Electroweak properties of Weakly-Bound Light Nuclei

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Introduction

- **Light nuclei are special:**
  - Weakly-bound:
    - Usually include few (if any) bound excited states.
    - Consequently, inelastic reaction often leads to break-up of the nucleus.

\[
\begin{array}{cccccc}
\text{2H} & \text{3H} & \text{3He} & \text{4He} & \text{5He} & \text{6He} \\
\text{B.E [MeV]} & -2.22 & -8.48 & -7.72 & -28.3 & \text{unbound} \\
\text{B.E/mass} & 1 \times 10^{-3} & 3 \times 10^{-3} & 3 \times 10^{-3} & 7 \times 10^{-3} & \text{unbound} \\
\end{array}
\]

\[
\begin{array}{cccccc}
\text{6Li} & \text{...} \\
\text{B.E [MeV]} & -32.0 \\
\text{B.E/mass} & 5 \times 10^{-3} \\
\end{array}
\]
Introduction

Why looking at electro-weak properties?

One can immediately relate *electro-weak reaction* of a probe with nucleus to currents inside the nucleus.

\[ \hat{H}_W \sim \int d^3 \vec{x} \hat{J}_\mu^+ (\vec{x}) \hat{J}_\mu^- (\vec{x}) \]

\[ \mathcal{O} \sim \langle \psi_i | \hat{\mathcal{J}}_\mu | \psi_f \rangle \]

Scattering operator

Currents in the nucleus

\[ W, Z \text{ propagator} = \frac{g_{\mu\nu} + \frac{q_\mu q_\nu}{M_W^2}}{q^2 + M_W^2} \quad q \ll M_W \rightarrow \frac{g_{\mu\nu}}{M_W^2} \]
Introduction

• **Why looking at electro-weak properties?**
  
  • One can immediately relate *electro-weak reaction* of a probe with nucleus to currents inside the nucleus.
  
  • The currents are *reflections* of the *symmetries* of the nuclear interaction.
  
  • Thus, one can *relate electro-weak properties and reaction rates* with non-trivial properties not only of the *target*, but also of the *fundamental theory* leading to its structure!
  
  • In addition, electro-weak properties are important as a microscopic input for simulations of astrophysical phenomena.
    
    • Solar fusion.
    
    • Supernovae.
  
  • These are often very challenging, or even impossible to measure, thus need accurate, parameter free predictions.
Introduction

- **Why looking at electro-weak properties?**
- **Why light Nuclei?**
  - Available methods for solving exactly the Schrödinger equation for few body systems, from nucleonic dof: no core shell model, expansions in Hyperspherical Harmonics, Green’s function Monte Carlo, ….
  - **Chiral effective field theory (χPT),** enables a connection between the fundamental theory of QCD and the nuclear interaction.
  - **Allow accurate, parameter free calculations, of weakly bound systems, and their properties,** from their nucleonic dof.
Chiral Effective Field Theory

- Symmetries are important **NOT** degrees of freedom.
- In QCD – an approximate chiral symmetry:
  - The $u$ and $d$ are (almost) massless.
    $\text{SU}(2)_L \times \text{SU}(2)_R \cong \text{SU}(2)_V \times \text{SU}(2)_A \rightarrow \text{SU}(2)_V$
  - The $\text{SU}(2)_V$ symmetry is the isospin symmetry.
  - However, no degenerate parity doublets are found in the spectrum.
  - The axial symmetry is spontaneously broken.
- Chiral EFT is based on this observation.
  - Identify $Q$ – the momentum scale of the process.
  - Choose $\Lambda$ – the theory cutoff.
  - In view of these-identify the effective degrees of freedom.

$$\lambda \sim \frac{1}{Q} \gg \frac{1}{\Lambda} \sim R$$
The pions are interpreted as the Goldstone bosons of the spontaneously broken SU(2)\textsubscript{A} symmetry.

Their mass is a result of the explicit symmetry breaking due to the finite \textit{u} and \textit{d} masses. This introduces an additional scale.

- If $Q<<\Lambda<<m_\pi$ then the effective theory is of point particles (pionless \textit{\chi}PT).
- If $Q\sim m_\pi<<\Lambda$ then the effective theory should consist of both pions and nucleons (pionfull \textit{\chi}PT), and even higher dof: Delta resonance, etc.

