The extent of universal physics in three-body collisions

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The extent of universal physics in three-body collisions

Outline

1. Introduction
   - Efimov effect in ultracold atomic gases
   - Theoretical Framework

2. Efimov physics near narrow Feshbach resonances

3. Efimov physics at finite energy
   - $a > 0$
   - $a < 0$

4. Summary & Outlook
In this section

1. Introduction
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   - \( a > 0 \)
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4. Summary & Outlook
Three-body Efimov effect

An infinite series of three-body bound states with
\[ E_n = E_0 e^{-2n\pi/s_0} \]
when two-body scattering length \( a \to \infty \) \((s_0 \approx 1.00624)\)

Efimov spectrum

Three-body Efimov effect leads to universal scattering length scaling in the three-body scattering processes which can be observed in ultracold atomic gases.
Universal three-body recombination rates:

\[
K_3^{(a<0)} = \frac{4590 \sinh 2\eta}{\sin^2 [s_0 \ln(|a|/r_0) + \Phi + 1.53] + \sinh^2 \eta} \frac{\hbar}{m a^4} (B + B + B \rightarrow B_2^* + B)
\]

\[
K_3^{(a>0)} = 67.1 e^{-2\eta} \left( \sin^2 [s_0 \ln \frac{a}{r_0} + \Phi] + \sinh^2 \eta \right) \frac{\hbar}{m a^4} (B + B + B \rightarrow B_2 + B)
\]

Theory

Experiment

Efimov effect in ultracold quantum gases

Universal three-body vibrational relaxation rates:

\[ V_{\text{rel}}^{(a>0)} = \frac{20.1 \sinh 2\eta}{\sin^2 [s_0 \ln(a/r_0) + \Phi + 1.47]} + \sinh^2 \eta \frac{\hbar}{m} (B_2^* + B \rightarrow B_2 + B) \]

Theory

Experiment

Smith-Whitten hyperspherical coordinates

- Hyperradius represents the overall size of the three-body system $R = \sqrt{\rho_1^2 + \rho_2^2}$
- Hyperangle $\theta$ characterizes the geometry of three-body triangle
- Hyperangle $\phi$ characterizes the permutations.

Three-body Schrödinger equation in the hyperspherical coordinates:

$$
\left[ -\frac{\hbar^2}{2\mu} \frac{\partial^2}{\partial R^2} + \frac{(\Lambda^2 + \frac{15}{4})\hbar^2}{2\mu R^2} + V_{12} + V_{23} + V_{31} \right] \psi_E = E \psi_E
$$
Adiabatic hyperspherical representations

Slow hyperradial motion ⇒ Choice of adiabatic representation:
- Intuitive picture and qualitative analysis from the potentials
- Quick convergence for numerical calculations

Generating adiabatic basis \( \Phi_\nu(R; \Omega) \) by treating \( R \) as a parameter:

\[
\begin{align*}
\left[ \frac{(\Lambda^2 + \frac{15}{4})\hbar^2}{2\mu R^2} + V_{12} + V_{23} + V_{31} \right] \Phi_\nu(R; \Omega) &= U_\nu(R) \Phi_\nu(R; \Omega)
\end{align*}
\]

Schrödinger equation reduces to coupled radial equations:

\[
\begin{align*}
\left[ -\frac{\hbar^2}{2\mu} \frac{\partial^2}{\partial R^2} + U_\nu(R) \right] F_{\nu E}(R) - \frac{\hbar^2}{2\mu} \sum_{\nu'} W_{\nu\nu'}(R) F_{\nu' E}(R) &= EF_{\nu E}(R)
\end{align*}
\]
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Solving three-body problems in the adiabatic hyperspherical representation

Scaling in the hyperradial wavefunction $\Rightarrow$ Analytic expressions for the inelastic rates

Matching $F(R)$ at:
- $R \sim r_0$
- $R \sim |a|$

$\Rightarrow$ Transmission coefficient $T$ into deeper channels

$K_3 \propto T$
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Solving three-body problems in the adiabatic hyperspherical representation

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\( \Rightarrow \) Transmission coefficient \( T \) into deeper channels

\[ K_3 \propto T \]

