Renormalization Group Techniques and Applications

Achim Schwenk

INT Weakly-Bound Systems Workshop, March 10, 2010
Strong interaction physics in the lab and cosmos

Matter at the extremes:

density $\rho \sim \ldots 10^{15}$ g/cm$^3$

proton-rich, neutron-rich,

$^8$He to Z/N $\sim 0.05$

temperatures $T \sim \ldots 100$ MeV
Outline

Effective field theory and renormalization group for nuclear forces

Applications to weakly-bound and neutron-rich nuclei

Similarity renormalization group for nuclei
Λ / Resolution dependence of nuclear forces

with high-energy probes: quarks+gluons
at low energies: complex QCD vacuum

lowest energy excitations: pions, nearly massless, $m_\pi=140$ MeV
“phonons” of QCD vacuum
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“phonons” of QCD vacuum

$\Lambda_{\text{chiral}}$

momenta $Q \sim \lambda^{-1} \sim m_\pi$

$\Lambda_{\text{pionless}}$

$Q \ll m_\pi$
Λ / Resolution dependence of nuclear forces

with high-energy probes: quarks+gluons

Effective theory for NN, 3N, many-N interactions and electroweak operators: resolution scale/Λ-dependent

\[ H(\Lambda) = T + V_{NN}(\Lambda) + V_{3N}(\Lambda) + V_{4N}(\Lambda) + \ldots \]

\( \Lambda_{\text{chiral}} \)

momenta \( Q \sim \lambda^{-1} \sim m_\pi \): chiral effective field theory (EFT)

neutrons and protons interacting via pion exchanges and shorter-range contact interactions

typical momenta in nuclei \( \sim m_\pi \)

\( \Lambda_{\text{pionless}} \)

\( Q \ll m_\pi \)
Λ / Resolution dependence of nuclear forces

with high-energy probes: quarks+gluons

Effective theory for NN, 3N, many-N interactions and electroweak operators: resolution scale/Λ-dependent

$$H(Λ) = T + V_{NN}(Λ) + V_{3N}(Λ) + V_{4N}(Λ) + \ldots$$

Λ\text{chiral}

momenta $Q \sim \lambda^{-1} \sim m_π$

universal properties of neutrons and cold atoms, reactions at astrophysical energies, loosely-bound halo nuclei,…

Λ\text{pionless}

$Q \ll m_π$: pionless effective field theory

large scattering length physics and corrections
Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Meissner,…

Chiral EFT for nuclear forces

Separation of scales: low momenta $\frac{1}{\Lambda} = Q \ll \Lambda_b$ breakdown scale $\sim 500$ MeV

limited resolution at low energies, can expand in powers $Q/\Lambda_b$

include long-range pion physics
details at short-distance not resolved
capture in few short-range couplings, fit to experiment once, $\Lambda$-dependent

systematic: can work to desired accuracy and obtain error estimates from truncation order and $\Lambda$ variation

several open problems
Chiral EFT for nuclear forces

Separation of scales: low momenta \( \frac{1}{\lambda} = \frac{Q}{\Lambda} \ll \Lambda_b \) breakdown scale \( \sim 500 \text{ MeV} \)

Accurate reproduction of low-energy NN scattering at \( N^3\text{LO} \)

Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Meissner, ...
First, exciting efforts to connect nuclear forces to underlying QCD

Long-range couplings: pion-nucleon coupling $g_A$ Edwards et al. (2006)

chiral EFT extrapolation to physical pion mass agrees with experiment

variation of nuclear forces with quark masses Beane et al. (2006)

Future possibility: access/constrain 3-neutron forces (3n exp difficult)
Nuclear forces and the Renormalization Group

RG evolution to lower resolution/cutoffs

\[ H(\Lambda) = T + V_{\text{NN}}(\Lambda) + V_{3\text{N}}(\Lambda) + V_{4\text{N}}(\Lambda) + \ldots \]

exact RG for NN interactions

\[ \frac{d}{d\Lambda} V_{\text{low}k}^\Lambda(k', k) = \frac{2}{\pi} \frac{V_{\text{low}k}^\Lambda(k', \Lambda) T^\Lambda(\Lambda, k; \Lambda^2)}{1 - (k/\Lambda)^2} \]

Bogner, Kuo, AS, Furnstahl, …

\[ ^1S_0 \text{ channel} \]

red = short-range repulsion
Nuclear forces and the Renormalization Group

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\[ H(\Lambda) = T + V_{\text{NN}}(\Lambda) + V_{\text{3N}}(\Lambda) + V_{\text{4N}}(\Lambda) + \ldots \]

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Bogner, Kuo, AS, Furnstahl, …

low-momentum interactions \( V_{\text{low } k}(\Lambda) \) with sharp or smooth regulators
decouples low-momentum physics from high momenta
red = short-range repulsion and short-range tensor parts
Low-momentum universality

