New developments in few-nucleon scattering

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INFN, Sezione di Pisa

INT workshop
“Weakly Bound Systems in Atomic and Nuclear Physics”
March 8, 2010
Outline

1. Introduction

2. Scattering calculations with the HH method ($A \leq 4$)

3. Integral relations

4. Conclusion
Collaborators

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Weakly bound systems in nuclear physics

“Few-nucleon Physics”:

- $A \leq 4$:
  - NN/3N forces & Weak/EM currents by EFT
  - Solution of $H\psi = E\psi$: quite accurate
  - First part of the talk: “recent results using the HH method”

- Problems:
  - $A_y$ “puzzle” in $N - d$ and $p - ^3\text{He}$ (see later)
  - Several problems in $N - d$ breakup and $A = 4$ scattering
  - Goal: NN/3N/4N observables $\leftrightarrow$ EFT

Weakly bound systems: $A > 4$:

- Extension of the numerical methods very difficult
- Many of these systems are unbound $\leftrightarrow$ need for scattering calculations
- Second part of the talk: “Integral Relations”
Weakly bound systems in nuclear physics

"Few-nucleon Physics":

- $A \leq 4$:
  - NN/3N forces & Weak/EM currents by EFT
  - Solution of $H\Psi = E\Psi$: quite accurate
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Weakly bound systems: $A > 4$:

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- Many of these systems are unbound $\leftrightarrow$ need for scattering calculations
- Second part of the talk: “Integral Relations”
NN & 3N interaction

NN potentials

- "Old models": Argonne V18, CD-Bonn, Nijmegen ($\chi^2 \approx 1$)
- Fit of 3N data using non-locality in P-waves (ISuj [Doleschall, 2008])
- Effective field theory
  - J-N3LO [Epelbaum and Coll, 1998-2006]
  - I-N3LO [Entem & Machleidt, 2003]
- Low-q interaction [Bogner and Coll., 2001-2007]

3N potentials

- Effective field theory
  - J-N2LO [Epelbaum et al, 2002]
  - N-N2LO [Navratil, 2007]
- Illinois [Pieper et al, 2001]
- Under progress: N3LO, $\Delta$, CSB, ...

Models under study: I-N3LO, AV18, I-N3LO/N-N2LO, AV18/UIX.
Study of the dynamics of 3 and 4 nucleon systems

\[ H = \sum_i \frac{p_i^2}{2M} + \sum_{i<j} V(i,j) + \sum_{i<j<k} W(i,j,k) + \ldots \]

Search for accurate solution of \( H\psi = E\psi \)

- Expansion of \( \psi \) on the basis of **Hyperspherical Harmonics**
- Problems: 1) convergence 2) antisymmetrization of the basis 3) boundary conditions for scattering states, \ldots
- **Accurate, state-of-the-art**, calculations of bound and elastic observables
- Treatment of non-local or projecting potential possible
- Hard-core potential \( \rightarrow \) inclusion of a correlation factor
- Still to be solved: proper treatment of breakup channels \((N + d \rightarrow N + n + p)\)

The HH method

HH functions

- hyperradius $\rho^2 = \frac{2}{A} \sum_{i<j} r_{ij}^2$
- hyperangles $\Omega = \{ \xi_1, \ldots, \xi_{A-1} \} \ (\xi_i \text{ Jacobi vectors})$
- $T = T_\rho + T_\Omega$
- The HH functions $\mathcal{Y}_{[K]}(\Omega)$ are the eigenstates of $T_\Omega$

$$\Phi_n = L_n^{(3A-4)}(\beta \rho) e^{-\beta \rho/2} \mathcal{Y}_{[K]}(\Omega)$$

Advantages

Simplified calculation of the matrix elements of

- local/non-local NN & 3N potentials
- coordinate/momentum space interaction
The HH method