Write a Lagrangian composed of ALL possible operators invariant under symmetries of the underlying theory.

Find a \textit{systematic} way to organize diagrams according to their contribution to the observable.
Weinberg’s Power Counting Scheme

- Each Feynman diagram can be characterized by: \( \left( \frac{Q}{\Lambda} \right)^n \)
- Weinberg showed that \( v \) is bound from below.
- In addition, expand in the inverse of the nucleon’s mass (take \( \Lambda \sim M_N \)) \( \Rightarrow \textbf{Heavy Baryon } \chi PT. \)
- This power counting is based on an expansion around the RG fixed point \( Q=0. \)
- There are indications that this is correct only for a limited range of \( \Lambda \), due to the existence of a non-trivial, unitary, fixed point.
- This is also evident in the abnormal size of the NN interaction induced by a pion exchange (however, expansions based on the latter seem to have convergence problems (KSW)).

20 years of debate led by: Weinberg, Kaplan, Savage, Wise, van-Kolck, Nogga, Timmermans, Birse, Meissner, Epelbaum…
The big deal in $\chi$PT

- A perturbation theory/expansion in small parameter of the observable, gives control over the accuracy of the calculation.
- Varying the cutoff gives estimate of the theoretical error-bar.
- Allows connection between \textit{a-priori} unrelated operators:
  - In particular the nuclear force and the electro-weak currents in the nucleus (that the $\text{Su}(2) \times \text{Su}(2)$ structure is a gauging of).
- When the low-energy constants are known: the calculations are predictions of QCD.
χPT approach for low-energy EW nuclear reactions:

QCD

Low energy EFT

Chiral Lagrangian

Nöther current

Nuclear potential

Wave functions

Nuclear Matrix Element

Weak current
Forces in $\chi$PT

- The leading order NNN forces are at $N^2$LO.
- They include 2 new contact parameters.
- No new parameters at $N^3$LO.
Current conservation leads to a connection between the spatial current and charge density.

At low energy transfer the scattering operator is simply the dipole operator (Siegert theorem).

This approximation is accurate to about 10% at 100 MeV.
Photo-dissociation of $^4$He

- Sensitivity to NN force model.
- Sensitivity vanishes when adding the 3NF.
- The theoretical prediction is much more accurate than the experimental measurement, and actually “chooses” the “correct” measurement.

Chiral EFT  S.Quaglioni and P.Navratil  PLB 652 (2007)
The structure of $^{4}\text{He}$

- $^{4}\text{He}$ is a spherical nucleus ($J=0$), but what is its symmetry in the body frame?

- The unretarded dipole approximation to the photodissociation cross-section can be related to the mean inter-nucleon distances:

$$\Sigma_{BSR} = \int_{0 \omega_h}^{\infty} \omega^{-1} \sigma_{\gamma}^{E\text{UR}} d\omega = \frac{3}{4\pi^2 \alpha} \langle 0 | \hat{D} \cdot \hat{D} | 0 \rangle$$

$$= \frac{3}{4\pi^2 \alpha} \left( Z^2 \langle r_p^2 \rangle - \frac{Z(Z-1)}{2} \langle r_{pp}^2 \rangle \right) =$$

$$= \frac{3}{4\pi^2 \alpha} \left( N^2 \langle r_n^2 \rangle - \frac{N(N-1)}{2} \langle r_{nn}^2 \rangle \right) =$$

$$= \frac{3}{4\pi^2 \alpha} \frac{NZ}{2} \left( \langle r_{np}^2 \rangle - \langle r_n^2 \rangle - \langle r_p^2 \rangle \right) =$$

- In $^{4}\text{He}$ this is enough to reconstruct the structure:

- Thus, $^{4}\text{He}$ has a slightly deformed internal tetrahedral symmetry…

\[ \frac{\langle r_{pp}^2 \rangle}{\langle r_p^2 \rangle} = \frac{\langle r_{nn}^2 \rangle}{\langle r_n^2 \rangle} = 2.78 \text{ fm}^2 \quad \frac{\langle r_{np}^2 \rangle}{\langle r_n^2 \rangle} = 2.62 \text{ fm}^2 \]