≈ universality from different phenomenological potentials

RG preserves NN observables and long-range parts
decouples low-momentum physics from high momenta
Low-momentum universality

≈ universality from different chiral N$^3$LO potentials

RG preserves NN observables and long-range parts decouples low-momentum physics from high momenta
Weinberg eigenvalue diagnostic

study spectrum of $G_0(z) V |\Psi_\nu(z)\rangle = \eta_\nu(z) |\Psi_\nu(z)\rangle$ at fixed energy $z$
governs convergence $T(z) |\Psi_\nu(z)\rangle = (1 + \eta_\nu(z) + \eta_\nu(z)^2 + \ldots) V |\Psi_\nu(z)\rangle$
can write as Schrödinger equation $\left(H_0 + \frac{1}{\eta_\nu(z)} V\right) |\Psi_\nu(z)\rangle = z |\Psi_\nu(z)\rangle$
large cutoffs lead to flipped-potential bound states of $-\lambda V$ for small $\lambda$
Weinberg eigenvalue diagnostic

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large cutoffs lead to flipped-potential bound states of \(-\lambda V\) for small \( \lambda \)
\( \rightarrow \) large \( \eta \) \( \rightarrow \) strong coupling to high momenta and Born series nonpert.

leads to slow convergence for all nuclei

\( ^3S_1 - ^3D_1 \)

\( E=0 \)

Born series becomes perturbative after RG except in channels with bound states

\( ^3S_1 \) (\( B_d = -2.223 \) MeV)
Outline

Effective field theory and renormalization group for nuclear forces

Applications to weakly-bound and neutron-rich nuclei

Similarity renormalization group for nuclei
Neutron-rich nuclei in the laboratory

500 MeV proton beam on specialized targets

highest power at TRIUMF-ISAC

a way to add neutrons to reach extreme neutron-rich nuclei

Novel forms of matter: halo nuclei

$^{3}\text{He}$, $^{4}\text{He}$, $^{5}\text{He}$, $^{6}\text{He}$, $^{7}\text{He}$, $^{8}\text{He}$

$^{8}\text{He}$: most neutron-rich nucleus in the lab
TITAN Penning trap see talk by S. Ettenauer

First direct mass measurement of helium and lithium halo nuclei

high precision $\delta m/m \sim 10^{-8}$
Neutron halos

New precision era for masses and charge radii (from isotope shifts)

poses extraordinary challenges for theory
Neutron halos

Hyperspherical Harmonics for $^6$He and Coupled-Cluster theory for $^8$He describe weakly-bound nuclei with correct asymptotics but compare convergence to stable $^4$He!

Based on $N^3$LO NN potential, RG cutoff variation → need 3N forces

Bacca et al. (2009)
Why are there three-nucleon (3N) forces?

Nucleons are finite-mass composite particles, can be excited to resonances

dominant contribution from $\Delta(1232 \text{ MeV})$

$+$ shorter-range parts

tidal effects leads to 3-body forces in earth-sun-moon system
Chiral EFT for 3N forces

Separation of scales: low momenta \( \frac{1}{\Lambda} = Q \ll \Lambda_b \) breakdown scale \( \sim 500 \text{ MeV} \)

consistent \( \text{NN-3N} \) interactions

3N, 4N: only 2 new couplings to \( N^3\text{LO} \)

\[
\begin{align*}
\text{LO} & \quad \mathcal{O} \left( \frac{Q^0}{\Lambda^0} \right) \\
\text{NLO} & \quad \mathcal{O} \left( \frac{Q^2}{\Lambda^2} \right) \\
\text{N}^2\text{LO} & \quad \mathcal{O} \left( \frac{Q^3}{\Lambda^3} \right) \\
\text{N}^3\text{LO} & \quad \mathcal{O} \left( \frac{Q^4}{\Lambda^4} \right)
\end{align*}
\]

leading 3N: \( N^2\text{LO} \)
van Kolck (1994), Epelbaum et al. (2002)

\( c_1, c_3, c_4, c_D, c_E \)

\( c_i \) from \( \pi N \) and NN from Meissner (2007)
\[
\begin{align*}
c_1 &= -0.9^{+0.2}_{-0.5} \\
c_3 &= -4.7^{+1.2}_{-1.0} \\
c_4 &= 3.5^{+0.5}_{-0.2}
\end{align*}
\]

single-\( \Delta \) excitation = particular \( c_i \)

\( c_D, c_E \) fit to \( ^3\text{H} \) binding energy and \( ^4\text{He} \) radius (or \( ^3\text{H} \) beta decay half-life)
Towards the limits of existence - the neutron drip-line