**HH functions**

- hyperradius $\rho^2 = \frac{2}{A} \sum_{i<j} r_{ij}^2$
- hyperangles $\Omega = \{ \frac{\xi_1}{\rho}, \ldots, \frac{\xi_{A-1}}{\rho} \}$ (\(\xi\), Jacobi vectors)
- $\mathcal{T} = \mathcal{T}_\rho + \mathcal{T}_\Omega$
- The HH functions $\Phi_{[K]}(\Omega)$ are the eigenstates of $\mathcal{T}_\Omega$

$$
\Phi_n = L_n^{(3A-4)}(\beta \rho) e^{-\beta \rho/2} \mathcal{Y}_{[K]}(\Omega)
$$

**Advantages**

Simplified calculation of the matrix elements of

- local/non-local NN & 3N potentials
- coordinate/momentum space interaction
Scattering calculation

Example: $A - B$ elastic scattering

\[
\Omega_{LS}^F(A, B) = \sqrt{\frac{1}{N}} D_{AB} \sum_{\text{perm.}=1}^N \left[ Y_L(\hat{r}_{AB})[\phi_A \phi_B]_S \right]_{JJ_z} \frac{F_L(\eta, q_{AB}r_{AB})}{q_{AB}r_{AB}}
\]

\[
\Omega_{LS}^G(A, B) = \sqrt{\frac{1}{N}} D_{AB} \sum_{\text{perm.}=1}^N \left[ Y_L(\hat{r}_{AB})[\phi_A \phi_B]_S \right]_{JJ_z} \frac{G_L(\eta, q_{AB}r_{AB})}{q_{AB}r_{AB}} (1 - e^{-\gamma r_{AB}})^{2L+1}
\]

\[
\Omega_{LS}^\pm(A, B) = \Omega_{LS}^G(A, B) \pm i\Omega_{LS}^F(A, B)
\]

\[
|\Psi_{LS}\rangle = \sum_n a_{LS,n} \phi_n + |\Omega_{LS}^F(p, ^3\text{He})\rangle + \sum_{L'S'} T_{LS,L'S'} |\Omega_{L'S'}^+(p, ^3\text{He})\rangle
\]

- $T_{LS,L'S'}$ = T-matrix elements
- $a_{LS,n}$ and $T_{LS,L'S'}$ determined using the Kohn variational principle (KVP)
Scattering calculation

Example: $A - B$ elastic scattering

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\Omega^{F}_{LS}(A, B) = \sqrt{\frac{1}{N}} D_{AB} \sum_{\text{perm.}=1}^{N} \left[ Y_L(\hat{r}_{AB})[\phi_A \phi_B]_S \right]_{JJ_z} F_L(\eta, q_{AB}r_{AB}) \frac{1}{q_{AB}r_{AB}} 
\]

\[
\Omega^{G}_{LS}(A, B) = \sqrt{\frac{1}{N}} D_{AB} \sum_{\text{perm.}=1}^{N} \left[ Y_L(\hat{r}_{AB})[\phi_A \phi_B]_S \right]_{JJ_z} G_L(\eta, q_{AB}r_{AB}) (1 - e^{-\gamma r_{AB}})^{2L+1} \frac{1}{q_{AB}r_{AB}} 
\]

\[
\Omega^{\pm}_{LS}(A, B) = \Omega^{G}_{LS}(A, B) \pm i\Omega^{F}_{LS}(A, B) 
\]

\[
|\Psi_{LS}\rangle = \sum_{n} a_{LS,n} \Phi_n + |\Omega^{F}_{LS}(p, ^3\text{He})\rangle + \sum_{L' S'} T_{LS,L'S'} |\Omega^{+}_{L'S'}(p, ^3\text{He})\rangle 
\]

- $T_{LS,L'S'} = T$-matrix elements
- $a_{LS,n}$ and $T_{LS,L'S'}$ determined using the Kohn variational principle (KVP)
Study of the 3N force (1)

Fix the parameters of the 3N force (∼5):

