(CORE COLLAPSE) SUPERNOVA SIMULATIONS
How is the supernova shock wave revived?

The most fundamental question in supernova theory

- Gravity
- Neutrino Heating
- Convection
- **Shock Instability**
- Nuclear Burning
- Rotation
- Magnetic Fields

*New Ingredient*
Components of a Supernova Model

- Neutrino Transport
- Fluid Instabilities
- Rotation
- Magnetic Fields
- Weak Interactions
- Equation of State
- Gravity

Equations To Be Solved

- Boltzmann Kinetic/Moment Equations
- Magnetohydrodynamics Equations
- Poisson or Einstein Equations
Neutrino heating depends on neutrino luminosities, spectra, and angular distributions.

\[ \dot{\epsilon} = \frac{X_n}{\lambda^3} \frac{L_{\nu_e}}{4\pi r^2} \left( E_{\nu_e}^2 \right) \left( \frac{1}{F} \right) + \frac{X_p}{\lambda^3} \frac{L_{\bar{\nu}_e}}{4\pi r^2} \left( E_{\bar{\nu}_e}^2 \right) \left( \frac{1}{F} \right) \]

Neutrino heating is sensitive to all three (most sensitive to neutrino spectra).

⇒ Must compute neutrino distributions.

\[ f(t, r, \theta, \phi, E, \theta_p, \phi_p) \]

\[ E_R(t, r, \theta, \phi, E) = \int d\theta_p \, d\phi_p \, f \]

\[ E_R(t, r, \theta, \phi) = \int dE \, d\theta_p \, d\phi_p \, f \]

Multifrequency

Multiangle

(MGFLD, MGVET)

Gray
The simulation of core collapse supernovae with fully general relativistic, multi-angle, multi-frequency, Boltzmann neutrino transport has been achieved for spherically symmetric cases.

What’s missing?
- Better weak interaction physics?
- Better EOS?
- Neutrino mixing?
- Multi-D effects.

Mezzacappa et al., PRL, 86, 1935 (2001)

Liebendoerfer et al., PRD, 63, 103004 (2001)
Agile-BOLTZTRAN

1D
Boltzmann Neutrino Transport
Exact GR
State-of-the-Art Weak Physics and EOS

CHIMERA

1D/2D/3D
MGFLD
Approximate GR
State-of-the-Art Weak Physics and EOS
Adaptive (fixed-zone-number) radial mesh.

GenASiS

3D
MGVET/Boltzmann Neutrino Transport
MHD
Exact GR (with Singularity Avoidance)
State-of-the-Art Weak Physics and EOS
Cell-by-Cell AMR
<table>
<thead>
<tr>
<th>Publication</th>
<th>Milestone</th>
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(3+1)D awaits! (3+2)D looms!
What can we expect from 3D?

Blondin, Mezzacappa, and DeMarino 2003 *ApJ* 584, 971

Blondin and Mezzacappa 2007 *Nature* 445, 58
1. Geometric Effects
2. Special Relativistic Effects
3. General Relativistic Effects

<table>
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<tr>
<th>Spatial Dimensions</th>
<th>Newtonian or GR</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>Partial Weak Interactions</th>
<th>Complete Weak Interactions</th>
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<tr>
<td>Liebendoerfer et al. (2004)</td>
<td>1</td>
<td>GR</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
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<tr>
<td>Ott et al. (2008)</td>
<td>2</td>
<td>Newtonian</td>
<td>X</td>
<td></td>
<td></td>
<td>X</td>
<td>1D Counterpart: No-Observer-Correctons Newtonian</td>
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Partial Weak Interactions:
(1) No electron capture on nuclei.
(2) Scattering is assumed isotropic and isoenergetic.
### Important Neutrino Emissivities/Opacities

**“Standard” Emissivities/Opacities**

1. \( e^{-(+)} + p(n), A \leftrightarrow \nu_e(\bar{\nu}_e) + n(p), A' \)
   \( e^+ + e^- \leftrightarrow \nu_{e,\mu,\tau} + \bar{\nu}_{e,\mu,\tau} \)

2. \( \nu + n, p, A \rightarrow \nu + n, p, A \)
   \( \nu + e^-, e^+ \rightarrow \nu + e^-, e^+ \)

- \( \star \)

**Notes**

  - Nucleons in nucleus independent.
  - No energy exchange in nucleonic scattering.

  - Include correlations between nucleons in nuclei.

  - Burrows and Sawyer, PRC, 59, 510 (1999)
    - (Small) Energy is exchanged due to nucleon recoil.
    - Many such scatterings.

    - New source of neutrino-antineutrino pairs.

- Janka et al. PRL, 76, 2621 (1996)
Correspondence between structure of integro-PDE and underlying linear systems...