Photodissociation of 6 body nuclei

From S. Bacca

χPT axial weak currents to fourth order

\[ \hat{d}_R \equiv \frac{M_N}{\Lambda_{\chi} g_A} c_D + \frac{1}{3} M_N (c_3 + 2c_4) + \frac{1}{6} \]

- **Single nucleon current**
- **1 pion exchange**
- **Contact term**

**Nucleon-pion interaction**, *NO new parameters*

**Contact term**

Axial MEC – remarks

• The MEC include “OπEC” and contact topologies.
• MEC involve only TWO nucleons.
• Thus, in principle $c_D$ can be calibrated using two-body weak processes.
• So – three nucleon force constrained at the two nucleon level!
• The most attractive process –
  • Muon capture on deuteron – known only at the 5% level. An experiment at PSI “MuD (MuSun)” aims to measure this process to 1%.
• However, many 3 nucleon processes are measured very well.
A calculation of $^3\text{H} \beta$ decay using consistent $\chi$PT interaction and currents

$$\hat{d}_R \equiv \frac{M_N}{\Lambda \chi g_A} c_D + \frac{1}{3} M_N (c_3 + 2c_4) + \frac{1}{6}$$

Step 1: use the trinuclei binding energies to find a $c_D$-$c_E$ relation

Step 2: calibrate $c_D$ according to the triton half life.
A prediction of $^4$He

<table>
<thead>
<tr>
<th></th>
<th>$E_{g.s.}$</th>
<th>$\langle r_p^2 \rangle^{1/2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NN</td>
<td>$-25.39(1)$</td>
<td>$1.515(2)$</td>
</tr>
<tr>
<td>NN + NNN</td>
<td>$-28.50(2)$</td>
<td>$1.461(2)$</td>
</tr>
<tr>
<td>Expt.</td>
<td>$-28.296$</td>
<td>$1.467(13)$</td>
</tr>
</tbody>
</table>
A closer look into the weak axial correlations in $^3H$

- Specific character of the force has minor effect
- Is this the origin for the success of EFT*?
EFT* approach for low-energy nuclear reactions:

- Phenomenological Hamiltonian
- Solution of Schrödinger equation
- Nuclear Matrix Element
- Wave functions

QCD

Chiral Lagrangian

Low energy EFT

Nöther current

Weak current

Applications of the EFT* approach

- $p+p$ fusion in the sun.
- $^3\text{He}+p$ fusion in the sun.
- The weak structure of the nucleon from $\mu$ capture on $^3\text{He}$.
- Neutrino scattering on light nuclei in core-collapse supernovae.
- $\beta$ decay of $^6\text{He}$ and the suppression of the axial constant in nuclear matter.

**References**

Solar Fusion II, Rev. Mod. Phys. [to be published].


Few Solar Fusion open problems

Solar Fusion II, Rev. Mod. Phys. [to be published].
The weak fusion process \( p + p \rightarrow d + \nu + e^+ \), is sun’s clock – it’s the process determining the sun’s evolution rate.

An open field for state of the art/benchmark calculations with immense prospects.

- Needed:
  - 3 nucleon currents in pionless EFT.
  - Consistent calculations in pionfull EFT.
hep process

- The weak fusion $p + ^3\text{He} \rightarrow ^4\text{He} + \nu + e^+$ is the source of the most energetic neutrinos.
- The single nucleon current is highly suppressed.
- The EFT* calculation depends strongly on the EFT cutoff.

$$S_{\text{hep}}(0) = 8.3 \pm 1.3 \text{keV} \cdot \text{b}$$
Summary and outlook

- Electro-weak reactions with light nuclei can be used to:
  - Constrain the Nuclear interaction.
  - Give information regarding the structure of the nucleus.
  - Extract microscopic information about the fundamental theory and its symmetries.
  - Predict, to a percentage level accuracy, reaction rates for astrophysical phenomena.
- Halo helium isotopes (\(^6\)He and \(^8\)He) are a great challenge for ab-initio calculation.
- Many challenges in the astrophysical sector. (pp fusion in a consistent manner as an important benchmark).