1. $^3$H and $^4$He binding energies
2. $n - d$ doublet scattering length ($J = 1/2$)
3. $n - d$ and $p - d$ elastic scattering ($A_y$)

<table>
<thead>
<tr>
<th>Potential</th>
<th>$B(^3H)$ (MeV)</th>
<th>$B(^4He)$ (MeV)</th>
<th>$^2a_{nd}$ (fm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AV18</td>
<td>7.624</td>
<td>24.22</td>
<td>1.258</td>
</tr>
<tr>
<td>I-N3LO</td>
<td>7.854</td>
<td>25.38</td>
<td>1.100</td>
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<tr>
<td>AV18/TM’</td>
<td>8.440</td>
<td>28.31</td>
<td>0.623</td>
</tr>
<tr>
<td>AV18/UIX</td>
<td>8.479</td>
<td>28.48</td>
<td>0.578</td>
</tr>
<tr>
<td>I-N3LO/N-N2LO</td>
<td>8.474</td>
<td>28.37</td>
<td>0.675</td>
</tr>
<tr>
<td>Exp.</td>
<td>8.482</td>
<td>28.30</td>
<td>0.645±0.003±0.007</td>
</tr>
</tbody>
</table>

n-d zero-energy scattering = first “excited” state of $^3$H
Study of the 3N force (2)

Urbana \[ W(1, 2, 3) = aW_{2\pi}^a(1, 2, 3) + cW_{2\pi}^c(1, 2, 3) + U_0 W_R(1, 2, 3) \].

<table>
<thead>
<tr>
<th>Potential</th>
<th>(a)</th>
<th>(c/a)</th>
<th>(U_0)</th>
<th>(B(\text{\textsuperscript{3}H})) (MeV)</th>
<th>(B(\text{\textsuperscript{4}He})) (MeV)</th>
<th>(a_{\text{nd}}) (fm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AV18</td>
<td>-0.0293</td>
<td>0.25</td>
<td>0.0048</td>
<td>7.624</td>
<td>24.22</td>
<td>1.258</td>
</tr>
<tr>
<td>AV18+URIX</td>
<td>-0.0293</td>
<td>0.25</td>
<td>0.0048</td>
<td>8.479</td>
<td>28.48</td>
<td>0.578</td>
</tr>
<tr>
<td>AV18+URIX-1</td>
<td>-0.0200</td>
<td>1.625</td>
<td>0.0176</td>
<td>8.484</td>
<td>28.33</td>
<td>0.644</td>
</tr>
<tr>
<td>AV18+URIX-2</td>
<td>-0.0250</td>
<td>1.25</td>
<td>0.0182</td>
<td>8.484</td>
<td>28.34</td>
<td>0.644</td>
</tr>
<tr>
<td>AV18+URIX-3</td>
<td>-0.0293</td>
<td>1.00</td>
<td>0.0181</td>
<td>8.484</td>
<td>28.33</td>
<td>0.643</td>
</tr>
<tr>
<td>Exp.</td>
<td></td>
<td></td>
<td></td>
<td>8.482</td>
<td>28.30</td>
<td>0.645(\pm)0.003(\pm)0.007</td>
</tr>
</tbody>
</table>

Significant modifications of the 3N force

We have found 3 families of the 3N force: the N-N2LO, Urbana, and TM- families

In each case, same operatorial structure (N2LO)

They differ in the short-range part (regularization)
Study of the $3N$ force (3)

Comparison with p-d: $A_y$ and $i T_{11}$

$N - d$ elastic scattering
solid curve: AV18+UIX
red band= AV18+N-N2LO-family
cyan band= AV18+Urbana-family
violet band= AV18+TM-family

work in progress
\( n - ^{3}\text{He} \) scattering lengths

\[
|\Psi_{LS}\rangle = \sum_n a_{LS,n}\phi_n + \Omega_{LS}^F(n, ^{3}\text{He}) + \sum_{L'S'} T_{LS,L'S'}^{el} \Omega_{L'S'}^+(n, ^{3}\text{He}) + \sum_{L'S'} T_{LS,L'S'}^{ex} \Omega_{L'S'}^+(p, ^{3}\text{H})
\]

\[a_{S} = -\lim_{T} T_{0S,0S}^{el}/q_{n^{3}\text{He}}\]