Ab-initio calculations:

- Solve the hyperangular equation in B-spline basis
- Solve the hyperradial equation as a standard multichannel scattering problem
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Resonance width $\Gamma$ connects effective range expansion:

$$k \cot(\delta) \simeq -\frac{1}{a} + \frac{1}{2} r_{\text{eff}} k^2 \quad \text{by} \quad r_{\text{eff}} \sim -\frac{1}{\Gamma}$$


For small $|r_{\text{eff}}|$ (broad resonance), corrections from $r_{\text{eff}}$ have been studied perturbatively:

Platter, Ji, and Phillips, PRA (2009);
Hammer, Länder, and Platter, PRA (2007);
Bratten and Hammer, PRA (2003).
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Modeling two-body Feshbach resonance

Two-channel Feshbach resonance $\Rightarrow$ One-channel shape resonance

Two-body potentials

Long-range two-body scattering wavefunctions can be made identical.
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Scaling of the adiabatic potentials

\[ \frac{c_0}{|r_{\text{eff}}| R} \]

\[ (U_v + \frac{\hbar^2}{2\mu}) 2\mu \text{(a.u.)} \]

\[ \frac{c_0}{|r_{\text{eff}}| R} \]

Adiabatic hyperspherical potentials \((a=\infty, r_{\text{eff}}=-5 \times 10^3)\)

- \[ \sim \frac{\hbar}{2\mu} \frac{c_0}{|r_{\text{eff}}| R} \text{ Coulomb-like } (\alpha|r_{\text{eff}}| \ll R \ll \beta a) \]
- Non-universal “effective charge” \(c_0/|r_{\text{eff}}|\)
- Universal rates

Scaling of the adiabatic potentials

Adiabatic hyperspherical potentials

\( (a=\infty, \, r_{\text{eff}}=-5 \times 10^3) \)

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Scaling for the three-body relaxation rates—bosons

Adiabatic hyperspherical potentials ($\beta a \gg \alpha |r_{\text{eff}}| \gg r_0$)
Scaling for the three-body relaxation rates–bosons

Adiabatic hyperspherical potentials \((\beta a \gg \alpha |r_{\text{eff}}| \gg r_0)\)

\[ V_{\text{rel}} \propto T \]

\[ V_{\text{rel}} = \frac{2\sqrt{3}\pi\beta \sin 2\varphi_0 \sinh 2\eta}{\sin^2[s_0 \ln(|a/r_{\text{eff}}|) + \Phi + \varphi] + \sinh^2\eta} \frac{\hbar}{ma} \]

where

\[ \tan \Phi = 2s_0 \left( \alpha - \text{Re}A/|r_{\text{eff}}| \right) / \left( \alpha + \text{Re}A/|r_{\text{eff}}| \right), \]

\[ \sinh \eta = |\text{Im}A/\alpha r_{\text{eff}}| \csc(2\varphi_0) \sin^2(\Phi + \varphi_0), \]

\[ \tan \varphi_0 = 2s_0. \]

Non-trivial short-range physics!

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Scaling for the three-body relaxation rates

- $1/|r_{\text{eff}}|$ suppression for $B_2^* + B \rightarrow B_2 + B$
- $|r_{\text{eff}}|^{3.33}$ enhancement for $(FF')^* + F \rightarrow FF' + F$

Implications in ultracold experiments

- Increased collisional stability for dimers of bosonic atoms
- Reduced collisional stability for dimers of fermionic atoms

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Finite energy recombination for $a > 0$ ($J = 0^+$)

Recombination pathways at finite energy

when $\frac{1}{a} \ll k \ll \frac{1}{r_0}$,

interference between the two pathways gives:

$$K_3 \propto \frac{1}{k^4} \sin^2(-s_0 \ln(kr_0) + \Phi)$$

Connection to zero-energy behavior

At finite energy:

\[ K_3 \propto \frac{1}{k^4} \sin^2(-s_0 \ln(kr_0) + \Phi) \]

when \( \frac{1}{a} \ll k \ll \frac{1}{r_0} \).

Near zero-energy:

\[ K_3 \propto a^4 \sin^2(s_0 \ln(a/r_0) + \Phi') \]

when \( k \lesssim \frac{1}{a} \).