**Discovery of $^{40}$Mg and $^{42}$Al suggests neutron drip-line slant towards heavier isotopes**

T. Baumann¹, A. M. Amthor¹, D. Bazin¹, B. A. Brown¹, C. M. Folden III¹, A. Gade¹, T. N. Ginter¹, M. Hausmann¹, M. Matos², D.J. Morrissey², M. Portillo¹, A. Schiller¹, B.M. Sherrill², A. Stolz¹, D.B. Tarasov³ & M. Thoennessen²

The oxygen anomaly

*Nature 459*, 1069-1070 (25 June 2009)

NUCLEAR PHYSICS

**Unexpected doubly magic nucleus**

Robert V. F. Janssens

Nuclei with a ‘magic’ number of both protons and neutrons, dubbed doubly magic, are particularly stable. The oxygen isotope $^{24}\text{O}$ has been found to be one such nucleus — yet it lies just at the limit of stability.
The oxygen anomaly - not reproduced without 3N forces

many-body theory based on two-nucleon forces: drip-line incorrect at $^{28}\text{O}$

(a) Forces derived from NN theory

(b) Phenomenological forces
Towards 3N forces in medium-mass nuclei

Coupled-Cluster theory with 3N forces Hagen et al. (2007)

first benchmark for $^4\text{He}$, work on $^{16}\text{O}$ in progress

normal-ordered 0-, 1- and 2-body parts of 3N forces dominate

2-body part

residual 3N interaction can be neglected: very promising
The oxygen anomaly - impact of 3N forces

include normal-ordered 2-body part of 3N forces (enhanced by core A)

leads to repulsive interactions between valence neutrons (repulsive based on the Pauli principle)

d_{3/2} orbital remains unbound

first microscopic explanation of the oxygen anomaly

Evolution to neutron-rich calcium isotopes

repulsive 3N contributions also key for calcium ground-state energies

Holt, Otsuka, AS, Suzuki, in prep.

fit to experiment

![Graph showing the energy of calcium isotopes as a function of mass number, with various models and experimental data points.](image)
Evolution to neutron-rich calcium isotopes

repulsive 3N contributions also key for calcium ground-state energies
Holt, Otsuka, AS, Suzuki, in prep.

3N mechanism important for shell structure: $2^+$ excitation energy in $^{48}\text{Ca}$

N=28 shell closure due to 3N and single-particle effects ($^{41}\text{Ca}$)
predict $2^+$ excitation energy in $^{54}\text{Ca}$ at $\sim3$ MeV
Weinberg eigenvalue diagnostic

study spectrum of \( G_0(z)V |\Psi_\nu(z)\rangle = \eta_\nu(z) |\Psi_\nu(z)\rangle \) at fixed energy \( z \)
governs convergence \( T(z) |\Psi_\nu(z)\rangle = (1 + \eta_\nu(z) + \eta_\nu(z)^2 + \ldots) V |\Psi_\nu(z)\rangle \)
can write as Schrödinger equation \( \left(H_0 + \frac{1}{\eta_\nu(z)} V\right) |\Psi_\nu(z)\rangle = z |\Psi_\nu(z)\rangle \)
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leads to slow convergence for all nuclei

\( ^3S_1 - ^3D_1 \)

Born series becomes perturbative after RG except in channels with bound states

bound states are lost at finite density

\( ^3S_1 (B_d = -2.223 \text{ MeV}) \)

short-range
Is nuclear matter perturbative with chiral EFT and RG?
conventional Bethe-Brueckner-Goldstone expansion (sums ladders): no, due to nonpert. cores (flipped-V bound states) and off-diag coupling
start from chiral EFT and RG evolution:
nuclear matter converged at $\approx 2$nd order, 3N drives saturation
weak cutoff dependence, but need to improve 3N treatment

see talk by K. Hebeler for neutron matter and more
Outline

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Similarity renormalization group for nuclei
Similarity RG

unitary transformations to band-diagonal $V_{\text{srg}}(\lambda)$ from flow equations


$$\frac{dH_s}{ds} = [\eta_s, H_s] = [[G_s, H_s], H_s]$$

evolution driven towards nonzero part of generator $G_s$

with flow operator $G_s = T_{\text{rel}}$ and resolution $\lambda = s^{-1/4}$

Bogner, Furnstahl, Perry,…

$$\frac{dV_s(k, k')}{ds} = -(k^2 - k'^2) V_s(k, k') + \frac{2}{\pi} \int_0^\infty dq \left(k^2 + k'^2 - 2q^2\right) V_s(k, q) V_s(q, k')$$