...Leads to Nonlinear Algebraic Equations
• Linearize
• Solve via Multi-D Newton-Raphson Method
⇒ Solve Exascale Sparse Linear Systems

Implicit Time Differencing...
• Extremely Short Neutrino-Matter Coupling Time Scales
• Neutrino-Matter Equilibration
• Neutrino Transport Time Scales

Conservative numerical formulation of these terms is challenging.
Lentz et al. (2010), in preparation.
Comparison of 1D Simulations; 15 W-H Progenitor

Shock Radii vs Post Bounce Time

Shock Radius [km]

Post Bounce Time [s]
Supernova Model

MHD
- Explicit Updates w/ AMR
  - Hydrodynamics/MHD
    - Finite Volume Methods
      - Hydrodynamics (PPM)
      - MHD (Central Scheme)
  - Gravity
    - Poisson Solve (Newtonian)
    - Einstein Solve (General Relativity)

Gravity

Radiation
- Implicit Updates w/ AMR
  - Sparse Solves
    - Local (to processor): Nuclear Burning
      - GPU Suitable
    - Distributed (across processors): Radiation
  - Physics-Based Preconditioners
  - Jacobian-Based Krylov Subspace Methods

Nuclear Burning

Libraries
The Need for Exascale Resources

Dominated by preconditioning of dense blocks.

FLOPS $\sim N_t N_s N_i f N_m^2 \sim 3.5 \times 10^{22} f$

$N_t$ = number of time steps $\sim 1 \times 10^6$
$N_s$ = number of spatial zones $\sim 512 \times 512 \times 512$
$N_i$ = number of iterations per time step $\sim 10$
$N_m$ = number of neutrino momentum zones
$f \in [1, N_m] = [1, 5120]$

$N_m = N_v \times N_E \times N_p \times N_a$

$N_v = 4$
$N_E$ = number of neutrino energy groups $\sim 20$
$N_p$ = number of neutrino polar direction angles $\sim 8$
$N_a$ = number of neutrino azimuthal direction angles $\sim 8$

Runtime: $\sim 4f$ days per run on a 1 EF machine (at 10% of peak).

Algorithms critical!

Memory Footprint: 27 PB
The Need for Exascale Resources

Dominated by preconditioning of dense blocks.

FLOPS \sim N_t N_s N_i f N_m^2 \sim 3.4 \times 10^{19} f

\( N_t \) = number of time steps \( \sim 1 \times 10^6 \)
\( N_s \) = number of spatial zones \( \sim 512 \times 512 \times 512 \)
\( N_i \) = number of iterations per time step \( \sim 10 \)
\( N_m \) = number of neutrino momentum zones
\( f \in [1,N_m] = [1,160] \)

\( N_m = N_v \times N_E \)

\( N_v = 4 \times 2 \)
\( N_E \) = number of neutrino energy groups \( \sim 20 \)

Runtime: \( \sim f \) hours per run on a 1 EF machine (at 10% of peak).

Memory Footprint: 27 TB
Recent (2+1)D models show promise. We are on track.

- Wilson delayed shock mechanism plus SASI yields explosions in models performed by several groups beginning with a range of progenitor masses.

A great deal remains to be done in (2+1)D and (2+3)D.

- There are presently very few published explosion models, and even they are incomplete.

Approximate (3+1)D models with multi-frequency neutrino transport will emerge soon.

(3+1)D models with GR, full weak interaction physics, and complete transport physics must be performed.

- Such simulations will require exascale resources.

Definitive (3+3)D simulations will likely require sustained exascale platforms.
• Solvers
  o Communication optimal, reduce data movement, multi-core aware, fault tolerant, ...

• AMR (See Martin Berzins’ talk.)
  o Will geometric (e.g., space-filling curve) approaches scale?
  o Are task-based approaches a viable alternative?
     ➢ All about the scheduling.

• Programming Models (See Brad Chamberlain’s talk.)
  o Maximize the use of extant code.
  o Facilitate a task-based approach.

• Compilers (See Rich Graham’s talk.)
  o Translation support for Chapel/MPI/OpenMP/Fortran/… with user hints to specify task dependencies.

• Debuggers (See Rich Graham’s talk.)
  o Debugging at $10^5$-$10^6$ cores.

• Data Analytics (See Scott Klasky’s talk.)
  o High-performance I/O.
  o In situ and post-processed visualization of multi-D scalar, vector, and tensor data.
Multi-institution, multi-investigator, multi-disciplinary effort.

Applied Math/CS Collaborators
- Closures, Preconditioners/Solvers: Hauck, D’Azevedo
- I/O, Data Management: Klasky, Shipman
- Compilers/Debuggers: Graham
- Visualization: Ahern, Meredith, Pugmire, Toedte
- Cray Center of Excellence: Wichmann

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