The oscillations are connected.

\[ K_3(E, a)E^2/\alpha^4 \]

Oscillations in different systems

BBB vs. BBX ($m_X/m_B < 1$)
- Smaller oscillatory period for BBX
- Bigger amplitude for BBX

Experimental possible: CsCsLi

\[ K_3(E)E^2 \text{ for CsCsCs and CsCsLi} \]

- CsCsCs
  - $a_{\text{CsCs}} = 8000$ a.u.
  - Oscillatory period ≈ 515

- CsCsLi
  - $a_{\text{CsLi}} = 5000$ a.u.
  - Oscillatory period ≈ 30

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Oscillations and the Efimov states

- The energies of the Efimov states $E_n$ are correlated with the oscillations.
- The number of the Efimov states can be counted from the oscillations.

$K_3(E)E^2$ for CsCsLi

$E_{2b}$

$E_0 = \frac{\hbar^2}{2\mu r_0^2}$
Smooth behavior of $K_3^{J>0}$ does not change the oscillatory structures.

Thermal average blurs the oscillations dramatically, but the structure can still be seen.

Bypassing thermal effect–BEC collision

- Advantage: Collisional energy is well-defined and is tunable.
- Question: How does condensate dynamics change the loss?

Solve coupled mean-field equation with loss terms:

\[
i \frac{\partial}{\partial t} \phi_{Cs}(r, t) = \left( \hat{h}_{Cs} + a_{CsCs}|\phi_{Cs}|^2 + a_{CsLi}|\phi_{Li}|^2 \right) \phi_{Cs} \\
- i \left( 3K_{CsCsCs}|\phi_{Cs}|^4 + 2K_{CsCsLi}|\phi_{Cs}|^2|\phi_{Li}|^2 + K_{CsLiLi}|\phi_{Li}|^4 \right) \phi_{Cs}
\]

\[
i \frac{\partial}{\partial t} \phi_{Li}(r, t) = \left( \hat{h}_{Li} + a_{LiLi}|\phi_{Li}|^2 + a_{CsLi}|\phi_{Cs}|^2 \right) \phi_{Li} \\
- i \left( 3K_{LiLiLi}|\phi_{Li}|^4 + K_{CsCsLi}|\phi_{Cs}|^4 + 2K_{CsLiLi}|\phi_{Cs}|^2|\phi_{Li}|^2 \right) \phi_{Li}
\]
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Efimov signatures in the loss

The losses for Cs and Li are 2:1
⇒ The CsCsLi recombination is the dominant loss process.

BEC collision at $E = 16 \mu K (a_{CsLi} = 5 \times 10^3$ Bohr)

The Efimov signature remains in the final losses.

Energy-dependent atomic losses

(Collisional time $\propto E^{\alpha/2}$, where $\alpha$ is in $[0, 1]$.)

Efimov oscillations in recombination for $a < 0$ at finite energy ($J = 0^+$)

Recombination pathways at finite energy

when $\frac{1}{a} \ll k \ll \frac{1}{r_0}$,

$$K_3 = \frac{\frac{384\sqrt{3}\pi^2}{mk^4}}{\cosh(\pi s_0) + \sin[-2s_0 \ln(kr_0) + 2\Phi - 2\varphi_0]} \sinh(\pi s_0) \sinh(2\eta)$$

where

$$\tan \varphi_0 = \frac{\text{Re}[\Gamma(is_0)] - \text{Im}[\Gamma(is_0)]}{\text{Re}[\Gamma(is_0)] + \text{Im}[\Gamma(is_0)]}$$
Oscillations in different systems

Three-body recombination rates for BBB and BBX

- Smaller $m_X/m_B$, smaller the oscillation period, smaller modulation.
- Thermal averaging has small effect.

Higher partial wave contributions are suppressed!
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Summary

- We use adiabatic hyperspherical representation to perform analytical and numerical studies on Efimov physics.

- In ultracold atomic systems, the Efimov physics is studied by the scattering processes (recombination, relaxation).

- The Efimov physics extends from broad to narrow Feshbach resonances and from zero-energy regime to finite-energy domain.
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