<table>
<thead>
<tr>
<th>Int.</th>
<th>Method</th>
<th>( a_0 ) (fm)</th>
<th>( a_1 ) (fm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AV18</td>
<td>HH</td>
<td>8.03 – i5.18</td>
<td>3.53 – i0.0076</td>
</tr>
<tr>
<td></td>
<td>RGM</td>
<td>7.79 – i4.98</td>
<td>3.47 – i0.0066</td>
</tr>
<tr>
<td></td>
<td>FY</td>
<td>7.71 – i5.25</td>
<td>3.43 – i0.0082</td>
</tr>
<tr>
<td>AV18/UIX</td>
<td>HH</td>
<td>7.89 – i3.34</td>
<td>3.37 – i0.0055</td>
</tr>
<tr>
<td></td>
<td>RGM</td>
<td>7.63 – i4.05</td>
<td>3.31 – i0.0051</td>
</tr>
<tr>
<td>I-N3LO</td>
<td>HH</td>
<td>7.49 – i5.05</td>
<td>3.45 – i0.0058</td>
</tr>
<tr>
<td></td>
<td>FY</td>
<td>7.82 – i4.51</td>
<td>3.47 – i0.0068</td>
</tr>
<tr>
<td></td>
<td>AGS</td>
<td>7.36 – i0.0042</td>
<td></td>
</tr>
<tr>
<td>I-N3LO/N-N2LO</td>
<td>HH</td>
<td>7.370(58) – i4.448(5)</td>
<td>3.278(53) – i0.001(2)</td>
</tr>
</tbody>
</table>

Triplet $n - ^3\text{He}$ scattering length vs. $B(^3\text{H})$

Also calculated by Deltuva & Fonseca, (2007)
Integral relations (IR) for the process $A + B \rightarrow A + B$

**Example**

$A = 2$, central potential, S-wave, no spin

$$(H - E)\psi(r) = \left(-\frac{\hbar^2}{2\mu} \nabla^2 + V - E\right)\psi(r) = 0 \quad E = \frac{q^2}{2\mu}$$

$$\Omega^F = \sqrt{\frac{2\mu q}{4\pi}} \frac{\sin(qr)}{qr}$$

$$\Omega^G = \sqrt{\frac{2\mu q}{4\pi}} \frac{\cos(qr)}{qr} \left(1 - \exp(-\gamma r)\right).$$

- $\gamma$ “regularization” parameter
- Normalization chosen so that

$$\langle \Omega^F | H - E | \Omega^G \rangle - \langle \Omega^G | H - E | \Omega^F \rangle = 1$$

$$\psi(r \rightarrow \infty) \longrightarrow A\Omega^F(r) + B\Omega^G(r) \quad \tan \delta = B/A$$
Exact relations

\[
\langle \Psi | H - E | \Omega^G \rangle - \langle \Omega^G | H - E | \Psi \rangle = A
\]

\[
\langle \Psi | H - E | \Omega^F \rangle - \langle \Omega^F | H - E | \Psi \rangle = -B
\]

If \( \Psi \) is the exact wave function: \( (H - E)\Psi = 0 \)

\[
\frac{B}{A} = -\frac{\langle \Psi | H - E | \Omega^F \rangle}{\langle \Psi | H - E | \Omega^G \rangle}
\]

These relations represent an efficient method to extract \( A \) & \( B \)

