Radius and Mass Determinations from Neutron Star Observations

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Astrophysical Transients: Multi-messenger Probes of Nuclear Physics
Institute for Nuclear Theory
Outline

- Neutron Star Limits from General Relativity and Causality
- Mass Measurements
  - $2 \, M_\odot$ Neutron Stars?
  - Limits to the Extent of Quark Matter
- Neutron Star Radii
  - Relation to the Nuclear Symmetry Energy
  - Thermal Emission from Cooling Neutron Stars
  - Photospheric Radius Expansion X-Ray Bursters
- The Universal Mass-Radius Relation and the Neutron Star EOS
  - Consistency with Neutron Matter Expectations
  - Implications for Other Laboratory Constraints
Neutron Star Structure

Tolman-Oppenheimer-Volkov equations

\[ \frac{dp}{dr} = -\frac{G}{c^2} \frac{(m + 4\pi pr^3)(\varepsilon + p)}{r(r - 2Gm/c^2)} \]

\[ \frac{dm}{dr} = 4\pi \frac{\varepsilon}{c^2} r^2 \]
The most compact and massive configurations occur when the low-density equation of state is "soft" and the high-density equation of state is "stiff" (Koranda, Stergioulas & Friedman 1997).

$\varepsilon_0$ is the only EOS parameter.

The TOV solutions scale with $\varepsilon_0$

$w = \varepsilon/\varepsilon_0$

$y = p/\varepsilon_0$

$x = r\sqrt{G\varepsilon_0/c^2}$

$z = m\sqrt{G^3\varepsilon_0/c^2}$
Extreme Properties of Neutron Stars

- $M_{\text{max}} = 4.1 \ (\varepsilon_s / \varepsilon_0)^{1/2} \ M_\odot$ (Rhoades & Ruffini 1974)
- $M_{B,\text{max}} = 5.41 \ (m_B c^2 / \mu_0)(\varepsilon_s / \varepsilon_0)^{1/2} \ M_\odot$
- $R_{\text{min}} = 2.82 \ GM/c^2 = 4.3 \ (M/M_\odot) \ \text{km}$
- $\mu_{B,\text{max}} = 2.09 \ \text{GeV}$
- $\varepsilon_{c,\text{max}} = 3.034 \ \varepsilon_0 \simeq 51 \ (M_\odot/M_{\text{largest}})^2 \ \varepsilon_s$
- $p_{c,\text{max}} = 2.034 \ \varepsilon_0 \simeq 34 \ (M_\odot/M_{\text{largest}})^2 \ \varepsilon_s$
- $n_{B,\text{max}} \simeq 38 \ (M_\odot/M_{\text{largest}})^2 \ n_s$
- $E_{\text{BE, max}} = 0.34 \ M$
- $P_{\text{min}} = 0.74 \ (M_\odot/M_{\text{sph}})^{1/2}(R_{\text{sph}}/10 \ \text{km})^{3/2} \ \text{ms} = 0.20 \ (M_{\text{sph, max}}/M_\odot) \ \text{ms}$
Maximum Energy Density in Neutron Stars

\[
p = s(\varepsilon - \varepsilon_0)
\]
Mass-Radius Diagram and Theoretical Constraints

GR:
\[ R > 2\frac{GM}{c^2} \]

\[ P < \infty : \]
\[ R > (9/4)\frac{GM}{c^2} \]

causality:
\[ R \gtrsim 2.9\frac{GM}{c^2} \]

— normal NS
— SQS

\[ R_\infty = \frac{R}{\sqrt{1-2GM/Rc^2}} \] contours

Radius (km)
Black hole? ⇒ Firm lower mass limit? ⇒ $M > 1.68 \, M_\odot$ {95% confidence}

Although simple average mass of w.d. companions is $0.27 \, M_\odot$ larger, weighted average is $0.08 \, M_\odot$ smaller

Freire et al. 2007 {w.d. companion? statistics?}

Demorest et al. 2010

Champion et al. 2008
PSR J1614-2230

3.15 ms pulsar in 8.69d orbit with 0.5 $M_\odot$ white dwarf companion. Shapiro delay tightly confines the edge-on inclination: $\sin i = 0.99984$

