Spatially Modulated Interaction and Dipolar Interaction Induced Resonances

Hui Zhai
Institute for Advanced Study
Tsinghua University
Beijing, China

INT Seattle
Fermion Workshop
2011
Three major recent topics:

1. Quantum gases in synthetic gauge field (with spin-orbit coupling)
2. New strongly interacting systems in quantum gases
3. Magnetism in quantum gases
Main Group Members

Ran Qi  Zeng-Qiang Yu  Xiaoling Cui

Chao Gao  Zheyu Shi  Chao-Ming Jian
Feshbach resonance is an important tool to achieve strong interactions in ultracold Fermi gases.
New Way to achieve scattering resonance in New cold atom systems with Novel features

A: Spatially modulated interaction induced resonance
   Alkali-earth-(like) atomic gases

B: Strong dipolar interaction induced resonance
   Polar molecular gases
New Way to achieve scattering resonance in New systems with New features

A: Spatially modulated interaction induced resonance

Alkali-earth-(like) atomic gases
Idea of Optical Feshbach resonance

\[ E = \sin(Kx) \] (1)

\[ V(r_1, r_2) = V(r_1 - r_2) \] (2)

\[ V(r_1, r_2) = V(r_1 - r_2, r_1 + r_2^2) \] (3)

\[ E = \int dx \left[ \frac{\hbar^2}{2m} \phi^* \nabla^2 \phi + \frac{4\pi \hbar^2}{m} a_{\text{loc}}(x) n^2(x) \right] \] (4)

\[ \lim_{r \to 0} \psi(r, x) = \frac{1}{r} - \frac{1}{a_{\text{loc}}(x)} \] (5)

\[ K a_{\text{eff}} \gg 1 \] (6)

\[ H = -\frac{\hbar^2}{4m} \nabla^2_R - \frac{\hbar^2}{m} \nabla^2_r + v(r) \] (7)

\[ v(r) = \begin{pmatrix} -V_0 & \hbar \Omega \\ \hbar \Omega & -V_c \end{pmatrix} \] (8)
Optical Feshbach resonance

\[ \mathcal{H} = -\frac{\hbar^2}{4m} \nabla^2_{\mathbf{R}} - \frac{\hbar^2}{m} \nabla^2_{\mathbf{r}} + v(\mathbf{r}) \]

\[ v(\mathbf{r}) = \begin{pmatrix} -V_0 & \hbar \Omega \\ \hbar \Omega & -V_c \end{pmatrix} \]

\[ a_s = a_{bg} \left( 1 - \frac{\Omega^2}{\Omega^2 - \Omega_0^2} \right) \]

Optical Feshbach resonance with Standing wave
Spatial dependent interaction

Two-body interaction potential:

\[ V(r_1, r_2) = V(r_1 - r_2) \]

Spatial independent
Spatial dependent interaction

Two-body interaction potential:

\[ V(r_1, r_2) = V \left( r_1 - r_2, \frac{r_1 + r_2}{2} \right) \]

Spatially periodically modulated

\[ \mathcal{H} = -\frac{\hbar^2}{4m} \nabla_R^2 - \frac{\hbar^2}{m} \nabla_r^2 + v(r, R) \]

\[ v(r) = \begin{pmatrix} -V_0 & \hbar\Omega(R) \\ \hbar\Omega(R) & -V_c \end{pmatrix} \]

\[ \Omega(R) = \Omega \cos(Kx) \]

\( a_s(x) \) is spatially dependent and modulates periodically in space
Experimental Realization

How $a_s(x)$ modulates in space?

$$a_s = a_{bg} \left(1 - \frac{\Omega^2}{\Omega^2 - \Omega_0^2}\right) \quad \Omega(R) = \Omega \cos(Kx)$$

$$a_s(x) = a_{bg} \left(1 - \frac{\Omega^2 \cos^2(Kx)}{\Omega^2 \cos^2(Kx) - \Omega_0^2}\right)$$

Any other physics effects?
What we have done:
Solve two-body problem of this Hamiltonian

\[ \mathcal{H} = -\frac{\hbar^2}{4m} \nabla_R^2 - \frac{\hbar^2}{m} \nabla_r^2 + \nu(r) \]

\[ \nu(r) = \begin{pmatrix} -V_0 & \hbar \Omega(R) \\ \hbar \Omega(R) & -V_c \end{pmatrix} \]

\[ \Omega(R) = \Omega \cos(Kx) \]

