
\end{array}
\end{array}
\]

Volume Integrals of Re \( \Sigma_0^\ell \) for neutrons in \(^{40}\text{Ca}\).

The horizontal, dashed lines are the volume integrals of \( \Sigma_0^\infty,\ell (E_F) \).

**The OP must dependence on angular momentum!**

\( \rightarrow \) **non locality.**

---

**FRPA**

**DOM (dispersive optical model)**

**Fig. 8.** Angular momentum dependence for the volume Integrals \( J_v = J_v^\ell (E_F) \) of \( \Sigma^\infty,\ell (E_F) \) excluding the contribution of the dynamic part of the self-energy. For each \( \ell \), results for protons are given by solid diamonds and neutrons by solid circles. Proton potentials are considerably less attractive due to the Coulomb energy. When the Coulomb interaction is suppressed (open diamonds) the proton results are close to the neutron results. The results shown are for \(^{40}\text{Ca}\) using the AV18 interaction.
• absorption away from $E_F$ is enhanced by the tensor force

• little effects from charge exchange (e.g. $^{48}$Ca <-> $^{48}$Sc)

$J_w$: integral over the imaginary opt. pot (overall absorption)

N/Z asymmetry dependence of $J_W$ and tensor force contribution

![Graph showing the asymmetry dependence of $J_W$ and tensor force contribution with various isotopes and energy levels.](image)

- Full FRPA (n) and Full FRPA (p) show the absorption for neutrons and protons, respectively.
- Without tensor force (n) and Without tensor force (p) illustrate the effects of removing the tensor component.
- Tensor contribution (n) and Tensor contribution (p) display the impact of the tensor force on the absorption.

The graphs compare different isotopes (e.g., $^{40}\text{Ca}$, $^{48}\text{Ca}$, $^{60}\text{Ca}$) at various energy levels ($E - E_F$) and demonstrate the correlation between nucleon types and energy changes.

Integrating the physics with spectroscopic factors and surface absorption, the graphs highlight the asymmetry's effect on the absorption for neutron and proton interactions.