1. Solve \( H\Psi = E\Psi \) with some method
2. extract \( A, B \) (and \( \tan \delta \)) using the IR

Similar methods

1. Harris PRL 19, 173 (1967)
The variational character of the IR (1)

- **exact** wave function \( \Psi = \Phi + A\Omega^F + B\Omega^G \)
  - \( \Phi \) short range part | \( A \) normalization (considered fixed in the following)
- **trial** wave function \( \overline{\Psi} = \overline{\Phi} + A\Omega^F + B\Omega^G \)

Typical case: \( \Phi_n, n = 1, \ldots \) complete set of square integrable functions

\[
\Phi = \sum_{n=1}^{\infty} a_n \Phi_n \quad \overline{\Phi} = \sum_{n=1}^{M} \overline{a}_n \Phi_n
\]

\[
\overline{\Psi} - \Psi \sim \epsilon \quad \epsilon \text{ “small”}
\]

**IR**

\[
\begin{bmatrix}
B \\
\overline{A}
\end{bmatrix}
= -\frac{\langle \overline{\Psi} | H - E | \Omega^F \rangle}{\langle \overline{\Psi} | H - E | \Omega^G \rangle}
\]

differs from \( B/A \) by \( \epsilon^2 \)

\[
(\overline{a}_n - a_n)_{[n=1,M]} \sim \epsilon \quad (a_n)_{[n=M+1,\infty]} \sim \epsilon \quad \overline{B} - B \sim \epsilon
\]
The variational character of the IR (2)

\[
\begin{bmatrix} B \\ A \end{bmatrix} = -\frac{\langle \Psi | H - E | \Omega^F \rangle}{\langle \Psi | H - E | \Omega^G \rangle} \quad \bar{\Psi} = \bar{\Phi} + A\Omega^F + B\Omega^G \quad \bar{\Phi} = \sum_{n=1}^{M} \bar{a}_n \Phi_n
\]

- Use \( \Omega^F = (1/A)(\Psi - \Phi - B\Omega^G) \) (\( \Psi \) = exact wave function)

\[
\begin{bmatrix} B \\ A \end{bmatrix} = \frac{B}{A} - \frac{1}{A} \frac{\langle \bar{\Psi} | H - E | \Phi \rangle}{\langle \bar{\Psi} | H - E | \Omega^G \rangle}
\]

1. \( \langle \bar{\Psi} | H - E | \Phi \rangle = \langle \Phi | H - E | \bar{\Psi} \rangle \) (\( \Phi \) is short-range)
2. Typically \( \bar{\Psi} \) is determined by \( \langle \Phi_n | H - E | \bar{\Psi} \rangle = 0 \quad n = 1, \ldots, M \)

\[
\langle \sum_{n=1}^{\infty} a_n \Phi_n | H - E | \bar{\Psi} \rangle = \langle \sum_{n=M+1}^{\infty} a_n \Phi_n | H - E | \bar{\Psi} \rangle = \langle \sum_{n=M+1}^{\infty} a_n \Phi_n | H - E | \epsilon \rangle \sim \epsilon^2
\]

3. It can also be derived from the KVP [PRL 103, 090402 (2009)]
Generalization to $A > 2$

$$\Omega_{LS}^F(A, B) = \sqrt{\frac{1}{N}} D_{AB} \sum_{\text{perm.}=1}^{N} \left[ Y_L(\hat{r}_{AB})[\phi_A \phi_B]_S \right]_{JJ_z} \frac{F_L(\eta, q_{AB}r_{AB})}{q_{AB}r_{AB}}$$

$$\Omega_{LS}^G(A, B) = \sqrt{\frac{1}{N}} D_{AB} \sum_{\text{perm.}=1}^{N} \left[ Y_L(\hat{r}_{AB})[\phi_A \phi_B]_S \right]_{JJ_z} \frac{G_L(\eta, q_{AB}r_{AB})}{q_{AB}r_{AB}}(1 - e^{-\gamma r_{AB}})^{2L+1}$$