Pulsar mass is $1.97 \pm 0.04 \, M_\odot$

Distance $> 1$ kpc, $B \simeq 1.8 \times 10^8 \, G$

Demorest et al. 2010
Black Widow Pulsar PSR B1957+20

1.6ms pulsar in circular 9.17h orbit with a $M_c \sim 0.03 \, M_\odot$ companion. Pulsar is eclipsed for 50-60 minutes each orbit; eclipsing object has a volume much larger than the companion or its Roche lobe. It is believed the companion is ablated by the pulsar leading to mass loss and an eclipsing plasma cloud. Companion nearly fills its Roche lobe. Ablation by pulsar leads to eventual disappearance of companion. The optical light curve does not represent the center of mass of the companion, but the motion of its irradiated hot spot.

![Pulsar radial velocity and eclipse diagram](NASA/CXC/M.Weiss)
Implications of Maximum Masses

\[ M_{\text{max}} > 2 \, M_{\odot} \]

\[ M_{\text{max}} > 2.4 \, M_{\odot} \]

- Upper limits to energy density, pressure and baryon density:
  - \( \varepsilon < 13.1 \varepsilon_s \)
  - \( p < 8.8 \varepsilon_s \)
  - \( n_B < 9.8 n_s \)

- Lower limit to spin period:
  \( P > 0.56 \, \text{ms} \)

- Lower limit to neutron star radius:
  \( R > 8.5 \, \text{km} \)

- Upper limits to energy density, pressure and baryon density in the case of a quark matter core (\( s = 1/3 \)):
  - \( \varepsilon < 7.7 \varepsilon_s \)
  - \( p < 2.0 \varepsilon_s \)
  - \( n_B < 6.9 n_s \)

- Upper limits to energy density, pressure and baryon density:
  - \( \varepsilon < 8.9 \varepsilon_s \)
  - \( p < 5.9 \varepsilon_s \)
  - \( n_B < 6.6 n_s \)

- Lower limit to spin period:
  \( P > 0.68 \, \text{ms} \)

- Lower limit to neutron star radius:
  \( R > 10.4 \, \text{km} \)

- Upper limits to energy density, pressure and baryon density in the case of a quark matter core (\( s = 1/3 \)):
  - \( \varepsilon < 5.2 \varepsilon_s \)
  - \( p < 1.4 \varepsilon_s \)
  - \( n_B < 4.6 n_s \)
Neutron Star Matter Pressure and the Radius

\[ p \simeq Kn^\gamma \]

\[ \gamma = d \ln p / d \ln n \sim 2 \]

\[ R \propto K^{1/(3\gamma-4)} M^{(\gamma-2)/(3\gamma-4)} \]

\[ R \propto p_f^{1/2} n_f^{-1} M^0 \]

\[ (1 < n_f / n_s < 2) \]

Wide variation:

\[ 1.2 < \frac{p(n_s)}{\text{MeV fm}^{-3}} < 7 \]

GR phenomenological result

(Lattimer & Prakash 2001)

\[ R \propto p_f^{1/4} n_f^{-1/2} \]

\[ p_f = n^2 dE_{sym} / dn \]

\[ E_{sym}(n) = E_{neutron}(n) - E_{symmetrical}(n) \]
The Uncertain $E_{\text{sym}}(n)$

The measurement of flux and temperature yields an apparent angular size (pseudo-BB):

\[ \frac{R_\infty}{d} = \frac{R}{d} \frac{1}{\sqrt{1 - \frac{2GM}{Rc^2}}} \]

Observational uncertainties include distance, interstellar H absorption (hard UV and X-rays), atmospheric composition.

Best chances for accurate radii:
- Nearby isolated neutron stars (parallax measurable)
- Quiescent X-ray binaries in globular clusters (reliable distances, low $B$ H-atmospheres)
Inferred M-R Probability Estimates from Thermal Sources

Steiner, Lattimer & Brown 2010
Photospheric Radius Expansion X-Ray Bursts

\[ F_{Edd} = \frac{GMc}{\kappa D^2} \sqrt{1 - \frac{2GM}{R_{ph}c^2}} \]

EXO 1745-248

\[ A = f_c^{-4} \left( \frac{R_{\infty}}{D} \right)^2 \]