Qi Ran and HZ, arXiv: 1101.4464
Results I: Bound States

![Graph showing bound states](image-url)
Results II: Scattering Resonances

\[ a_{\text{eff}} = \lim_{k \to 0} \frac{\tan \delta(k)}{k} \]

Strongly interacting many-body system !!
Universal Behavior ?
Results III: Local Scattering Length
--- related to local interaction energy

Bethe-Peierls condition:

$$\lim_{r \to 0} \psi(r, x) = \frac{1}{r} - \frac{1}{a_{loc}(x)}$$

Local scattering length

$$a_{loc}(x) = -\lim_{r \to r_0} \frac{r\psi_0(x, r)}{\partial_r(r\psi_0(x, r))}$$

The mean-field energy for a BEC:

$$\mathcal{E} = \int dx \left[ -\frac{\hbar^2}{2m} \varphi^* \nabla^2 \varphi + \frac{4\pi \hbar^2}{m} a_{loc}(x)n^2(x) \right]$$
Results III: Local Scattering Length

Exact formula:

\[ a_{\text{loc}}(x) = \frac{1 - \sum_{m \neq 0} U_m \cos(mKx)/U_0}{\frac{1}{a_{\text{eff}}} - \sum_{m \neq 0} U_m |m|K \cos(mKx)/(2U_0)} \]

Simplified formula

- \( Ka_{\text{eff}} \ll 1 \):
  \[ a_{\text{loc}}(x) = a_{\text{eff}} \left[ 1 - \frac{2U_2}{U_0} \cos(2Kx) \right] \]

- \( Ka_{\text{eff}} \gg 1 \):
  \[ a_{\text{loc}}(x) = \frac{1}{K} \left[ 1 - \frac{U_0}{2U_2 \cos(2Kx)} \right] \]
Results III: Local Scattering Length

\[ a_s(x)/a_{bg} \]

(a) 

(b) 

(c) 

(d)
### Summary: Take Home Message

<table>
<thead>
<tr>
<th>New Mechanism</th>
<th>New System</th>
<th>New Features</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two-body interaction potential has center-of-mass dependence</td>
<td>Alkali-earth-(like) atomic gases: Sr, Ca, Yb</td>
<td>Spatially dependent local scattering length</td>
</tr>
</tbody>
</table>
New Way to achieve scattering resonance in New systems with New features

B: Strong dipolar interaction induced resonace

Polar molecular gases
Polar molecular gases

large dipole moment: $d$

Hard to cool it directly !!
Polar molecular gases

Difficulties:
1. Large energy detuning: 10-100 THz
2. Small transition matrix element
Polar molecular gases

Feshbach molecule

Ground state molecule

STImulated Raman Adiabatic Passage

D. S. Jin and Jun Ye’s group
Polar molecular gases

KRb+KRb $\rightarrow$ K2+Rb2  Chemically unstable !!

<table>
<thead>
<tr>
<th></th>
<th>Na</th>
<th>K</th>
<th>Rb</th>
<th>Cs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Li</td>
<td>$-328(2)$</td>
<td>$-533.9(3)$</td>
<td>$-618(200)$</td>
<td>$-415.38(2)$</td>
</tr>
<tr>
<td>Na</td>
<td>74.3(3)</td>
<td>45.5(5)</td>
<td>236.75(20)</td>
<td></td>
</tr>
<tr>
<td>K</td>
<td></td>
<td>$-8.7(9)$</td>
<td>37.81(13)</td>
<td>29.1(1.5)</td>
</tr>
<tr>
<td>Rb</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Zuchowski and Hutson, PRA (2010)

\[ E_D \sim \frac{D}{\langle r \rangle^3} \sim k_F^3 D \]
\[ E_F \sim k_F^2 \]

\[ k_F D \sim 1 \]
\[ E_D \sim E_F \]