- $D_{AB}$ = normalization factors chosen so that

$$\langle \Omega^F (A, B) | H - E | \Omega^G (A, B) \rangle - \langle \Omega^G (A, B) | H - E | \Omega^F (a, B) \rangle = 1 \quad D_{AB} = \sqrt{2\mu_{AB}q_{AB}}$$

- $i \equiv LS, AB$: $i = 1, \ldots, N$ asymptotic channels

$$\overline{\Psi}_i \rightarrow A_{ij} \Omega^F_j + \overline{B}_{ij} \Omega^G_j \quad i = 1, \ldots, N \quad K = A^{-1}\overline{B} \quad K - \text{matrix}$$

Generalized IR

$$A_{ij} = < \overline{\Psi}_i | H - E | \Omega^G_j > \quad \overline{B}_{ij} = - < \overline{\Psi}_i | H - E | \Omega^F_j > \quad [K] = A^{-1}\overline{B}$$
Applications

Adiabatic HH

- Calculation of $\Psi$ using the adiabatic HH expansion
- See next talk by Eduardo
- IR first derived for this case P. Barletta et al., PRL 103, 090402 (2009)
- Direct application of the boundary condition ($A_{ij} = \delta_{ij}$) only for $\rho \to \infty$

Use of the short range character of the IR

- “interacting region” $\equiv \mathcal{V}_i =$ region where all particles are close together ($\rho \leq 30$ fm)
- $(H - E)\Omega_i^{F,G} = 0$ for $r_{AB}$ outside $\mathcal{V}_i$
- $\bar{\Psi}_i$ need to be known only in $\mathcal{V}_i$

1. Use of “bound-state”–like wave function
2. Phase-shifts of scattering between charged particles using a screened Coulomb interaction
## Applications

### Adiabatic HH

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### Use of the short range character of the IR

- "interacting region" $\equiv \mathcal{V}_i = \text{region where all particles are close together} (\rho \leq 30 \text{ fm})$
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- $\Psi_i$ need to be known only in $\mathcal{V}_i$

1. Use of “bound-state”—like wave function
2. Phase-shifts of scattering between charged particles using a screened Coulomb interaction
"Bound-state"–like wave function

- Expansion over a square integrable function basis

\[ \Psi = \sum_{n=1}^{M} a_n \Phi_n \quad \Phi_n \to 0 \text{ outside } V_1 \]

- Examples: HO basis, HH basis, etc.

Eigenvalue problem

\[ \langle \Phi_n | H - E | \Phi_{n'} \rangle a_{n'} = 0 \]

- The lowest eigenvalues: \( \rightarrow \) bound states
- The others eigenvalues: \( \rightarrow \) scattering states (wrong behaviour outside \( V_1 \))
- \( \rightarrow \) use them in the IR!
Example (1): A=2 case, S-wave

\[ V(r) = -V_0 \exp\left(-\frac{r^2}{r_0^2}\right) \quad V_0 = 51.5 \text{ MeV} \quad r_0 = 1.6 \text{ fm} \]

\[ \bar{\Psi} = \sum_{n=1}^{M} a_n \Phi_n \quad \Phi_n = L_n^{(2)}(\beta r) \exp\left(-\frac{\beta r}{2}\right) \]

\[
\begin{array}{c|c|c|c|c}
M & 10 & 20 & 30 & 40 \\
\hline
E_0 & -0.395079 & -0.397740 & -0.397743 & -0.397743 \\
E_1 & 0.536349 & 0.116356 & 0.048091 & 0.026008 \\
\tan \delta & -1.507280 & -0.622242 & -0.392005 & -0.286479 \\
[B/A] & -1.522377 & -0.621938 & -0.392021 & -0.286480 \\
\end{array}
\]
Example 2: $A = 3$ case