Radius and Mass Determinations from Neutron Star Observations
Systematics with $R_{ph} = R$

$$F_{Edd} = \frac{GMc}{\kappa D^2} \sqrt{1 - 2 \frac{GM}{R_{ph} c^2}} = \frac{GMc}{\kappa D^2} \sqrt{1 - 2\beta}$$

$$\kappa \simeq 0.2(1 + X) \text{ cm}^2\text{g}^{-1}$$

$$A = \frac{F_\infty}{\sigma T_\infty^4} = f_c^{-4} \left( \frac{R_\infty}{D} \right)^2$$

$$\alpha = \frac{F_{Edd}}{\sqrt{A}} \frac{\kappa D}{c^3 f_c^2} = \beta (1 - 2 \beta)$$

$$\gamma = \frac{Ac^3 f_c^4}{F_{Edd} \kappa} = \frac{R}{\beta (1 - 2 \beta)^{3/2}}$$

$$\beta = \frac{1}{4} \pm \frac{1}{4} \sqrt{1 - 8\alpha}$$

$$R_\infty = \alpha \gamma$$

If $R_{ph} \gg R$, $\alpha < 1/\sqrt{27} \simeq 0.192$
$M - R$ Probability Estimates from PRE Bursts

EXO 1745-248 $R_{ph} = R$
$\alpha = 0.14 \pm 0.01$

4U 1820-30 $R_{ph} > R$
$\alpha = 0.14 \pm 0.01$

EXO 1745-248 $R_{ph} = R$
$\alpha = 0.18 \pm 0.02$

4U 1820-30 $R_{ph} > R$
$\alpha = 0.18 \pm 0.02$

4U 1608-52 $\alpha = 0.26 \pm 0.10$


4U 1608-52 $\alpha = 0.21 \pm 0.06$

"Steiner, Lattimer & Brown 2010, 2011"

""
Bayesian TOV Inversion

- $\varepsilon < 0.5 \varepsilon_0$: Known crustal EOS
- $0.5 \varepsilon_0 < \varepsilon < \varepsilon_1$: EOS parametrized by $K, K', S_v, \gamma$
- $\varepsilon_1 < \varepsilon < \varepsilon_2$: $n_1$; $\varepsilon > \varepsilon_2$: Polytropic EOS with $n_2$

- EOS parameters $(K, K', S_v, \gamma, \varepsilon_1, n_1, \varepsilon_2, n_2)$ uniformly distributed
- $M$ and $R$ probability distributions for 7 neutron stars treated equally.
Inferred Model EOS Parameters

Steiner, Lattimer & Brown 2010

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Consistency with Neutron Matter and Heavy-Ion Collisions

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Neutron Matter and Mass Fit Symmetry Correlations

Radius and Mass Determinations from Neutron Star Observations

Gravitational Mass [$M_\odot$] vs. Radius [km]

- Hebeler, K. et al.
- Steiner, A. et al.

Graph showing various models with different colors and line styles, indicating different neutron matter symmetry fits.
With More Extreme Assumptions

Strange Quark Stars

$M_{\text{max}} \geq 2.4M_{\odot}$

Larger $f_c$

Fiducial SLB 2010

Skyrme forces with too large symmetry energy

Radius and Mass Determinations from Neutron Star Observations
Radius and Maximum Mass Limits

Hebeler et al. 2010

M_max implied by R of 1.4 M⊙ star.
Neutron Matter and the Symmetry Energy

- Fits to nuclear binding energies result in a strong, nearly linear, correlation between volume and surface symmetry energy coefficients of the liquid droplet model.
- This correlation is dependent on the nature of the liquid droplet model and how it treats the interaction between the Coulomb effects on the nuclear surface, and does not translate directly into a correlation between $S_v$ and $L = 3(dS_v/d\ln n)_n$.
- Finite nucleus models, such as Thomas-Fermi and Hartree or Hartree-Fock, for a particular nuclear interaction, can be fit to binding energies to obtain the correlation between $S_v$ and $L$.
- Neutron matter studies (Hebeler & Schwenk; Carlson et al.) indicate that $E_{\text{sym}}$ and $(dE_{\text{sym}}/d\ln n)_n$ are also correlated.
- Comparing these correlations could constrain the properties of the symmetry energy. It could be dependent on the nature of the nuclear interaction model, but this has not been thoroughly explored.