SRG decouples high momenta with similar low-momentum universality
SRG evolution of 3N forces

start from chiral NN and 3N interactions,
SRG evolution in harmonic oscillator basis
Jurgenson, Navratil, Furnstahl (2009)

induced many-body interactions consistent with truncation error in EFT
In-medium SRG for nuclei

\[ H = \sum_{12} T_{12} a_1^\dagger a_2 + \frac{1}{(2!)^2} \sum_{1234} \langle 12|V|34 \rangle a_1^\dagger a_2^\dagger a_4 a_3 + \frac{1}{(3!)^2} \sum_{123456} \langle 123|V^{(3)}|456 \rangle a_1^\dagger a_2^\dagger a_3^\dagger a_6 a_5 a_4 \]

normal-order Hamiltonian with respect to reference state (e.g., Hartree-Fock ground state)

\[ H = E_0 + \sum_{12} f_{12} \{ a_1^\dagger a_2 \} + \frac{1}{(2!)^2} \sum_{1234} \langle 12|\Gamma|34 \rangle \{ a_1^\dagger a_2^\dagger a_4 a_3 \} + \frac{1}{(3!)^2} \sum_{123456} \langle 123|\Gamma^{(3)}|456 \rangle \{ a_1^\dagger a_2^\dagger a_3^\dagger a_6 a_5 a_4 \} \]

with 0-, 1- and 2-body normal-ordered parts

\[ E_0 = \langle \Phi | H | \Phi \rangle = \sum_1 T_{11} n_1 + \frac{1}{2} \sum_{12} \langle 12|V|12 \rangle n_1 n_2 + \frac{1}{3!} \sum_{123} \langle 123|V^{(3)}|123 \rangle n_1 n_2 n_3 \]

\[ f_{12} = T_{12} + \sum_i \langle 1i|V|2i \rangle n_i + \frac{1}{2} \sum_{ij} \langle 1i;j|W|2i;j \rangle n_i n_j, \]

\[ \langle 12|\Gamma|34 \rangle = \langle 12|V|34 \rangle + \sum_i \langle 12i|V^{(3)}|34i \rangle n_i, \]
In-medium SRG for nuclei

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with 0-, 1- and 2-body normal-ordered parts and in-medium SRG eqns e.g., for nuclear matter with \( \eta = [f, \Gamma] \) see Bogner et al., Kehrein (2006)

\[ \frac{dE_0}{ds} = \frac{1}{2} \sum_{1234} (f_{12} - f_{34}) |\Gamma_{1234}|^2 n_1 n_2 \bar{n}_3 \bar{n}_4 , \]

\[ \frac{df_1}{ds} = \sum_{234} (f_{41} - f_{23}) |\Gamma_{4123}|^2 (\bar{n}_2 \bar{n}_3 n_4 + n_2 n_3 \bar{n}_4) , \]

\[ \frac{d\Gamma_{1234}}{ds} = - (f_{12} - f_{34})^2 \Gamma_{1234} + \frac{1}{2} \sum_{ab} (f_{12} + f_{34} - 2f_{ab}) \Gamma_{12ab} \Gamma_{ab34} (1 - n_a - n_b) + \sum_{ab} (n_a - n_b) \]

\[ \times \left\{ \Gamma_{a1b3} \Gamma_{b2a4} [(f_{a1} - f_{b3}) - (f_{b2} - f_{a4})] - \Gamma_{a2b3} \Gamma_{b1a4} [(f_{a2} - f_{b3}) - (f_{b1} - f_{a4})] \right\} , \]

approx. includes many-body forces and sums pp, hh, ph diagrams
In-medium SRG for nuclei

To decouple 1p1h, 2p2ph, ... ApAh sectors from reference state, we want to suppress pphh and ph couplings, while all other (normal-ordered) couplings annihilate the reference state. The minimal choice is:

$$\eta(s) = [H^d(s), H(s)] = [H^d(s), H^{od}(s)]$$

$$H^{od}(s) = g^{od}(s) + \Gamma^{od}(s)$$

$$\Gamma^{od}(s) = \sum_{pp'h'hh'} \Gamma_{pp'h'h'}(s) a_p^\dagger a_{p'}^\dagger a_h a_{h'} + h.c.$$
In-medium SRG for nuclei

Tsukiyama, Bogner, AS, in prep.

first results for closed-shell nuclei
very promising convergence,
results comparable to CCSD(T)

...can be used to derive nonperturbative valence-shell effective interactions...
Thanks to collaborators

S. Bacca, S. Baroni, K. Hebeler, J. Holt

S.K. Bogner

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A. Nogga

D.J. Dean, G. Hagen, T. Papenbrock

T. Otsuka, K. Tsukiyama, Y. Akaishi

T. Suzuki

C.J. Pethick

B. Friman
Summary
development of effective field theory and the renormalization group with approaches from light to heavy nuclei to matter in astrophysics based on the same interactions
3N forces are a frontier for weakly-bound and neutron-rich nuclei exciting intersections with problems in many related areas exciting experiments to study neutron-rich matter in the laboratory at rare isotope beam (RIB) facilities worldwide