- Elastic $N - d$ scattering $E_{c.m.} < 2.2$ MeV
- For fixed $J$ and parity
  - $J^\pi = \frac{1}{2}^+ : LS = 2S_{\frac{1}{2}}, 4D_{\frac{1}{2}}$ (1 bound state)
  - $J^\pi = \frac{1}{2}^- : LS = 2P_{\frac{1}{2}}, 4P_{\frac{1}{2}}$ (no bound state)
  - $J^\pi = \frac{3}{2}^+ : LS = 4S_{\frac{3}{2}}, 2D_{\frac{3}{2}}, 4D_{\frac{3}{2}}$ (no bound state)
  - etc.
- Expansion basis
  $$\Phi_n = L_m^{(5)}(\beta \rho) \exp(-\beta \rho/2) \times \mathcal{Y}_K$$
- $\rho =$ hyperradius, $\mathcal{Y}_K = $ HH functions
- Typically $m \rightarrow 20$

There are 2 non-linear parameters

- $\beta$ in $\Phi_n$  $\gamma$ in $\Omega^G$

$$\Omega_{LS}^G(A, B) = \sqrt{\frac{1}{N}} D_{AB} \sum_{\text{perm.} = 1}^N \left[ Y_L(\hat{r}_{AB})[\phi_A \phi_B]_S \right]_{JJ_z} \frac{G_L(\eta, q_{AB} r_{AB})}{q_{AB} r_{AB}} \left(1 - e^{-\gamma r_{AB}}\right)^{2L+1}$$
$n - d$ low energy scattering

Central potential: Malfliet-Tjon I-III

\[
V_{S=0}(r) = \frac{1438.72}{r} e^{-3.11r} - \frac{513.968}{r} e^{-1.55r} \quad \text{and} \quad V_{S=1}(r) = \frac{1438.72}{r} e^{-3.11r} - \frac{626.885}{r} e^{-1.55r}
\]

\[
K(E^0) = k \cot \delta
\]

\[
K(E^0) = C_0^2(\eta) k \cot \delta + 2k\eta h(\eta)
\]
Example 3: $\frac{1}{2}^+$, AV18 potential

Problem: how to obtain 2 or more $\Psi$ at the same $E$?

1. Look at the eigenvalues $-2.2 < E < 0$ MeV
2. Vary $\beta$
Example 4: Preliminary results for $A = 4$

PRELIMINARY

$K = 34 \quad E = -3.59 \quad E_p = 3.55$

$K = 40 \quad E = -4.26 \quad E_p = 2.87$

$K = 46 \quad E = -4.55 \quad E_p = 2.58$
Conclusions

Integral Relations

- The IR could allow for a variety of applications for \( A > 4 \) systems
- They can be used with different methods (GFMC, NCSM, EIHH, AHH, ...)
  - Solution of \( H\Psi = E\Psi \) in \( \mathcal{V}_i \)
  - Calculation of the overlap integrals (via Monte Carlo also for “large” \( A \))
- Tests in \( A = 3, 4 \) (elastic channels) OK

Future work

- A better treatment of coupled channels
- Extension to breakup channels

In progress

- Extension of our HH method to \( A > 4 \)
- Work in progress with M. Gattobigio, INL, Nice (France)
Non-symmetrical basis

\[ \psi = \sum_k a_k \Phi_k \]

- It would be easy to use states \( \tilde{\Phi}_k \) constructed without any particular symmetry.
- In fact: to construct antisymmetric states very difficult as \( A \) increases
  \[ \Phi_k = \sum_{k'} b_{k'} \tilde{\Phi}_k \]
- Idea: solve \( H \sum_k a_k \tilde{\Phi}_k = E \sum_k a_k \tilde{\Phi}_k \)
- since \( H \) is a symmetric operators, the eigenstates are also eigenstates of the symmetric group \( S_A \)
- Last step: select the antisymmetrical eigenstates
  - multiplicity
  - diagonalizing some operator

“transposition operator” \[ [(2)] = \sum_{i<j} (i \leftrightarrow j) \]
Application for 5 bosons

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Eigenvalues from $H\Psi = E\Psi$

$$\Psi = \sum_k a_k \tilde{\Phi}_k$$

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Other publications 2008/2009