Dipole moments of vibrational ground state: $\nu=0$

<table>
<thead>
<tr>
<th>Mixture</th>
<th>[Debye]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Li-Na</td>
<td>0.56</td>
</tr>
<tr>
<td>Li-K</td>
<td>3.6</td>
</tr>
<tr>
<td>Li-Rb</td>
<td>4.2</td>
</tr>
<tr>
<td>Li-Cs</td>
<td>5.5</td>
</tr>
<tr>
<td>Na-K</td>
<td>2.8</td>
</tr>
<tr>
<td>Na-Rb</td>
<td>3.3</td>
</tr>
<tr>
<td>Na-Cs</td>
<td>4.6</td>
</tr>
<tr>
<td>K-Rb</td>
<td>0.6</td>
</tr>
<tr>
<td>K-Cs</td>
<td>1.9</td>
</tr>
<tr>
<td>Rb-Cs</td>
<td>1.2</td>
</tr>
</tbody>
</table>

\[ \text{M. Aymar and O. Dulieu, J. Chem.Phys., 122, 204302 (2005)} \]
s-wave resonances with strong dipolar interactions

Kanjilal and Blume PRA (2008)

\[
V_D = \frac{D(1 - 3 \cos^2 \theta)}{r^3}
\]

\[
\langle Y_{00} | V_D | Y_{00} \rangle = 0
\]
s-wave resonances with strong dipolar interactions

\[ \langle Y_{20}|V_D|Y_{00} \rangle \neq 0 \]

\[
V_{\text{eff}} = - \frac{\langle Y_{00}|V_D|Y_{20} \rangle \langle Y_{20}|V_D|Y_{00} \rangle}{l(l+1)/r^2} \propto - \frac{D^2}{r^4}
\]

Kanjilal and Blume PRA (2008)
How well can we tune a (positive) effective range in cold atoms? -- from the wiki

\[ k_F r_{\text{eff}} \ll 1 \]

\[ k_F r_{\text{eff}} \gg 1 \]

\[ r_{\text{eff}} < 0 \]

\[ r_{\text{eff}} > 0 \]

Narrow resonance!

Controlled by D, D can be tuned by electronic field
Positive effective range from a dipole induced resonance

\[
\frac{k}{\tan \delta(k)} = -\frac{1}{a_s} + \frac{1}{2} r_{\text{eff}} k^2
\]
What is the physical effect of effective range being positive or negative?

**High temperature regime:**

\[ b_2 = \int_0^{+\infty} \frac{dk}{\pi} \frac{d\delta(k)}{dk} e^{-\lambda^2 k^2/(2\pi)} \]

\[ E_{\text{int}} = \frac{3k_B T n}{2} (n \lambda^3) \left[ -\frac{b_2}{\sqrt{2}} + \frac{\sqrt{2}}{3} T \frac{\partial b_2}{\partial T} \right] \]
What is the physical effect of effective range being positive or negative?

\[-\frac{1}{a_{\text{avg}}} = -\frac{1}{a_s} + \frac{1}{2} r_{\text{eff}} \langle k^2 \rangle \]

| $a_s < 0$ | $r_{\text{eff}} < 0$ | $a_{\text{avg}} < a_s$ |
| $a_s < 0$ | $r_{\text{eff}} > 0$ | $a_s < a_{\text{avg}}$ |
| $a_s > 0$ | $r_{\text{eff}} < 0$ | $a_{\text{avg}} < a_s$ |
| $a_s > 0$ | $r_{\text{eff}} > 0$ | $a_s < a_{\text{avg}}$ |

$k_F r_{\text{eff}} > 0$

$k_F r_{\text{eff}} \sim 0$

$k_F r_{\text{eff}} < 0$
Summary: Take Home Message

<table>
<thead>
<tr>
<th>New Mechanism</th>
<th>New System</th>
<th>New Features</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two-body interaction potential has center-of-mass</td>
<td>Alkali-earth-(like) atomic gases: Sr,</td>
<td>Spatially dependent local scattering</td>
</tr>
<tr>
<td>of-mass dependence</td>
<td>Ca, Yb</td>
<td>length</td>
</tr>
<tr>
<td>Strong long range dipolar interactions</td>
<td>Polar molecules</td>
<td>positive and sizable effective range</td>
</tr>
</tbody>
</table>

Thank you very much for your attention!